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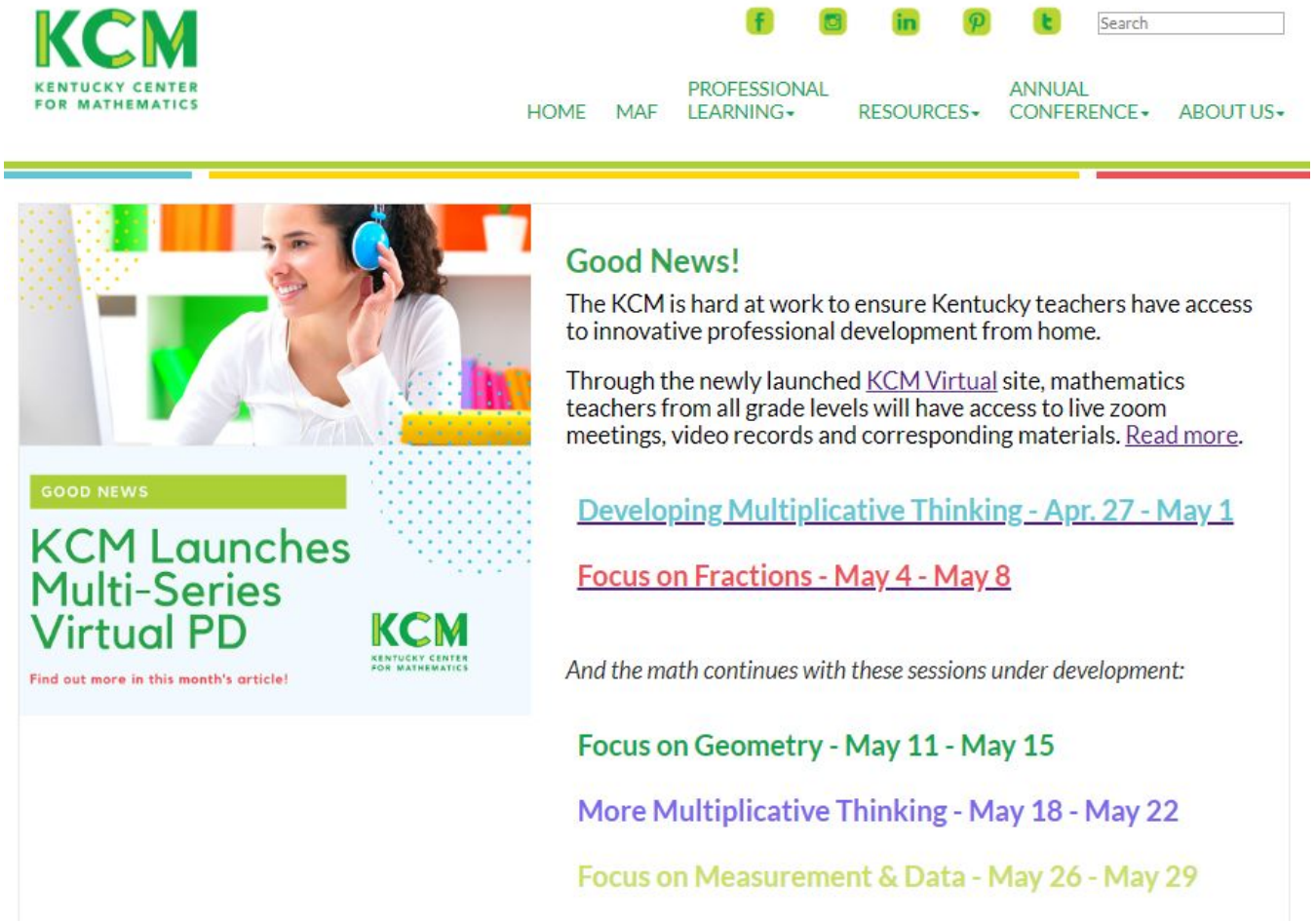
Developing Multiplicative Thinking-

Monitoring and Assessing Multiplication with Tonda Thompson

KCM Website

www.kentuckymathematics.org



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Welcome!

