One-to-One Math

The University of Kentucky Partnership for Mathematics and Science Education Reform (PIMSER)

Contents

List of the Materials in the Appendix

Hundreds chart	Subtraction Tic-Tac-Toe
Part – Part – Whole Mat	Mat for Adding and Subtracting with Base Ten Blocks
Sample Addition and Subtraction Problems	Recording Sheet for Adding and Subtracting with Base Ten Blocks
Five Frame	
Ten Frame	Sample Multiplication and Division Problems
Double Ten Frame	10x10 Multiplication Array
Sample Journal Page	Doubles Match
Make Five – circle combinations that have a sum of 5	Multiplying by 0, 1, 2 Bingo
Make Ten – circle combinations that have a sum of 10	Clock Face
Bingo – One Less Than and One More Than	Grid Paper Template
Bingo – Two More Than and Two Less Than	Multiplication Chart
Die with 0, 1, 2 and Die with More, Less – to be cut and taped together	Product Game (directions and game board)
Rules for Adding/Subtracting 0, 1, 2 Bingo	Mat for Multiplying with Base Ten Blocks
Adding/Subtracting 0, 1, 2 Bingo	Recording Sheet for Multiplying with Base Ten Blocks
Bingo Adding 0, 1, 2 – version 1	
Bingo Adding 0, 1, 2 – version 2 (problems on the board)	Math Basketball Die – to be cut and taped together
Adding 9 Bingo	Math Basketball Directions
Adding/Subtracting 8, 9 Bingo	Math Baseball Polyhedron – to be cut and taped together
Missing Numbers Template	Math Baseball Directions
Recording template for 10 More Than	Math Baseball Diamond
Recording template for 10 Less Than	Number Capture (3 versions)
Addition Four In a Row (adding doubles and near-doubles)	Math Race (Directions and Game Board)
Addition Chart	Catch Me If You Can (Directions and Game Board)
Addition Tic-Tac-Toe	

Research and Background

What are the "basic facts"?
Most research agrees that basic facts for addition refer to combinations where both addends are less than 10 and basic facts for subtraction refers to the corresponding addition fact; for example, 17 – 9 = 8 is a basic fact for subtraction while 17 – 4 = 13 is not a basic fact. Likewise, basic facts for multiplication have both factors less than 10 and division facts are those that refer to corresponding multiplication facts.

What does it mean to "know" a fact?
The rule of "3" – if a student can consistently give a quick response (in about 3 seconds) to a fact without resorting to an non-efficient method, such as counting, then they have mastered that fact.

Why is knowing the basic facts important?
"Fluency with basic facts allows for ease of computation, especially mental computations, and therefore, aids in the ability to reason numerically in every number-related area." Van de Walle, John A. (2007). Helping Children Master the Basic Facts. *Elementary and Middle School Mathematics, Teaching Developmentally*, 165

Which of our students should we expect to know their basic facts?
"All children are able to master the basic facts – including children with learning disabilities. All children simply need to construct efficient mental tools that will help them." Van de Walle, John A. (2007). Helping Children Master the Basic Facts. *Elementary and Middle School Mathematics, Teaching Developmentally*, 165

I learned my facts by just memorizing them; we drilled until we knew them; what's wrong with that?
"Students who memorize facts or procedures without understanding often are not sure when or how to use what they know, and such learning is often quite fragile. Learning with understanding also makes subsequent learning easier. Mathematics makes more sense and is easier to remember and to apply when students connect new knowledge to existing knowledge in meaningful ways. Well-connected, conceptually grounded ideas are more readily accessed for use in new situations." National Council of Teachers of Mathematics (2000), *Principles and Standards for School Mathematics*, 20

Drill is by far the most popular method for working on basic facts in our school, but, if that worked for all students, we would not have students in the next grade level still struggling with the facts or students in middle school and beyond that do not know their facts.

Drill is appropriate and beneficial for students that have an efficient strategy that they understand, like and know how to use - but have just not become proficient with it.

How can I help students learn the basic facts?

Children learn math the same way they learn everything else - by constructing their own knowledge. Another way of saying this is, even if you teach or ‘feed’ a child math facts, they will have to experiment with, explore and think about numbers before they really learn about and understand numbers and how they work. Research has proven that math facts can, in fact, be learned by young children using "two kinds of activities: situations in daily living … and games.” Kamii, Constance (1985). *Young Children Reinvent Arithmetic*

So, what is One-to-One Math?

One-to-One Math was developed based on the following beliefs:

1. All students can and should learn math.
2. If children like math and feel successful at math, they will learn math.
   a. For children to like math, it must be fun! Students like to play games, so One-to-One Math is a game based program.
   b. For children to feel successful at math they must be successful. One-to-One Math starts students with concepts they can quickly master and then builds on that success.
3. Students need to go through three stages as they learn math: 1. concrete or manipulative, 2. mental representational, 3. abstract or symbolic

One-to-One Math was developed to help teachers, parents, para-professionals, community members, etc. work with students on math facts and critical concepts.

One-to-One Math is a non-profit program – a CD is provided with all of the materials that we use in the training. Please share these activities, materials, and information with others who are involved in helping our children become life-long learners of mathematics!

Suggested Questions/Prompts

To make sense of mathematics:
- How did you get your answer?
- Tell me what you are thinking.
- How would you explain this to a student who doesn't understand?

To foster predicting, inventing, and problem solving:
- What would happen if…?
- Is there a pattern? What is it? Why not?
- What decisions can you make from this pattern?
- Can you do it a different way?
- What is the same or different about your two ways of doing this?
- Will it be the same if we use different numbers? Why or why not?

To rely more on themselves:
- Does it make sense to you? Why or why not?
- What do you think?
- What would seem more reasonable to you? Why?
- How can you check to see for yourself?
- What do you want to do next?
- Can you draw a picture or build a model to illustrate the problem?

To foster reasoning:
- Will what you did always work this way? How do you know?
- Do you see a pattern in this? What is it?
- How could it be done a different way?
- Can you explain your reasoning?
- Could you explain this in another way?
- What other numbers will work?
- Are there some numbers for which it will not work? How do you know?
- Write a new problem that is different in some ways but the same in others.
- Why do you want to change your answer?

To help connect and apply mathematics:
- Have you ever solved a problem like this before?
- Tell (or write) a story problem that uses this kind of mathematics.

Understanding Addition and Subtraction

Researchers have separated addition and subtraction problems into three categories: join problems, separate problems, and part-part-whole problems. These categories are based on the different types of relationships involved. Each category can then be divided into sub-categories depending upon which of the three quantities in the problem is unknown.

In most mathematics curricula, the major emphasis is on the easier "join" and "separate" problems with the result as the unknown part. This leads to the definitions of addition as "put together" and subtract as "take away". These definitions are limited and if these are the only exposure students have, they will have difficulty when the situation calls for something other than "put together" or "take away". Take for example, the following problem: *Bob has 3 nickels and Bill has 7 nickels. How many more nickels does Bill have than Bob?*

Students need exposure to all the different types of addition and subtraction problems.

Examples of Join Problems

<u>Join Problem: the result is unknown</u>

Katie has 8 baseball cards. Mason gave her 4 more. How many baseball cards does Katie have altogether?

<u>Join Problem: the amount of change is unknown</u>

Katie has 8 baseball cards. Mason gave her some more. Now Katie has 12 baseball cards. How many baseball cards did Mason give her?

