AVMR2 Differentiation Worksheet

After assessing students, input names into appropriate columns for each strand of the assessment. Each column is headed with the AVMR2 construct and the areas in which instruction needs to be targeted.
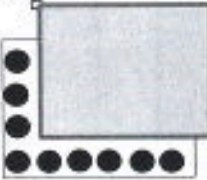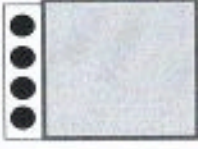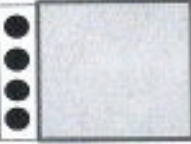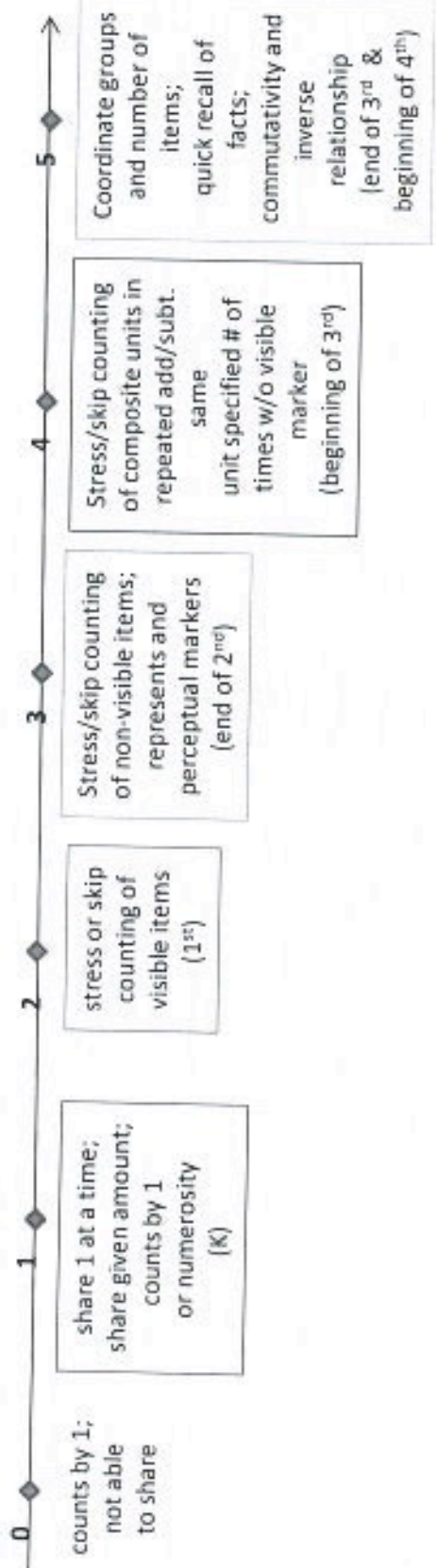
Multiplication and Division

Construct 0	Construct 1	Construct 2	Construct 3	Construct 4-5
Needs to work on putting items in equal groups; sharing items equally	Needs to work on counting visible, pre-grouped items and associate stress and skip counting with quantities	Needs to work on counting groups of items where individual items are not visible	Needs to work on counting groups within a group of non-visible items; different ways to break a whole group down without perceptual markers	Needs to work on recall or quick computation of basic mult/div facts; work on recognizing inverse relationship of mult/div and commutativity of mult

Work on stress or skip counting throughout

"You have 20 cookies. If you shared them equally among five people, how many cookies would each person get?"	"How many dots all together?"	"If you know that there are six rows of four dots, how many dots are there all together?"	"If you know that there are six rows of four dots, how many dots are there all together?"	"How many rows like this would you need to uncover to show 8 dots? What about 16? What about 32?"

AVMR 2

#1 _____ #2 _____ #3 _____ Name _____

MULT/DIV

0 — 1 — 2 — 3 — 4 — 5

0:
counts by 1; not able to share

1:
share 1 at a time; share given amount; counts by 1 or numerosity
(K)

2:
stress or skip counting of visible items
(1st)

3:
Stress/skip counting of non-visible items; represents and perceptual markers
(end of 2nd)

4:
Stress/skip counting of composite units in repeated add/subt. same unit specified # of times w/o visible marker
(beginning of 3rd)

5:
Coordinate groups and number of items; quick recall of facts; commutativity and inverse relationship
(end of 3rd & beginning of 4th)

Place Value: Base Ten

0 — 1 — 2 — 3 — 4 — 5

0:
Quantities are treated as individual items; add/subt. using count by 1 strategies; no tens awareness

1:
Tens and ones do not exist at the same time; reconstructs 10 by counting single units; counts forward and backward by 1 for add/subt.
(K)

2:
Ten is a single unit but realizes there are 10 ones; two digit add/subt. using jump, split, and/or jump-split strategies
(1st)