Your host

Tonda Thompson

Regional Consultant
Kentucky Center for Mathematics
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My Educational Experience

Classroom teacher for 16 years

MIT for 11 years (MAF-2006-2017)

KCM - since 2006

Math Recovery - 2006-2017

Teacher learner - FOREVER

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Agenda

*Research
*Standards
*Major categories of assessment
*6 types of formative assessments
*Observations
*Interviews
*Progress monitoring

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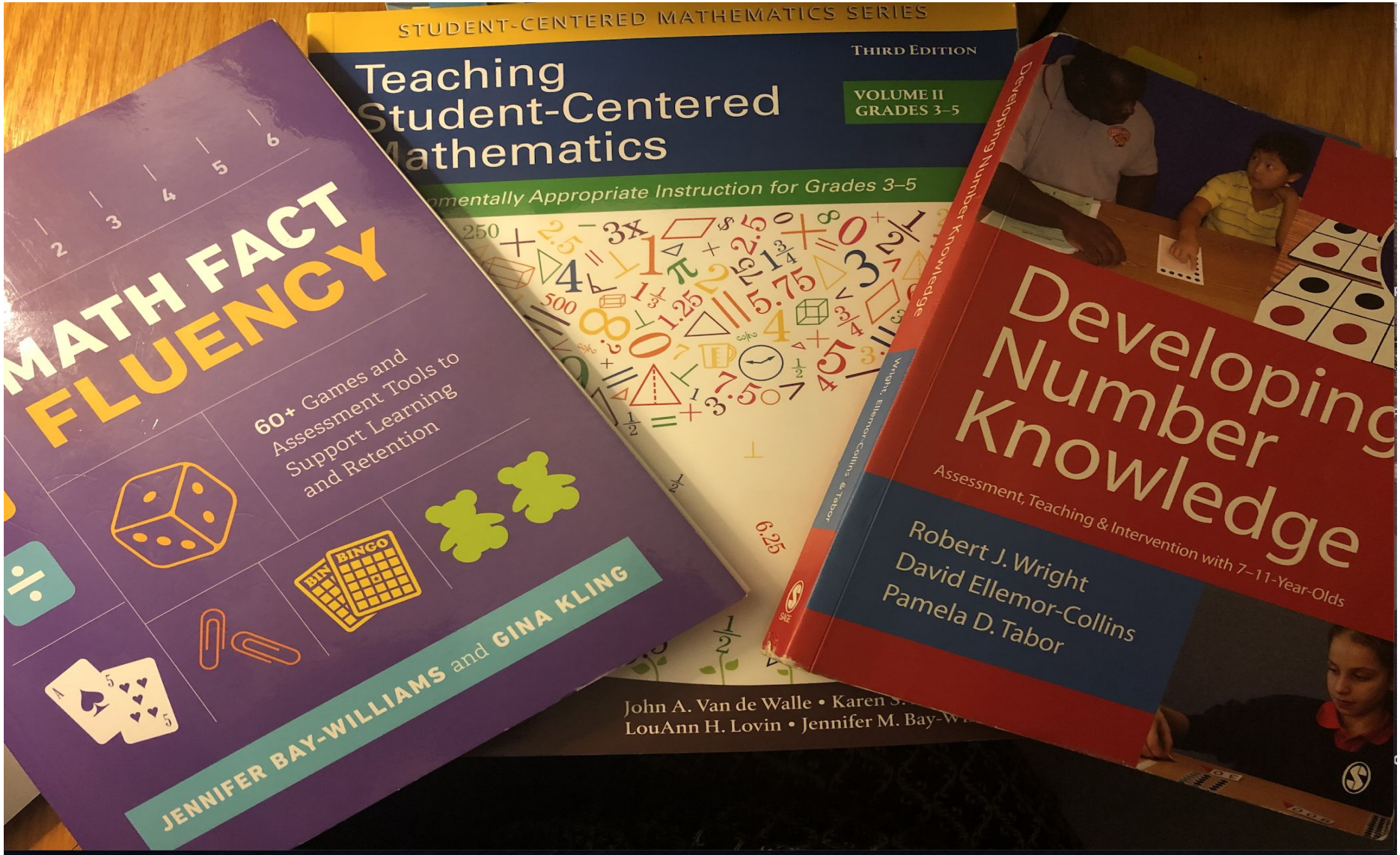
This work comes from the following books:



Operations and Algebraic Thinking	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Represent and solve problems involving multiplication and division.