<u>Join Problem: the initial amount is unknown</u>

Katie has some baseball cards. Mason gave her 4 more. Now Katie has 12 baseball cards. How many baseball cards did Katie have to begin with?

Examples of Separate Problems

<u>Separate Problem: the result is unknown</u>

Katie had 12 baseball cards. She gave 4 baseball cards to Mason. How many baseball cards does Katie have now?

<u>Separate Problem: the amount of change is unknown</u>

Katie had 12 baseball cards. She gave some to Mason. Now she has 8 baseball cards. How many baseball cards did she give to Mason?

Separate Problem: the initial amount is unknown
Katie had some baseball cards. She gave 4 to Mason. Now she has 8 baseball cards left. How many baseball cards did Katie have to begin with?

Examples of Part-Part-Whole Problems
Part-Part-Whole Problem: the whole is unknown
Mason has 4 baseball cards and 8 basketball cards. How many cards does he have?

Mason has 4 baseball cards and Katie has 8 baseball cards. They put their baseball cards together in a notebook. How many baseball cards did they put into the notebook?

Part-Part-Whole Problem: one of the parts is unknown
Mason has 12 cards. Eight of his cards are baseball cards, and the rest are basketball cards. How many basketball cards does Mason have?

Mason and Katie put 12 baseball cards into a notebook. Mason put in 4 baseball cards. How many baseball cards did Katie put in?

Examples of Compare Problems
Compare Problem: the difference is unknown
Mason has 12 baseball cards and Katie has 8 baseball cards. How many more baseball cards does Mason have than Katie?

Mason has 12 baseball cards and Katie has 8 baseball cards. How many fewer baseball cards does Katie have than Mason?

Compare Problem: the larger amount is unknown
Mason has 4 more baseball cards than Katie. Katie has 8 baseball cards. How many baseball cards does Mason have?

Katie has 4 fewer baseball cards than Mason. Katie has 8 baseball cards. How many baseball cards does Mason have?

Compare Problem: the smaller amount is unknown
Mason has 4 more baseball cards than Katie. Mason has 12 baseball cards. How many baseball cards does Katie have?

Katie has 4 fewer baseball cards than Mason. Mason has 12 baseball cards. How many baseball cards does Katie have?

Using 5 and 10 as Benchmark Numbers

5 and 10 are powerful numbers that can be used as anchors to "build" other numbers. We want children to be able to be able to recognize the combinations that make these numbers.

Start the process with a simple game called *I Wish*

> *I have 3 cars in my parking lot; I wish I had 5 cars. How many more cars do I need?* Be sure to ask them how they know they are correct. You may want to model your thinking by using the five-frame to build and draw the problem.

*	*	*		

5-frame

This is a very good formative assessment of your student. Watch to see how they find the missing added – do they know the amount or do they have to unit-count to find the missing number.

A second game to play with the student is *Make Five Go Fish*. This game uses the (0-10) number cards. Pull out all of the cards above 5and the wild cards from the deck before you begin play. On the CD there is a deck of cards that have 5-frames on them instead of 10-frames if you would like to use them instead.

Make Five "Go Fish"

Materials: number cards 0, 1, 2, 3, 4, 5 (four of each) from the number card deck
Objective: make sets of 2 cards with a sum of 5.

1. Each player is dealt five cards. The rest of the cards are placed down in the center of the table.
2. If you have any pairs of cards that total 5, put them down in front of you and replace those cards with cards from the deck.
3. Take turns. On your turn, ask the other player for a card that will go with a card in your hand to make 5.
4. If you get a card that makes 5, put the pair of cards down. Your turn is over. If you do not get a card that makes 5, take the top card from the deck. Your turn is over.

If the card you take from the deck makes 5 with a card in your hand, put the pair down. Your turn is over.

5. If there are no cards left in your hand but still cards in the deck, you take two cards from the deck.
6. The game is over when there are no more cards.
7. At the end of the game make a list of the number pairs you made.

When you feel that the student is comfortable with all of the combinations for 5, move on to the combinations for 10. Again, start with the 10-frame and the *I Wish* game: *I have 3 necklaces in my jewelry box; I wish I had 10*.

*	*	*		

Change the rules to where they are now looking for combinations that make 10 and play *Go Fish*. These cards are designed so that the student can count the missing part for 10. You may want to model out loud how to use them: *I have a 7; what I should ask you for; oh, I'm missing 3. Do you have a 3?*

When you notice that they are getting a fairly good grasp on the combinations that make 10, you may want to try the 10-beads with them. Notice that the beads are designed with 5 of each color – this is to help them "quick count" numbers such as 8 by seeing that 8 is 5 and 3 more. Hide the beads behind your back, move some of them into the palm of your hand, show the student the rest of the beads and ask them, *"How many are hiding in my hand?"*

For variety, you can use the 10-frame cards or the 10-beads and one of the board games. Take turns; turn over one of the cards; the missing addend is the number of spaces that the player gets to move.

Combinations for 10 are crucial for other strategies that we will use with the rest of the addition and subtraction facts. Keeping that in mind, we have tried to give you a variety of games to play with your students to practice that concept. Two additional games to play with your students are *Snappo* and *Rummy*.

SNAPPO

Materials: 0-10 number cards

Objective: to recognize pairs of addends with a sum of 10 and to "capture" those cards.

1. Deal out all the number cards (0-10) face down into 2 stacks.
2. Player #1 lays the top card from his/her stack <u>face up</u> on the table.
3. Player #2 lays the top card down from his/her stack face up on the table. If that card makes the sum of 10 with the other card that is already down

on the table, he/she should place it face up beside the other card and call out SNAPPO. He/she has captured the two cards and should place them in his/her collection of captured cards. If the card does not make a SNAPPO, it is still placed face up in the center of the table.

4. As play continues, the new card that is turned over can be matched with any card that is already on the table that makes a sum of 10.
5. Any person recognizing a match may call SNAPPO and capture the cards.
6. The game continues until there are no matching cards remaining.

"Make 10" RUMMY

Materials: 0-10 number cards
Players: 2 or more
Objective: Make sets of two cards that add up to 10.

1. Deal out 7 cards to each player.
2. Turn over the next card and start a discard pile - lay it down on the table so everyone can see the number.
3. Place the remaining deck of cards face down in the center of the table.
4. Check to see if you have any sets – a set is 2 cards that have a sum of 10. If you have a set, you may play it (lay it down) when it is your turn.
5. Each player in turn can either draw a card from the remaining deck or pick up the top card from the discard pile. If you wish to pick up more than the top card from the discard pile, you must be able to make a set with the last card in the stack that you picked up.
6. The game ends when a player "goes out" (has no more cards) or when there are no more cards left in the deck. The winner is the player with the most cards in his or her "captured" pile.
7. Calculate you score by adding 5 points per card for any sets that you have laid down and subtracting 5 points per card for any cards that remain in your hand when the game is over.

Don't feel like you have to play all of these games an equal amount of time. We have found that some students will like a certain game and ask to play it over and over – that's fine! As long as they are having fun, feeling successful, and learning math, we are accomplishing our goal!

Feel free to change the rules of any of these games to make them better suited for your students!