3:
Ten as conceptual structure w/o materials; can mentally solve two-digit add/subt. tasks using J, S, JS
(2nd and beginning of 3rd)

4:
Tens and ones are flexible conceptual structures w/o materials; chooses from a range of mental strategies to solve two-digit add/subt. tasks
(end of 3rd and Beginning of 4th)

5:
Hundreds, tens, and ones are conceptual structures w/o materials; one hundred is treated as ten groups of ten; chooses from a range of mental strategies to solve reasonable three-digit add/subt. tasks
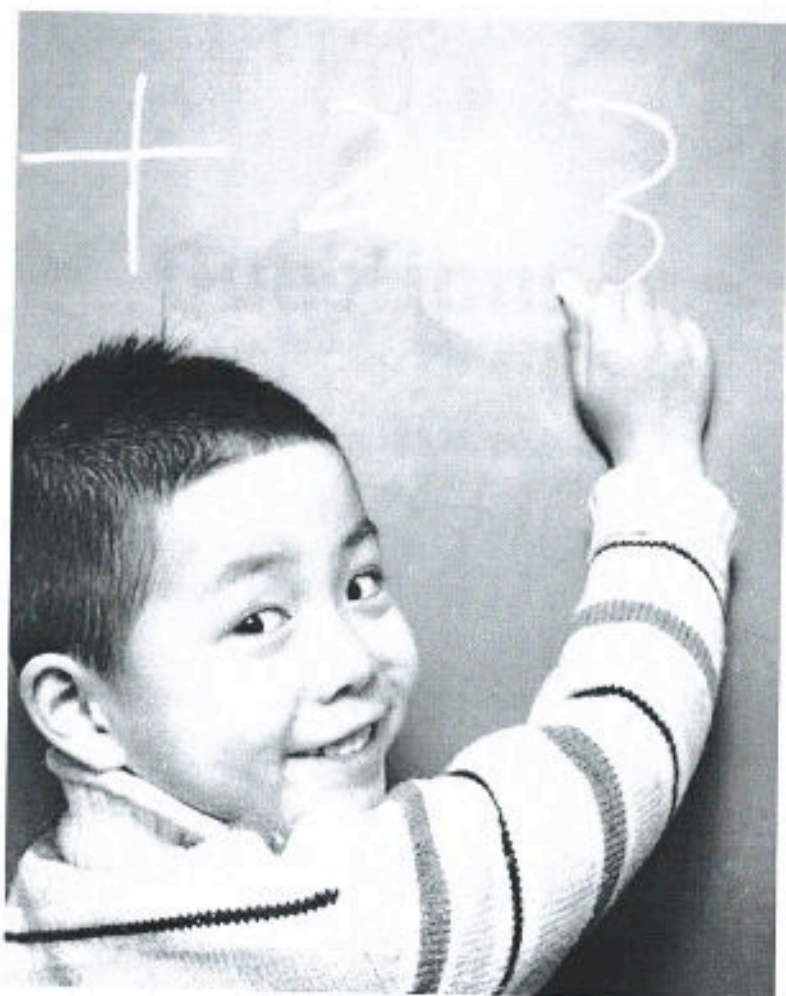(end of 4th)

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A Union of Professionals

Questions
for Math Class

ELEMENTARY

An AFT Common Core Resource

A Union of Professionals

Randi Weingarten
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The **American Federation of Teachers** is a union of
professionals that champions fairness; democracy; economic
opportunity; and high-quality public education, healthcare
and public services for our students, their families and our
communities. We are committed to advancing these principles
through community engagement, organizing, collective
bargaining and political activism, and especially through the
work our members do.

Questions
for **Math Class**

An AFT Common Core Resource

Questions like these can be helpful as teachers implement the Common Core State Standards for Mathematics.

Some questions are useful in helping students develop the habits of mind referred to in the Standards for Mathematical Practice.

Although these questions appear under different headings, the habits are intertwined, and many questions overlap in the kind of thinking and performance they elicit.

Other questions are meant to guide students to the heart of the mathematics. Both kinds of questions are necessary to develop mathematical proficiency.

Questions and prompts to engage and build mathematical practices

1. Make sense of problems and persevere in solving them.
   - Retell this problem in your own words.
   - What do you have to find out?
   - Can you think of similar problems we have solved? How might that help?
   - What have you already tried?
   - Where in the problem are the numbers you used?
   - Can you think of another strategy to try?
   - Where could you start?
   - Could you use objects or pictures to help you solve the problem?
   - Is this answer reasonable? Does it make sense? Why or why not?

   (Add your own)

2. Reason abstractly and quantitatively.
   - Can you write an equation?
   - What do these numbers represent? What does _____ stand for?
   - What is the relationship between the quantities in this problem? (What is the relationship between _____ and _____?)
   - Does your solution answer the question?
   - What is the unit being used?
   - How do you know that?