Standards	Clarifications
KY.3.OA.1 Interpret and demonstrate products of whole numbers. **MP.2, MP.5**	Students use models for multiplication situations. For example, students interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. Coherence KY.2.OA.4→KY.3.OA.1→KY.4.OA.1
KY.3.OA.2 Interpret and demonstrate whole-number quotients of whole numbers, where objects are partitioned into equal shares. **MP.2, MP.5**	Students use models for division situations. For example, students interpret 56 ÷ 8 as the number of 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 object each. Coherence KY.3.OA.1→KY.3.OA.2→KY.5.NF.3
KY.3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays and measurement quantities, by using drawings and equations with a symbol for the unknown number to represent the problem. **MP.1, MP.4**	Students flexibly model or represent multiplication and division situations or context problems (involving products and quotients up to 100). Note: Drawings need not show detail, but accurately represent the quantities involved in the task. **See Table 2 in Appendix A.** Coherence KY.3.OA.3→KY.4.OA.2
KY.3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. **MP.6, MP.7**	Students determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = □ ÷ 3, 6 x 6 = ?. Coherence KY.3.OA.4→KY.4.MD.3

Attending to the Standards for Mathematical Practice

Students recognize the numbers and symbols in an equation such as 5 x 8 = 40 are related to a context using groups or arrays (**MP.2**). For example, a student analyzes this equation and tells a story about walking 8 blocks round-trip to and from school each day, connecting to the equation by saying: 5 days x 8 blocks each day is 40 total blocks walked. To represent the problem, they show 5 jumps of 8 on an open number line or show five 8-unit long Cuisenaire Rods (**MP.5**). When reading story situations, students seek to make sense of the story and its quantities (**MP.1**). They do not just lift numbers out or use keywords. To help make sense of the problem, students decide to write an equation or use a number line. In other words they 'mathematize' the situation (**MP.4**). In missing value problems, students attend to what value is unknown and what operation is represented (**MP. 6**)

So why do we assess students in mathematics?

To enhance students learning

For making instructional decisions

To provide feedback to help learners progress

Van de Walle, Karp, Lovin, Bay-Williams; Teachering Student-Centered Mathematics, Volume II Grades 3-5

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Assessments fall into one of two major categories:

Summative Assessments - Cumulative evaluations that take place usually after instruction is completed. They commonly generate a single score, such as an end-of-unit test or a standardized test that is used in your state or school district.

Formative Assessments - assessments used to check students development during instructional activities, to pre-assess, or to attempt to identify students understandings or misconceptions.

Van de Walle, Karp, Lovin, Bay-Williams; Teachering Student-Centered Mathematics, Volume II Grades 3-5

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6 TYPES OF FORMATIVE ASSESSMENTS:

1. Observations
2. Questions
3. Interviews
4. Tasks
5. Students' Self-Assessment and Reflection
6. Rubrics

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Teaching Student-Centered Mathematics; Vol. II; Grades 3-5

OBSERVATIONS

During the observation, 2 valuable results occur:

1. Information that may have gone unnoticed is suddenly visible and important

2. Observation data gathered systematically can be combined with other data fused in planning lessons, providing feedback to students, conducting parent conferences and determining grades.

Van de Walle, Karp, Lovin, Bay-Williams; Teachering Student-Centered Mathematics, Volume II Grades 3-5

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ANECDOTAL NOTES (Observations)

The act of professional noticing where you observe learners through a focus on 3 phases:

1. Attending: if the child nods head, uses fingers to count, etc.

2. Interpreting: students gestures, comments, drawings and actions by making notes of possible strengths and the level of sophistication of their conceptual understanding.

3. Deciding: Where to go next with instructional actions.

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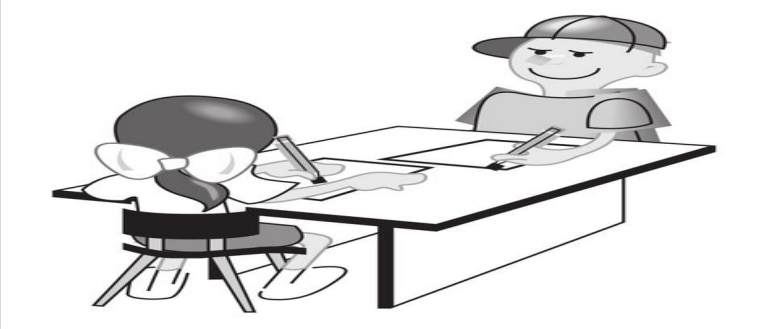
CHECKLIST (Observations)



To help focus your attention, a checklist with several specific processes, mathematical practices, or content objectives can be devised

OR

Checklist that involves listing all students in a class on one to three pages. Across the top of the page are specific abilities or common misconceptions to look for, possibly based on learning progression.