As you do the above games, don't forget to model your thinking, have the students build and draw the problems, and always ask the student to say the addition problem two ways:

Example: *2 + 3 = 5* and *3 + 2 + 5* or *3 and 2 make 5* and *2 and 3 make 5*

Make Five and *Make 10* are games where the student circles pairs of addends that have a sum of 5 or 10. On the CD, there are two additional versions of each game. You may want to time them and see if they can improve on their time each week.

Make Five

1	4	2	3	0
5	3	2	4	5
0	4	1	1	1
3	2	5	3	4
4	1	0	2	0
2	3	1	4	5

Make Ten

1	9	2	3	6	4
5	4	8	7	9	7
5	6	0	6	1	3
3	7	10	4	5	5
2	2	8	0	1	9
8	3	7	0	6	4

It will probably take longer that you had thought (or planned) for the student to become proficient at the combinations for 5 and 10 and you might be tempted to move on before they have mastered these concepts. PLEASE DON'T! Combinations for 5 and 10 are critical concepts. We will build other strategies and concepts around these as we work through the rest of the addition and subtraction facts.

<u>Make It Fun!!</u>

Addition and Subtraction Facts

Mastery of a basic fact means that a child can give a quick response (in about 3 seconds) without having to resort to an inefficient method such as counting.

An efficient strategy is one that can be done mentally and quickly. Our goal is to help the student develop an efficient strategy and then provide practice of that strategy through games.

Adding and Subtracting 0, 1, and 2

Adding and subtracting zero does not require any strategy; just a good understanding of the meaning of zero and addition/subtraction. Even though it does not require a strategy for adding and subtracting zero – don't neglect it! Be sure to give them several story problems to model for adding and subtracting zero.

For adding one and two, focus on the strategy of "more than" rather than "counting on"; for example: 7 is 1 more than 6; 2 more than 7 is 9. If a student uses the "counting on" strategy for these two addends, don't try to stop that use, but do discourage that strategy for larger addends. Likewise, for subtracting one and two, focus on "less than" rather than "take away".

Start with working on the relationship of numbers that are 1 more or less than a given number before you do 2 more or less than a number. The number line is a great tool to help students see this relationship of numbers. You can cover all of the numbers on the number line except for one number and ask the student, "*What is 1 more than 8"?* or *"What is 1 less than 8?* and "*What is 2 more than 8"?* or *"What is 2 less than 8?"*

Some students will have to unit-count from 1 to 9 to answer the question about one more than 8 and won't be able to quickly tell you what number is one less than 8 – don't panic, that just lets you know where the student is developmentally – with practice, they will develop that understanding.

After using the number line for these questions you can try different ways to practice this concept – turn over one of the 10-frame cards and ask the same type questions, or let the student turn over a number card and then spin the less than/more than spinner or roll the less than/more than die to generate the problem. There is also bingo games to practice one/two more than and one/two less than.

Adding and Subtracting 10

Adding and subtracting 10 to any number without having to unit count is an extremely important concept – not only in learning the basic facts but later when we work with different strategies to add and subtract 2-digit numbers.

To work on adding ten, have the student use the double 10-frame and build problems such as 10 + 3 and 4 + 10. Ask questions such as, "What's ten more than 3?" We want the student to see the pattern of what happens when we add 10 – please don't tell them the pattern, just do enough problems and keep asking them if they see a pattern. There is a recording chart that should be helpful for the student to see the pattern for adding ten.

Number	Ten More Than The Number

The 0-10 number cards, 0-9 die, 0-9 spinner, 0-10 spinner can all be used to help generate problems. Turn over one of the cards, spin the spinner, or roll the die and ask the student, "What is 10 more than ??".

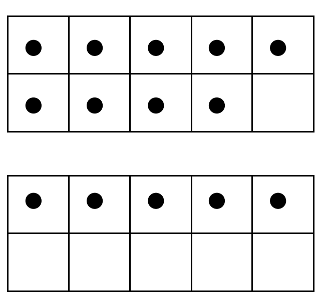
To work on subtracting 10, ask the student to build a number such as 17 on the double 10-frame and then ask them to subtract 10. There is a recording chart for ten less than a number and number cards from 11 through 20 made with double 10-frames on the cards to help with the visualization of subtracting 10.

A hundreds chart is a great tool to use to look for patterns.

You can use *Math Basketball*, *Math Baseball*, *Math Race*, or *Catch Me If You Can* as a game to practice these concepts.

Adding 9

Ask the student to build 9 + 5 on the double 10-frame.

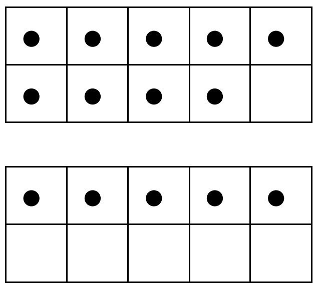


Now challenge them to find a quick way to determine how many counters are on the board. If you see them unit-counting, tell them that will work but that you are looking for a quicker way to determine how many counters there are.

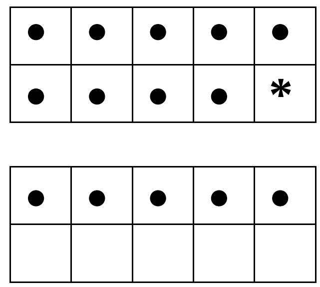
There are 2 very efficient strategies for adding 9:

1. One strategy is to pretend that the 10-frame with 9 in it is full and has 10; find that number; then subtract 1 for the one we pretended was there.
   For example:

   9 + 5

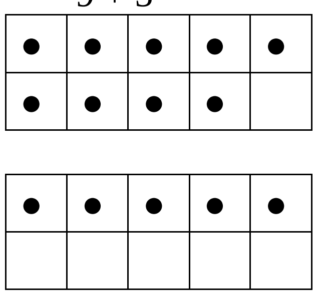
   

   becomes 10 + 5 – 1

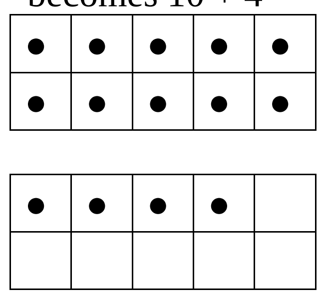
   

2. Another strategy is to take one of the counters from the smaller number and fill up the other 10-frame
   For example:

   9 + 5

   becomes 10 + 4

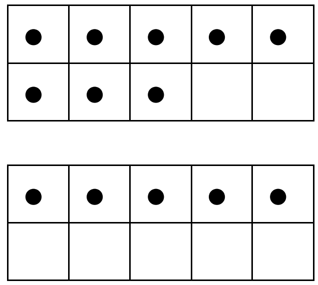
   

You will probably need to build several of these before either strategy becomes a habit. Make sure you build the commutative facts – build 5 + 9 as well as 9 + 5.

Pull the cards that have 9 as one of the addends from the *Adding 8 and 9 Cards* and use them to be the problems while you play *Math Basketball*, *Math Baseball*, *Math Race*, or *Catch Me If You Can*. For a little more abstraction, use the 0-10 number cards, 0-10 spinner, 0-9 spinner, or 0-9 die to generate a number and then ask the student to add 9 to this number. Model building the problems on the double 10-frames.

Adding 8

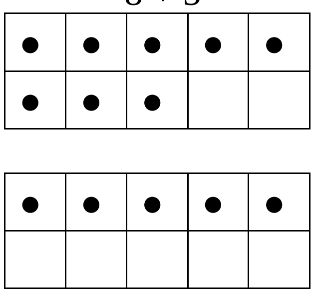
The strategies for adding 8, like adding 9, are built around being able to add 10. Start with asking the student to build a problem such as 8 + 5 using the double 10-frames and to find a quick way to determine how many counters are on the board.



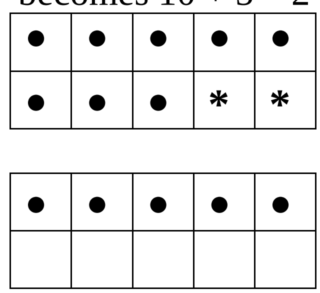
1. One strategy is to pretend that the 10-frame with 8 in it is full and has 10; find that number; then subtract 2 for the two we pretended was there.

For example:

8 + 5

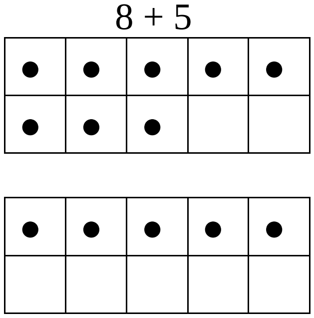


becomes 10 + 5 – 2

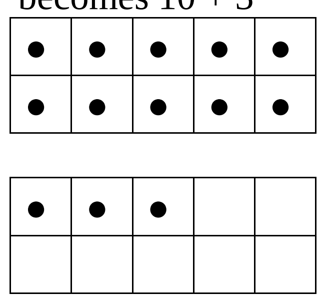


2. A second strategy for adding 8 is to use 2 of the counters from the 5 to finish filling up one of the 10-frames:

8 + 5



becomes 10 + 3



Again, you will probably need to build several of these before either strategy becomes a habit. Make sure you build the commutative facts – build 5 + 8 as well as 8 + 5.

Pull the cards that have 8 as one of the addends from the *Adding 8 and 9 Cards* and use them to be the problems while you play *Math Basketball*, *Math Baseball*, *Math Race*, or *Catch Me If You Can*. You can also use the 0-10 number cards, 0-10 spinner, 0-9 spinner, or 0-9 die to generate a number and then ask the student to add 8 to this number. Model building the problems on the double 10-frames.

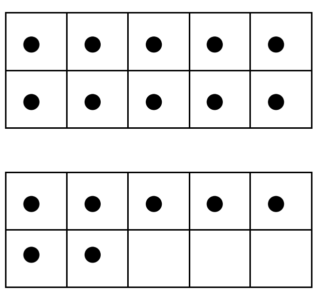
Subtracting 9 and 8

Subtracting 9 and 8 is also built around the number 10.

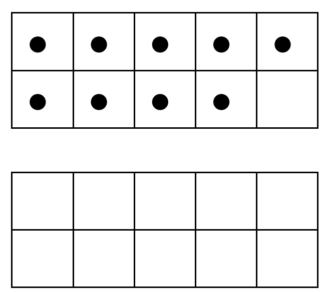
For example; let's look at the problem 17 – 9. There are three quick and efficient strategies for doing this:

1) start with 17 on the double-10 frame; remove 10 and put 1 back.
2) start with 17 on the double-10 frame; remove the 7 (to get down to 10) and then remove 2 more.
3) start with 9 on the double 10-frame and think addition – if I have 9, how many more will I need to have 17?

How can I quickly remove 9?



How many more do I need to make 17?



Similar strategies for working 17 – 8 are to:

1) start with 17 on the double-10 frame; remove 10 and put back 2.
2) start with 17 on the double-10 frame; remove 7 (to get down to 10) and then remove 1 more.
3) start with 8 on the double 10-frame and think addition – if I have 8, how many more will I need to have 17?

Use the 11-18 cards from the 11-20 set of cards to generate the first number and ask the student to subtract 8 and/or 9.

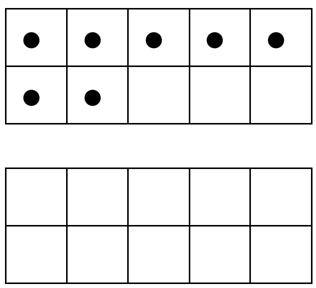
There is an 8, 9, 10 spinner and an +, – spinner that can be used along with the 11-20 number cards to practice adding and subtracting 8, 9, and 10 in a mixed sitting.

Using Ten as a Bridge

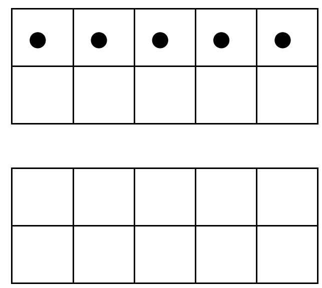
The "hard" addition and subtraction facts are those facts that have a sum greater than 10. We are going to use 10 as our "bridge" to help with these facts – in fact we did this with 8 and 9 in the second and third strategy listed.

For addition of these facts, we will "count up" but not by unit-counting but by "chunking" our counting around 10. Let's look at 7 + 5

If I start with 7, how many does to take to fill up the 10-frame? How many will go in the second frame?

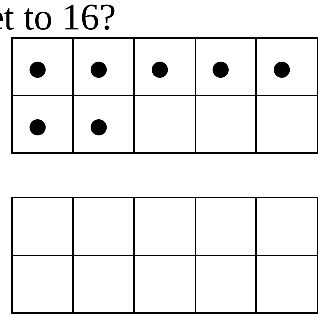


If I start with 5, how many does to take to fill up the 10-frame? How many will go in the second frame?

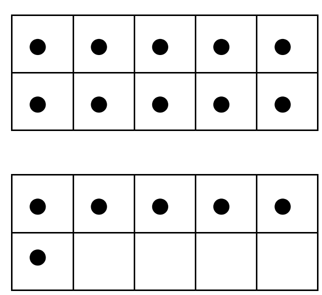


For subtraction, we will "count up" or "count down" but not by unit-counting but by "chunking" our counting around 10. Let's look at 16 – 7

If I start with 7, how many does to take to fill up the 10-frame? How many will go in the second frame to get to 16?



If I start with 16, how many does to take to get back to 10? How many more to get to 7?



You will need to have the students build a lot of these problems with the double 10-frame before you ever ask them to do this mentally. Understanding and being able to use the strategy of using 10 as a bridge will take a lot of time but is well worth the time spent!

Building around ten is POWERFUL! If we look at the facts with a sum of 10 and then the facts that can use 10 as a bridge, it is over half of all of the addition and subtraction facts!

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

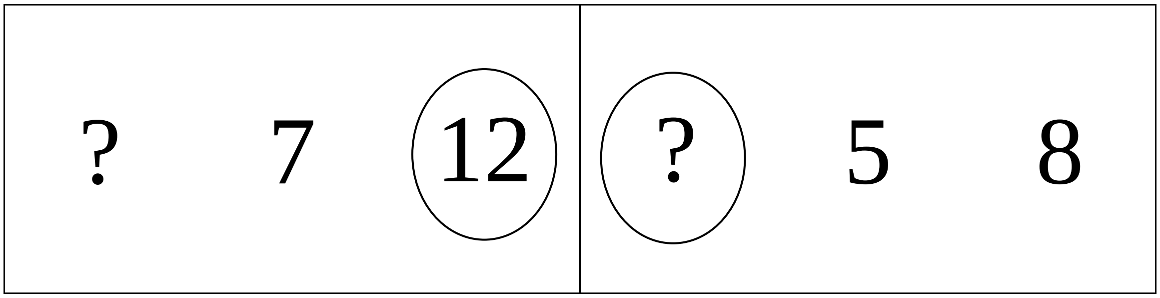
There is a set of Missing Number Cards in the materials – all of these cards are designed to practice the strategy of using 10 as a bridge.