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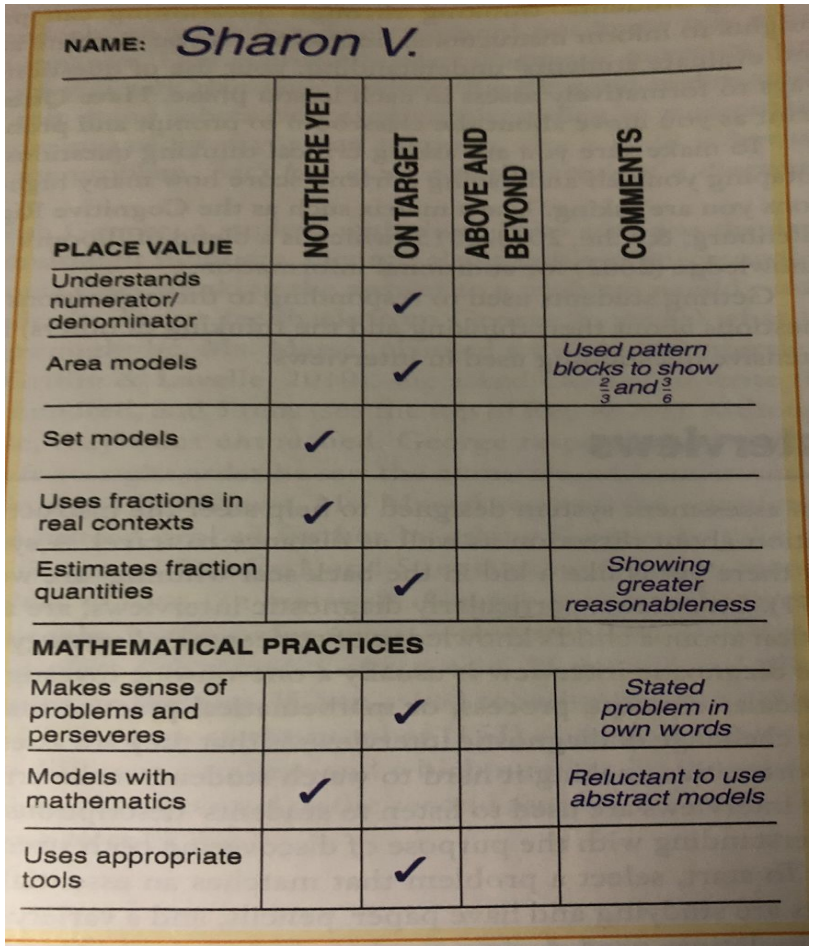
Van de Walle, Karp, Lovin, Bay-Williams; Teachering Student-Centered Mathematics, Volume II Grades 3-5

Checklist

Names	Foundational Facts Sets: Multiplication					
	2s	10s	5s	1s	0s	Squares



NAME: Sharon V.

PLACE VALUE	NOT THERE YET	ON TARGET	ABOVE AND BEYOND	COMMENTS
Understands numerator/ denominator		✓		
Area models		✓		Used pattern blocks to show $\frac{2}{3}$ and $\frac{3}{6}$
Set models	✓			
Uses fractions in real contexts	✓			
Estimates fraction quantities		✓		Showing greater reasonableness
MATHEMATICAL PRACTICES				
Makes sense of problems and perseveres		✓		Stated problem in own words
Models with mathematics	✓			Reluctant to use abstract models
Uses appropriate tools		✓		

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Observation Tool: Multiplication Strategy/Foundational Facts Sets

Names	Multiplication Strategy Selected						
	Foundational Fact (known)	Doubling	Adding a Group	Subtracting a Group	Near Square	Break Apart Strategy	Other (e.g., skip counting)

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Observations and Anecdotal Records

https://youtu.be/ed9EqWXO4Jg

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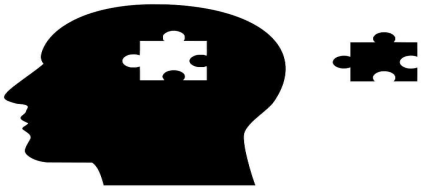
Questions



2. Questions: Probing students thinking through questioning can provide better data and more insights to inform instructional next steps.

Have high level questions on a tablet or in print as you move about the classroom to prompt and probe students thinking.

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Interviews

Interviews, particularly diagnostic interviews, are a means of getting in-depth information about a child's knowledge of concepts and strategy use to provide needed navigation. The diagnostic interview is usually a one-on-one investigation of a child's thinking about a particular concept, process, or mathematical practice that lasts from 3-10 minutes.