Show the student, without any explanation, families of numbers with the sum circled as in the cards below.

4 9 (13)	(11) 3 8

Ask them why they think the numbers go together and why one number is circled.

When this number family idea is understood, show them some families with one of the numbers replaced with a question mark and ask them what number is missing.



When the student understands this activity, tell them you have some missing number cards based on this idea. Each card has two of the three numbers that go together in the same way. Sometimes the circled number (the sum) is missing and sometimes one of the other numbers (a part) is missing. The object is to name the missing number.

The cards can be cut out and used as problems for the *Math Race, Catch Me If You Can, Math Basketball, and/or Math Baseball* games. Blank cards are also in the notebook so you can make other problem sets.

Doubles and Near Doubles

It is well documented that students seem to know the doubles facts (both addends alike) better than most other combinations. Maybe it is because of the sing-song rhythm when they say the problem – I don't know, but children like the doubles!

Start by working on the doubles before you do the near doubles. Pull just the doubles from the *Doubles and Near Doubles Cards* and have the student use the dry-erase marker to draw an example of a double problem on the cards before you use them in a game. This helps them to "see" the problem and solution.

When you get ready to use the near doubles cards, sort them into groups – for example 3 + 4, 4 + 3, and 3 + 3 (or 4 + 4). Ask the student how the cards are alike and how are they different. Let them draw the same type of design on the near double cards as they did on the double card. These cards can then be used to generate the problems for one of the games in the back of the notebook.
There is also a game called *Four in a Row* to use for practice of doubles and near doubles addition facts.

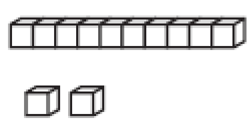
Using Base Ten Blocks

Comparing the Blocks

1. Ask the student if they have ever worked with base ten blocks in their classroom.
2. Ask them to identify the value of each block.
3. Ask them questions such as:
    a. "How many of these 'ones blocks' does it take to make one of these 'tens blocks'?"
    b. "If I wanted to trade you a 'tens block' for some 'ones', how many should you give me for it to be fair?"
    c. "How many of these 'tens blocks' does it take to make one of these 'hundreds block'?"
    d. "If I wanted to trade you a 'hundreds block' for some 'tens', how many should you give me for it to be fair?"
    e. "How many of these 'ones blocks' does it take to make one of these 'hundreds block'?"
    f. "If I wanted to trade you a 'hundreds block' for some 'ones', how many should you give me for it to be fair?"
4. Make sure they can trade both directions – from a smaller unit to a larger unit and from a larger unit to a smaller unit.
5. If they struggle with these questions, ask them to build a model by placing the different blocks beside each other and compare them.

What's My Number?

1. Tell the student you are going to play a game called "What's my mystery number?"
2. Use the base ten blocks and lay out a number such as the one below

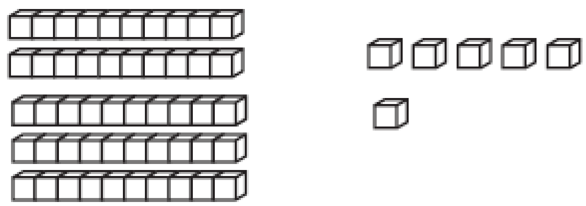


3. Ask the student "What's my mystery number?" Watch how they count to get their answer – do they count "10, 11, 12" or do they count "1, 2, 3, …, 12"? If they count by ones, ask them if they could count it a different way. Share how you would count by starting with the 10.

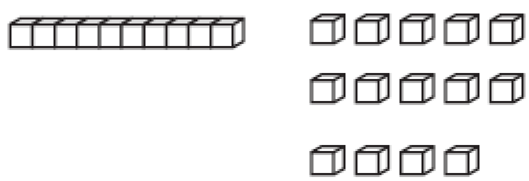
4. Lay out other numbers such as the examples below:

a. 

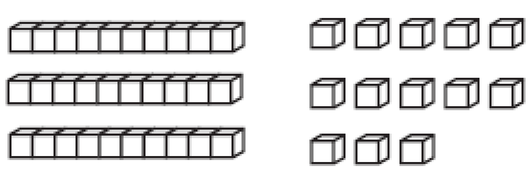
b. 

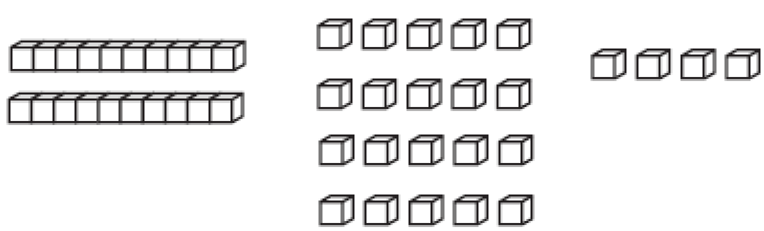
c. 

5. Work with problems like the ones above until you are confident the student can count the blocks efficiently (starting with 10's).
6. When they are comfortable with these type numbers, tell them that it's time for a challenge! Lay out the following:

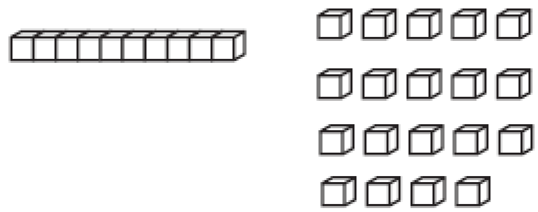


7. When they have figured out that it is 24, ask them to show you 24 a different way (see if they can show you 2 tens and 4 ones). Tell them that both ways are correct ways to show 24 but one way is just faster.
8. Give them some examples like the ones below – always ask them to show you the number a different way.

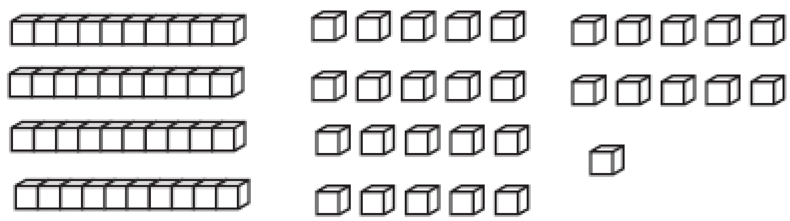
a. 

b. 

c.



d.



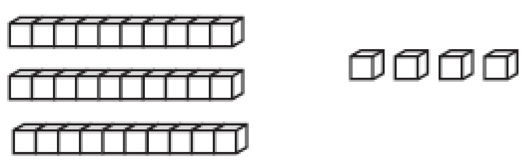
9. There is a set of 24 cards that you can use to play one of the games in the back of the notebook asking kids to identify the number – maybe tell them they get an extra turn if they can build the number a different way.

Being able to efficiently count the blocks is very important for adding and subtracting two-digit numbers.

Putting Numbers Together and Taking Numbers Apart

This concept of making numbers with many representations is absolutely essential for adding and subtracting with regrouping!