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Let's practice an Interview

https://youtu.be/EOIt5sN1J3c

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Where can I find Interview assessment screeners for Multiplication

https://sites.google.com/site/mathscreeners/home/number-sense-screeners

BVSD Math Screeners

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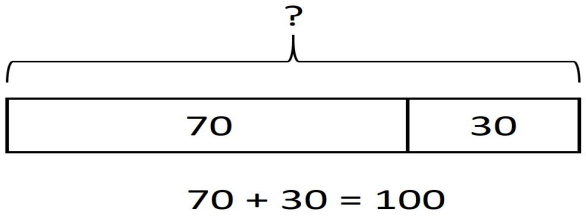
Tasks



Problem-Based Tasks: tasks that are connected to actual problem-solving activities used in instruction.

High-quality tasks permit every child to demonstrate his or her abilities

Ex: Write a multiplication problem that has a product that falls between the answers to these two problems: 49X25 and 45X30. Write an explanation of how you came up with your solution.

Translation Tasks: Write a word problem that matches the equation.

Illustrate the equation with materials or drawings

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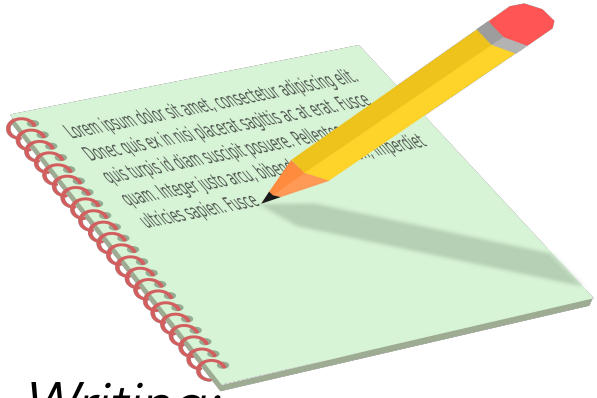

Writing:

As an assessment tool, writing in journals, exit slips, or other formats provide a unique window to students perceptions and the way they are thinking about an idea.

"I think the answer is… I think this because…..

Explain to a student who was absent today what you learned about multiplication.

If you got stuck today in solving a problem, where in the problem did you have trouble?

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Students self-assessment and reflection

Discussions of how students can improve when they analyze their own mistakes.

How well do you think you understand the work we have been doing in multiplication of a two-digit number by a one-digit number?

Which problem(s) on the activity sheet did you find most challenging? Which were easiest?

Rubrics

	4 Thoroughly meets standards	3 Meets standards	2 Approaching standards	1 Not yet approaching standards	0 No attempt
#1 7.G.4 7.G.6	Student correctly finds the ***area*** of the basketball key, with organized work that clearly shows their thinking, including a correct and labeled equation, with no calculation errors, and using correct units.	Student uses a correct strategy to find the ***area*** of the basketball key, with work that shows their thinking, including an equation. May include minor calculation errors or incorrect units.	Student uses a partially correct strategy to find the ***area***, but does not correctly find the area of the basketball key. Or student has correct answers but shows no work.	Student attempts to find the area but does not correctly find the ***area*** of any part of the basketball key. Or student has incorrect answers and shows no work.	No evidence of attempting the problem.
#2 7.G.4 7.G.6	Student correctly finds the ***perimeter*** of the basketball key, with work that clearly shows thinking, including a correct and labeled equation, with no calculation errors, and using correct units.	Student uses a correct strategy to find the ***perimeter*** of the basketball key, with work that shows their thinking, including an equation. May include minor calculation errors or incorrect units.	Student uses a partially correct strategy to find the ***perimeter***, but does not correctly find the area of the basketball key. Or student has correct answers but shows no work.	Student attempts to find the perimeter but does not correctly find the ***perimeter*** of any part of the basketball key. Or student has incorrect answers and shows no work.	No evidence of attempting the problem.

A rubric is a scale based on predetermined criteria with two important functions: (1) It permits students to see what is central to excellent performance, and (2) it provides you with scoring guidelines that support equitable analysis of students work.

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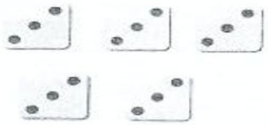
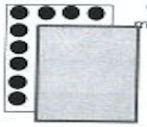
Now what, after assessment?

AVMR2 Differen..ation Worksheet

After assessing students, input names into appropriate columns for each strand of the assessment. Each column is headed with the AVMR2 construct and the areas in which instruction needs to be targeted.

Multiplication and Division				
Construct 0	**Construct 1**	**Construct 2**	**Construct 3**	**Construct 4-5**
Needs to work on putting items in equal groups; sharing items equally	Needs to work on counting visible, pre-grouped items and associate stress and skip counting with quantities	Needs to work on counting groups of items where individual items are not visible	Needs to work on counting groups within a group of non-visible items; different ways to break a whole group down without perceptual markers	Needs to work on recall or quick computation of basic mult/div facts; work on recognizing inverse relationship of mult/div and commutativity of mult
← Work on stress or skip counting throughout →				
"You have 20 cookies. If you shared them equally among five people, how many cookies would each person get?" 	"How many dots all together?" 	If you know that there are six rows of four dots, how many dots are there all together? 	"If you know that there are six rows of four dots, how many dots are there all together?" 	"How many rows like this would you need to uncover to show 8 dots? What about 16? What about 32?" 

Created by Kris Jarboe and Becky Reister NKU and EKU Regional Coordinators- KY Center for Math

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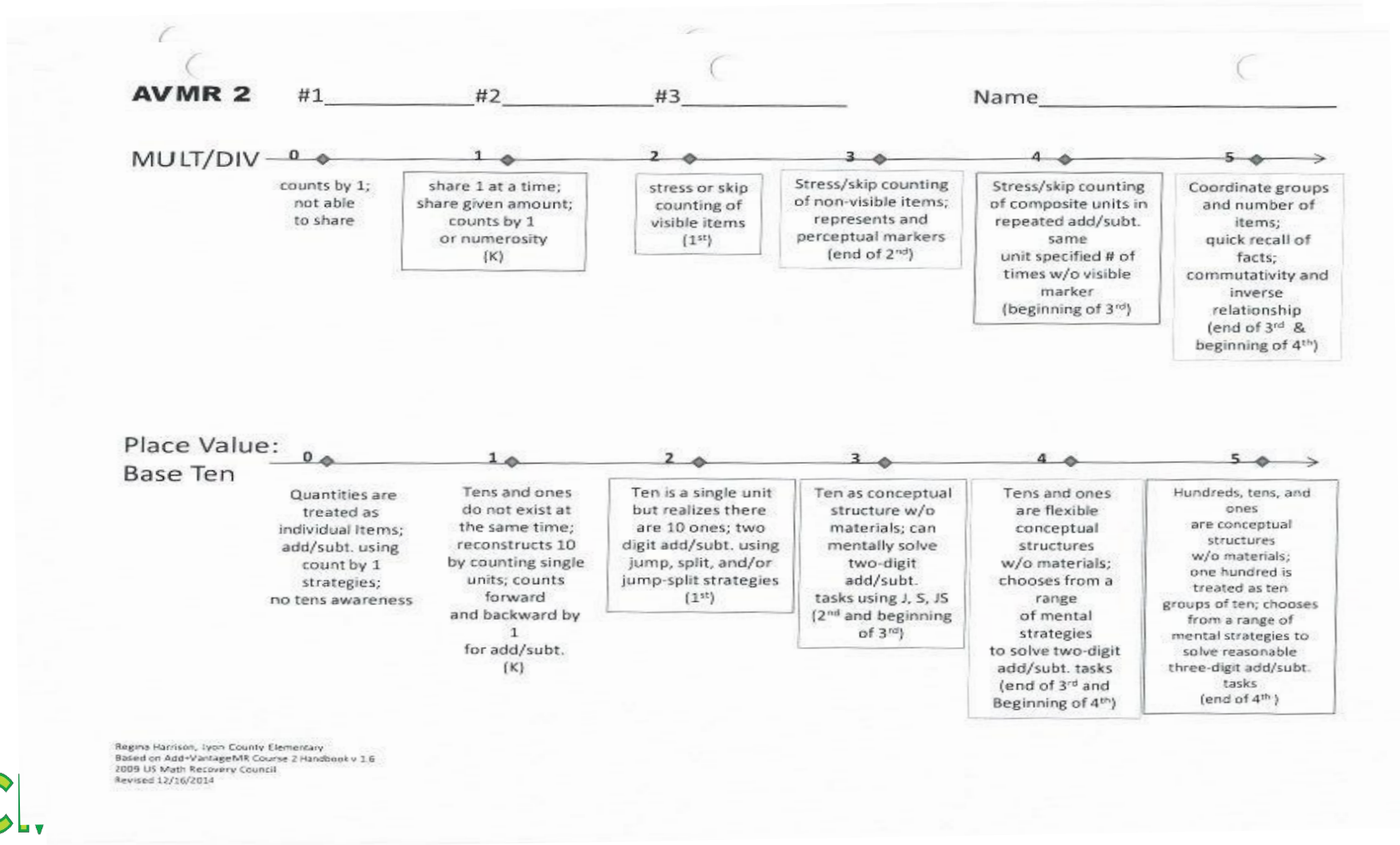
MULTIPLICATION AND DIVISION PROGRESS MONITORING