1. Ask the student to show you 34 with the base ten blocks. Most will show you like the one below:



2. Ask the student "Can you make this number a different way?" Don't be surprised if they don't think that they can do it any different. Let them think about it before you offer any hint such as, "I wonder if I could do it with 2 tens?"

   Ask them again to find a different way to show 34. When they have a third way, ask them for another way. Continue until they have discovered all four ways. I would also ask again for a different way to see if they realize that they have found all possible ways.

Record the different ways as the student discover them.

34	
3 tens	4 ones
2 tens	14 ones
1 ten	24 ones
0 tens	34 ones

I record them like the example above so we can identify a pattern later. Keep this recording so you can ask about a pattern after you have a few examples! I don't say anything about a pattern for the first few problems.

Emphasize how ALL of these are correct representations for 34!

3. Give them a few other numbers to show as many ways as possible:

42	28	57
4 tens 2 ones	2 tens 8 ones	5 tens 7 ones
3 tens 12 ones	1 ten 18 ones	4 tens 17 ones
2 tens 22 ones	0 tens 28 ones	3 tens 27 ones
1 ten 32 ones		2 tens 37 ones
0 tens 42 ones		1 ten 47 ones
		0 tens 57 ones

4. Ask the student to identify any pattern they see. Ask questions until they can explain how a ten can be exchanged (traded) for 10 ones and how 10 ones can be exchanged for a ten.
5. Again - emphasize that all of these are correct answers. Ask them to tell you which way is the quickest way to show the numbers. (I start calling this the "quick way" to show a number.)
6. Give them more of these type problems until they are VERY comfortable exchanging the ones and tens.

Adding and Subtracting Two (or more) Digit Numbers

We often lie (*but we do mean well*) to our students and tell them that they must start on the right side of the problem to add and/or subtract 2-digit numbers!

Let's take a minute to explore: work 56 + 38 without using the standard algorithm of regrouping.

Other Methods:

Now let's try subtraction: work 62 – 37 without using the standard algorithm of regrouping.

Other Methods:

If we want to move our students towards the traditional method of adding and subtraction, the base ten blocks are an excellent manipulative to use. Please avoid telling them how to do the problem with the blocks – such as "You need to trade…." – rather let them explore and find ways to do the problem.

Tell the students a story such as, "I have 32 baseball cards – can you show me the number 32 with the base ten blocks by using the "quick way" (3 tens and 2 ones)? Billy has 16 baseball cards – show me 16 the "quick way". If we put all of our cards together, how many will we have?"

Encourage them to tell you how many tens and how many ones they have. Ask them to make this number a different way – ask them what is the "quick way" to show this number?

Give them a problem that would have to be "re-grouped" but go through the same procedure – how many tens; how many ones; what's my number; show it to me a different way; what's the quick way to show this number?

When you think it is time to start recording the problem, there is a recording sheet made for this – encourage the students to just draw a segment for 10 and a dot or a star for 1 – if they draw the ten as a bar with 10 markings, they may have to "unit-count" to get the number.

Some possible ways to model the symbolic recording of 27 + 16 are

```
  27   2 tens   7 ones                              27    this method
+ 16   1 ten    6 ones          OR                + 16    is often
       3 tens 13 ones  or 4 tens 3 ones             30    referred to as
                                                  + 13    the partial-
                                                    43    sum method
```

For subtraction, pose a problem such as 23 – 8 and let them explore how to work it with the blocks. Don't be surprised when a student works it by "taking away" a ten and "putting back" 2.

A possible way to model the symbolic recording of 34 – 17 is to record the trades:

```
 34   3 tens  4 ones        2 tens 14 ones
-17   1 ten   7 ones        1 ten   7 ones
```

Multiplication and Division

Understanding Multiplication

We CANNOT expect students to memorize multiplication facts if they do not understand what multiplication really is! Time spent on developing the understanding of the concept of multiplication is time well spent.

Our plan is to look at multiplication 4 different ways: equal sets, repeated addition, arrays, and rectangular area.

Containers and Cubes

Have a discussion with your student about number sentences (equations). Discuss that a number sentence (equation) will have an equal sign with something on each side. Practice writing number sentences such as:

> *Bill has 3 apples in one basket and 2 apples in another basket. How many apples does he have altogether?*
>
> Discuss the two addition equations that can be made: 3 + 2 = 5 or 2 + 3 = 5

Give the student some small cups and unit cubes. Pose a situation such as *"Sally has 3 baskets and in each basket she has 2 apples. How many apples does Sally have altogether?"* Tell the student that you would like for them to use the cups and cubes to model this problem. (Watch to see how they determine the number of cubes – counting by ones, skip counting, etc.)

"Can you make me up a number sentence (problem) that tells the story of what we did?" Don't be surprised when the student does not write either a repeated addition and/or a multiplication problem but writes something such as 5 + 1 = 6. Don't tell them the problem but do ask them to explain how the numbers in their problem tell the story. If they do write a repeated addition sentence, emphasize how well that sentence fits the story and then show them a multiplication problem for the same story – do NOT emphasize that the number of groups must come first – we want them to realize that 2 x 3 and 3 x 2 are the same problem! If they do not write a repeated addition problem, work with them using smaller numbers such as two baskets until they are comfortable writing repeated addition problems and then show them a multiplication problem.

Repeat with other examples until the student is comfortable writing both a repeated addition problem and a multiplication problem.

Use a number cube (1-6) and a self-made die that has 2, 3, and 4 dots (two each) on the sides. Have them to roll the number cube – that tells them how many cups they need - and the dot die – that tells them how many unit cubes to put in each cup. On his/her white boards, have the student write as many number sentences as they can that tells the story (example: 4 cups with 3 cubes in each cup could be written as 3 + 3 + 3 + 3 = 12, 3 x 4 = 12, and 4 x 3 = 12). Ask the student to make up and tell you a story about their problem.

After the student is comfortable using the cups and cubes for multiplication problems, have them use circles and stars to draw multiplication problems.

Array Model of Multiplication

Multiplication can also be represented as an array: 4 x 3 can be shown as 4 rows of 3 items or 3 rows of 4 items. Some books will always have the first factor as the number of rows but it doesn't matter because we want the student to recognize that the arrays are basically the same – just rotations of each other!

Use the base ten ones or the two-color squares to show the array model of multiplication.

Pose problems such as the following and ask the student to build a model using an array. Have them write as many number sentences as they can for each problem.

Mary plants 5 rows of trees. She puts 4 trees in each row. How many trees did she plant?

Jack picked 6 apples. Jill picked 4 times as many apples as Jack. How many apples did Jill Pick?

Bill walked for 4 hours at 3 miles per hour. How far did he walk?

There is a 10x10 array of circles (a large one and small laminated ones) in the training material. Have the students build and record a model for several problems similar to the ones above.

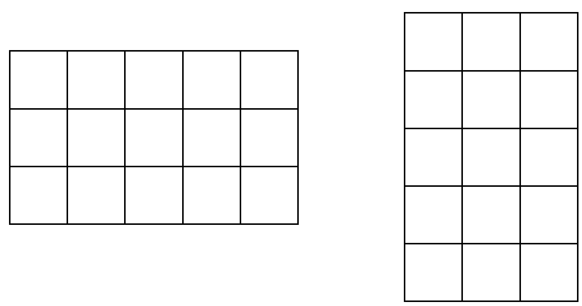
Also in the training material, there is a set of cards for the array model of each multiplication fact. Each array is built on a 10x10 grid to help the students visualize the magnitude of each product – they need to be able to mentally "see" that 6 x 8 is larger than 2 x 3. One activity that can be done with these cards is to turn over one of the cards and ask the student to give as many number sentences as they can for this card (repeated addition, row x column, and column x row). They can use their dry erase marker to write the multiplication problem on the front or the back of the card.

These cards can be separated into smaller subsets and be used later when you are working with the student on certain multiplication facts such as the doubles, five, etc. They can be used as flash cards by having the student write the multiplication problem(s) on the back and then turn it over if they need help finding the product.

Rectangular Area Model of Multiplication

A version of the array model is the rectangular area model for multiplication. Students need to build and draw rectangular models for multiplication problems. Three-quarter inch two-colored tiles are included in the materials for building these rectangles. There is also grid paper for the student to record the models of the problems.

Both the area model and the array model of multiplication are great ways to illustrate the commutative (order) property of multiplication. While 5 sets of 3 objects and 3 sets of 5 objects cannot immediately be seen to have the same results, a 5 by 3 rectangle and a 3 by 5 rectangle powerfully illustrates this property.



Activity 1: Give the student 12 square tiles. Ask them to build (and record on grid paper) a rectangle from the tiles. Discuss the two multiplication problems that can be made. Be sure to discuss the dimensions of the rectangle and how these dimensions are the factors for the multiplication problems. Ask them to make as many different rectangles from the tiles as they can.

Repeat for 18, 24, 30, and 36 tiles.

Activity 2: Repeat the activity listed above but for a set of numbers such as 1-15, or 10-25. Group together all of the arrays for the same number of squares – this helps the student to compare. Look for patterns such as: Which numbers have the least number of arrays? Which numbers have a factor of 2? Which numbers have an array that forms a square? What can you say about the factors of all the even numbers? Do even numbers always have two even factors? What about odd numbers?

Multiplication Match

The multiplication match cards show five different representations for a multiplication problem: repeated addition, equal sets, circles and stars, an array of dots, and a rectangular area model.

One possible way to use the cards is:

- Lay out the 8 cards that have the multiplication problems so that each multiplication problem can be seen.
- Shuffle the remaining cards and turn them face down in a pile between the players.
- Take turns turning over a card and matching it with the multiplication problem that it represents. Ask the students to describe the problem and to give the product. You may want to ask for another equivalent multiplication problem (3x5 is equivalent to 5x3 but the representation may look different – for example the repeated addition: 5+5+5 instead of 3+3+3+3+3).

A second way is to give the students all of the cards except the ones with multiplication problems on them; ask them to sort the remaining cards into sets that belong together. They may sort them into sets of all the repeated addition, all the circles and stars, etc. That's fine – just ask them to find another way of sorting them. When they have them sorted, ask if they can write a multiplication problem that would fit or give them the multiplication problems and let them match those cards with their sorted sets.

Understanding Division

In a lot of traditional mathematics programs, multiplication and division are taught separate with multiplication taught first. If this tradition is followed, a lot of children fail to see the connection between the two operations. It is important to combine the two operations as soon as possible to help students see this connection.

Division can be thought of 4 different ways. *A* divided by *B* can mean:

How many times can *B* be subtracted from *A*?
A divided into *B* equal groups.
A divided into equal groups of size *B*.

What number times *B* gives the product of *A*?

Look at the following problems:

A rope is 24 feet long. How many 6 feet long jump ropes can be made?

Sally has 24 pieces of bubble gum. She wants to share them equally among herself and five friends. How many pieces of gum will each person get?

Bill has 24 apples. He wants to put them into bags containing 6 apples each. How many bags will he need?

Mark has 24 baseball cards. He has 6 times as many cards as Sally. How many cards does Sally have?

Each of these problems can be represented by the problem 24 ÷ 6 but the wording of each problem demonstrates a different type of division problem. Students need practice with all of these types of problems.

Then we have the different symbolism for division: 24 ÷ 6, $\frac{24}{6}$, and $6\overline{)24}$. The last form would probably not exist if it were not for the standard paper-and-pencil procedure that utilizes it! Students have trouble reading that problem because it must be read right-to-left rather than our normal left-to-right reading procedure.

More often than not, division will not produce a whole number quotient – for example: problems that use 5 for the divisor will "come out even" only one time out of five. So now we have to deal with remainders in our division problems. A remainder can be dealt with several ways: it can remain a quantity left over, be partitioned into fractions, discarded leaving a smaller whole number answer, force the answer to the next larger whole number, or be rounded to the nearest whole number for an approximate result.

Think about the remainder in each of these problems:

Mason has 11 cookies and wants to share them with his sister Katie. How many cookies should each person get?

A rope is 25 feet long. How many 6 feet jump ropes can be made?

A van will hold 7 passengers. How many vans will be needed to take 22 people to town?

5 students are sharing 32 pieces of bubble gum. About how many pieces will each student get?

Students should not just think about remainders as in the standard notation of $25 \div 4 = 6$ R1. By providing students with real world problems, it gives us the opportunity for remainders to be put into context and dealt with accordingly.

Introducing Division

Provide the student with unit cubes or other counters and several soufflé cups. Have the student count out a number of counters, such as 12, to be the whole or total set. Next tell them a story that requires the 12 tiles to be divided into 3 equal-sized groups or equal sets of 3. Have them write the multiplication sentence(s) for their problem and then write the division problem.

Repeat for different numbers of counters as the whole. Start with quantities that are multiples of the divisors but be sure to soon include situations that do not "come out even."

Vary the activity by changing the model. Have the student use one-inch tiles to build rectangles. Tell them how many tiles should be in the rectangle and one of the dimensions (row or column) of the rectangle. Have them to record their rectangles on grid paper, give the multiplication problem, and the corresponding division problem.

Using the Rectangular Area Cards

Label each rectangular area card with the multiplication problem on the grid side. Write the product and one of the dimensions of the grid on the blank side.

Game 1:

1. Spread out all of the cards – some should show the multiplication problem and some should show the product.

2. Choose a card by placing a finger on it. If the multiplication problem is showing, you must give the product. If the product is showing, you must give the missing factor.
3. Turn the card over to check the answer. If the answer is correct, you earn the card.
4. The player with the most cards wins.

Game 2:

1. Deal out all the cards so that each player has the same number of cards – discard any leftovers.
2. Each player needs to stack his/her cards with the total side down in front of them.
3. At the same time, each player places his/her top card in the center of the table – with the multiplication problem showing.
4. Together the players decide which array has a larger total. The player with the larger array wins both cards.

Multiplication and Division Facts

Doubles

The multiplication double facts are equivalent to the addition doubles. The first activity that you might want the students to do is to sort the *Multiplication Doubles Cards*. Use these cards to play some of the games in the notebook – if an addition fact is turned over, you could ask the student to give the corresponding multiplication fact and vice versa. There is also a *Doubles Match* page in the notebook where the student matches up the corresponding addition and multiplication problem.