AVMR 2 #1_____ #2_____ #3_____ Name_____

MULT/DIV

0	1	2	3	4	5
counts by 1; not able to share	share 1 at a time; share given amount; counts by 1 or numerosity (K)	stress or skip counting of visible items (1st)	Stress/skip counting of non-visible items; represents and perceptual markers (end of 2nd)	Stress/skip counting of composite units in repeated add/subt. same unit specified # of times w/o visible marker (beginning of 3rd)	Coordinate groups and number of items; quick recall of facts; commutativity and inverse relationship (end of 3rd & beginning of 4th)

Place Value: Base Ten

0	1	2	3	4	5
Quantities are treated as individual items; add/subt. using count by 1 strategies; no tens awareness	Tens and ones do not exist at the same time; reconstructs 10 by counting single units; counts forward and backward by 1 for add/subt. (K)	Ten is a single unit but realizes there are 10 ones; two digit add/subt. using jump, split, and/or jump-split strategies (1st)	Ten as conceptual structure w/o materials; can mentally solve two-digit add/subt. tasks using J, S, JS (2nd and beginning of 3rd)	Tens and ones are flexible conceptual structures w/o materials; chooses from a range of mental strategies to solve two-digit add/subt. tasks (end of 3rd and Beginning of 4th)	Hundreds, tens, and ones are conceptual structures w/o materials; one hundred is treated as ten groups of ten; chooses from a range of mental strategies to solve reasonable three-digit add/subt. tasks (end of 4th)

Regina Harrison, Lyon County Elementary
Based on Add+VantageMR Course 2 Handbook v 1.6
2009 US Math Recovery Council
Revised 12/16/2014

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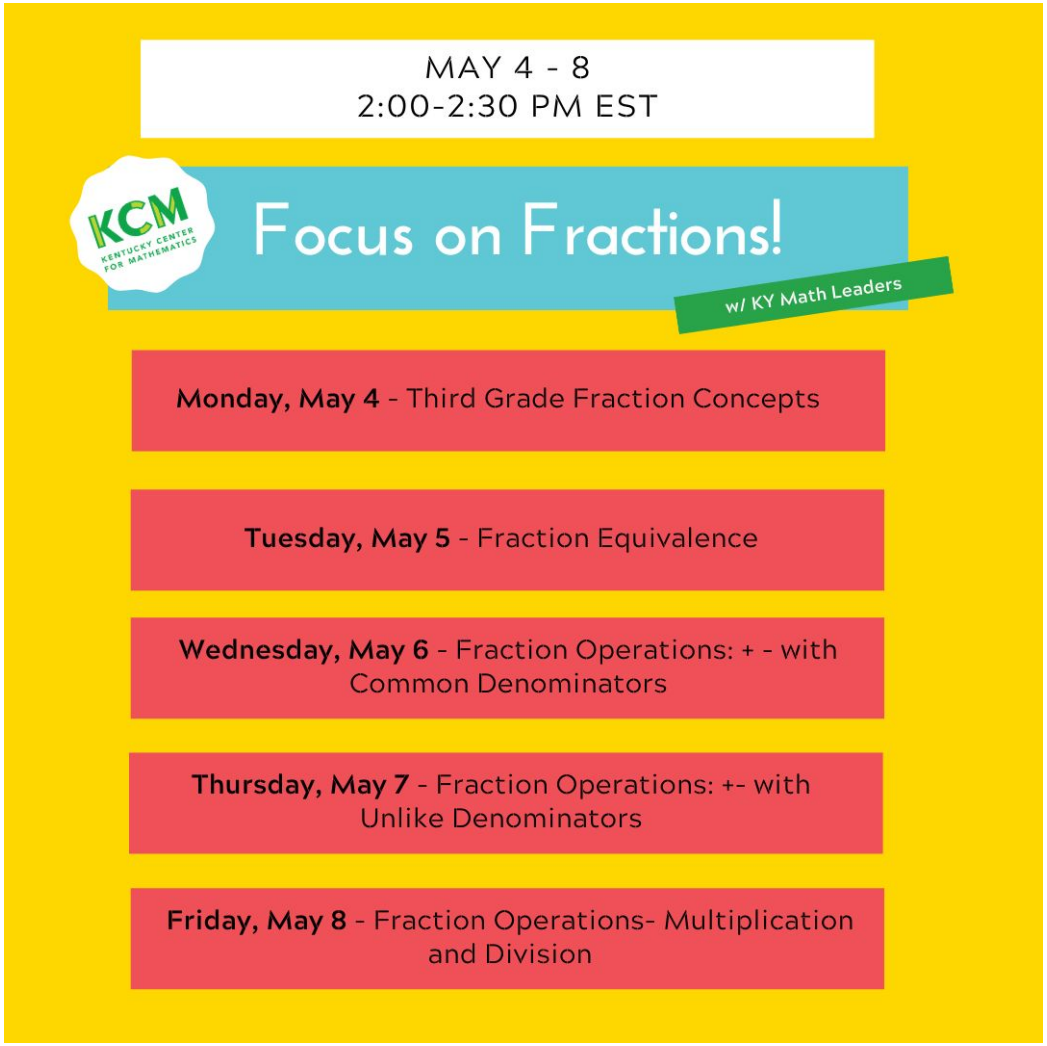
Upcoming Sessions