Multiplying by Zero and One

Forty of the 121 multiplication facts have zero or one for at least one factor. While these facts within themselves are "easy", they are sometimes mixed up with the addition "rules" for adding zero and one. The fact that 5 + 0 stays the same but 5 x 0 is always 0 and the fact that 1 + 5 is a one-more than idea and 1 x 5 stays the same can be confusing for some students. The concepts behind these facts are best developed through using story problems. Please, avoid using the "rules" – your saying to the students "Any number multiplied by 1 stays the same" and/or "Any number multiplied by 0 is always 0" is meaningless. If they develop and verbalize the rules, they will come closer to remembering them.

Multiplying by Five

Most students can quickly skip-count by fives. Help them to connect this skill with multiplying by five by asking them to build several problems that have 5 as one (or both) of the factors – you can use containers and cubes, the area model, the array model, etc. Ask them then to tell you how many counters there are and how can they "quick count" them.

The clock is another tool to use for the five facts. When the minute hand points to a number, it tells how many minutes after the hour it is. There is a large clock in the notebook and a page of smaller clocks on the CD.

Multiplying by Ten

Skip counting by tens will give a student the tens facts if they see the relationship between skip counting and multiplication. Multiplying by 10 is a powerful concept – we want every student to be able to quickly multiply any number by ten. <u>PLEASE</u> don't use the rule – *to multiply by ten just add a zero to the end of the number* – this is a band-aid fix for whole numbers that leads to misconceptions – it will show up later when students are asked to multiply number such as 12.34 x 10.

Multiplying by Nine

Have the students build and write the nines multiplication table.

9 x 0 = 0
9 x 1 = 9
9 x 2 = 18
9 x 3 = 27
9 x 4 = 36
9 x 5 = 45
9 x 6 = 54
9 x 7 = 63
9 x 8 = 72
9 x 9 = 81

Ask them to look especially at the problems where the second factor is 2 – 9 for patterns. "*Is there a pattern in the tens place? How is the tens place number related to the other factor?"* When they see that patterns ask them to identify what the tens place number will be for some random problems: "*What are the tens place number for 6 x 9? How about 9 x 3?"* When they are comfortable with this, ask them to look for a relationship between the tens place digit and the ones place digit. This method of multiplying by nine helps a lot of students learn the nines facts.

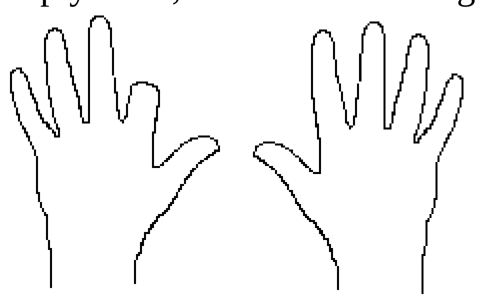
A second strategy for multiplying by nine involves thinking of 9 as (10 – 1) and applying the distribute property. For example: 7 x 9 can be thought of as 7 x 10 – 7 x 1 or 7 x 10 – 7 and 6 x 9 can be thought of as 6 x 10 – 6 x 1 or 6 x 10 – 6. This strategy is very helpful later on when we want students to work on some of the other multiplication facts. It can also be used to multiply larger numbers that involves repeated nines: 6 x 99 is 6 x 100 – 6 x 1 and 7 x 999 is 7 x 1000 – 7 x 1.

A third strategy for multiplying by nines is using "finger math".

Place both hands palm down in front of you.

Starting with your left pinkie, number your fingers (and thumb) 1, 2, 3 … 10.

If you want to multiply 9 x 4, turn the fourth finger down.



The fingers to the left of the turned-down finger represent the tens place digit and the fingers to the right represent the ones digit.

Multiplication War

To practice multiplying by 5, 9, 10, you can play Multiplication War – here are the rules:

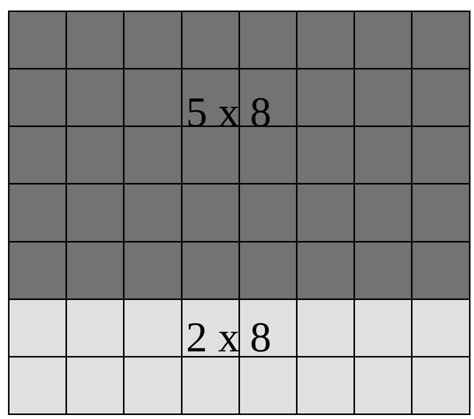
1. This is a game for 2 players.
2. Deal out all of the number cards (0-10) face down in two piles. Each player gets a stack of cards. Do not look at the numbers on the cards.
3. Deal out all of the 5's, 9's, and 10's face down in two separate piles.
4. At the same time, each player turns over one of his/her number cards and one of the 5's, 9's, and 10's cards and finds the product.
5. The player with the higher product wins all four cards and places them on the bottom of his/her stack of cards.
6. Play continues until time is called.
7. The winner is the player with the most cards.

Other Multiplication Facts

All of the other multiplication facts can be built around knowing the facts for 0, 1, 2, 5, 10 (we just put the 9's in for fun and because they have such great patterns!)

For example, let's look at the 7's. How can I get the product of 7 x 8 if I don't know my 7's or 8's?

Give the students a sheet of grid paper, some crayons, and scissors. Ask them to draw and color a 5 by 8 rectangle; write the multiplication problem 5 x 8 over the grids; then cut it out. Now ask them to draw and color (using a different color) a 2 by 8 rectangle. Have them write 2 x 8 over the grids and cut out this rectangle. Their next job is to tape the two rectangles together to form a new rectangle. Ask them, *"What are the dimensions of the new rectangle (7 x 8 or 8 x 7)?"* Ask them to write these problems on the opposite side (no grid marks) of the new rectangle. *"What's the product of 7 x 8 or 8 x 7? How can you use the 2 rectangles to figure out the product?"*

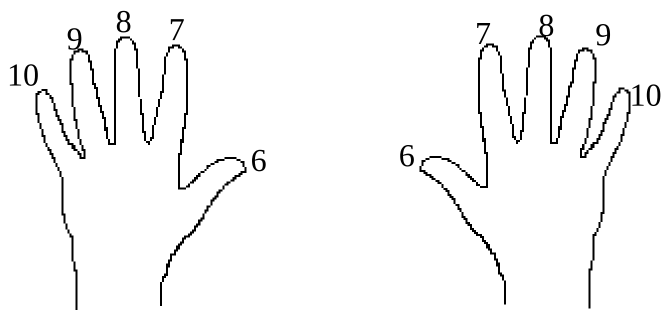


All of the 7's can be quickly figured out if students know their 5's and 2's.

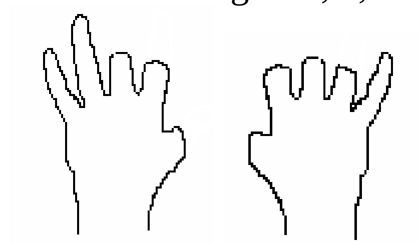
What facts can you use to get the 3's? 4's? 6's? 8's?

Just for Fun – Finger Math for Sixes through Tens

Place both hands palm down in front of you.
Starting with each thumb, number your thumb and fingers 6, 7, 8, 9, 10



Example: For 8 x 9, fold down the fingers 6, 7, 8 on one hand and fingers 6, 7, 8, 9 on the other hand.



All the fingers that are folded down get multiplied by ten (7 x 10).
The fingers that are not folded are multiplied together (2 x 1).
The product of 8 x 9 is 7 x 10 + 2 x 1.

Try these: 8 x 7 7 x 6 6 x 9