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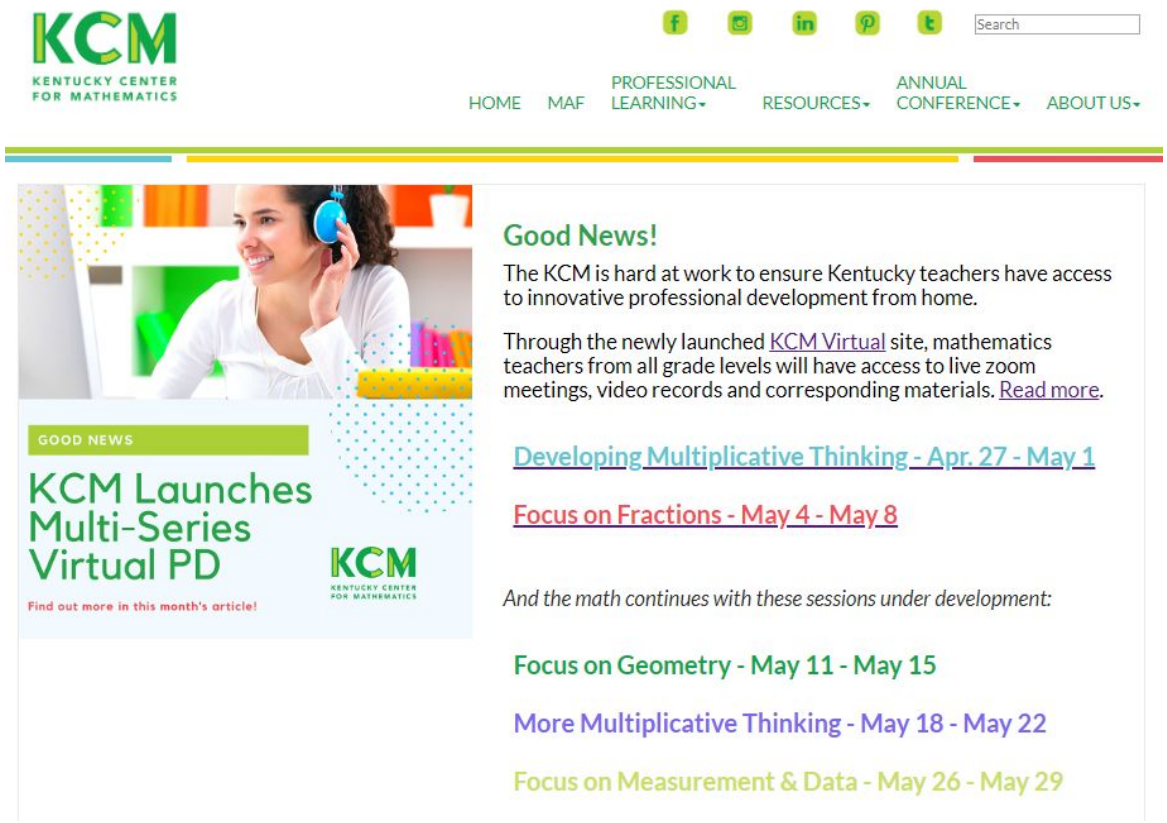

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Thank you for your time

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