   (Add your own)

3. Construct viable arguments and critique the reasoning of others.
   - Convince us that your solution is correct.

- Did you test whether your method worked? How?
- Can anybody explain it a different way?
- Does _____'s solution make sense to you? Why or why not? Did you ask a question about what is unclear?
- Can you explain what _____ did?
- How are these solutions alike? How are these solutions different?
- Tell us why you think the solution is wrong.

(Add your own)

4. Model with mathematics.
- Show me what you mean so I can see it.
- Which representation is most helpful to you (objects, diagram/picture, table, graph, equation, etc.)? What about it helped?
- Can you write an equation to represent this situation?
- What are the important quantities/numbers in the problem?

(Add your own)

5. Use appropriate tools strategically.
- What mathematical tools can you use to solve the problem?
- Which tool would be most helpful for you? Would a _____ or a _____ make more sense?
- Can that tool model this problem?
- How could estimation help you with this problem?
- Which of the tools takes less time?
- Why did you decide to use _____?

(Add your own)

6. Attend to precision.
   - Use precise math language and definitions in your explanation.
   - What does the equal sign tell you here?
   - Label quantities and axes so we all know what they refer to.
   - Relate your answer to the problem's context or story.
   - How can you test your solution?
   - Were you able to follow _____'s explanation? Can you explain it now? What wasn't clear?

   (Add your own)

7. Look for and make use of structure.
   - Do you see a pattern as you work? What is it? Can it help you?
   - Can you think of a rule or a property that could help us?
   - We said some things in mathematics are always true. Can one of those relationships or structures help here?

   (Add your own)

8. Look for and express regularity in repeated reasoning.
   - What does that pattern/repetition tell you?
   - What pattern do you see in the calculation?
   - Can you predict the next one? The nth one? Is it true in all cases?
   - Can you make a conjecture about _____? Will it always be true?
   - Why does that always work?

   (Add your own)

> The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the … years.
>
> —CCSSM, p. 8

Questions and prompts focusing on elementary mathematics content

Counting, Addition and Subtraction

1. What number is 10 and four more?
2. How do you know there are 12 here?
3. Here's 6. How many more to 10?
   How do you know?
4. How can you make 7 + 6 easier to add?
   Tell us why.
5. What does it mean to solve 14 − 8 as a
   missing addend problem. Talk us through it.
6. Can you solve this a different way?
7. How can you check your subtraction
   (or addition) problem? Why does that work?

Place Value

1. How many ways can you show the number
   with base-10 blocks?
2. What would happen to the number
   (e.g., the 4) if I moved it here?
3. What is the difference between 25 tens
   and 250?
4. How could decomposing 3000 make it easier
   to subtract 3000 − 642. How would you
   decompose it?
5. How is it possible to subtract 60 − 18 when
   there are no ones to subtract from?

Multiplication

1. What does a × b mean? How else can you
   explain it?
2. How are multiplication tables built? Explain
   using one table as an example.
3. How does modeling multiplication differ from
   modeling addition? Why?

4. Why do we say that multiplication and division are inverse operations? How can that help you make division easier?

5. Compare 4, 9 and 36. Use multiplication language.

6. If the divisor doubles, what happens to the quotient? What if the divisor is cut in half?

7. If the multiplier is cut in half and the number being multiplied is doubled, what happens to the product? Use a model to explain.

8. Write a real-life problem to go with the equation.

9. Explain how each symbol relates to the story.

Fractions

1. What's the whole for this fraction?

2. What's the unit for the question/answer?

3. What does each fraction refer to?

4. Show this fraction on the number line.

5. How does the size of the fraction change if only the denominator is increased? Decreased? If only the numerator is increased? Decreased? Why?

6. Is this fraction $>$, $=$, or < 1? How about $\frac{1}{2}$? How do you know?

7. You added the numerators but not the denominators. Why?

8. How did you get this denominator?

9. Why did you multiply the denominators? Describe what is happening when you do that.

10. Explain how to create equivalent fractions.

11. Explain the difference between array and area models for multiplication. Which is easier for you? Why?

12. What does it mean to multiply a fraction less than one by another fraction less than one?

13. Explain why you do or do not multiply the denominator when multiplying a fraction by a whole number.

14. When would you multiply denominators when multiplying two fractions? Why? Show what is happening to those numbers when you multiply the denominators.

Number Systems

1. Explain, using a model, what happens to a number when you multiply it by 10. Why? What happens when you multiply a number by 100? Why?

2. When two whole numbers that do not begin with zero are compared, the longer one is always greater. Does that hold true for numbers with decimals smaller than 1? Why or why not? Use examples.

3. (Given worked problems) Can you reason about where the decimal point should go without counting? Can you estimate the answer?

4. Will your answer be closer to 6, 60 or 600? How do you know?

5. What happens to the number (e.g., the 4) when I move it one place to the left? One place to the right? Why?

6. What is the relationship of the places in our number system?

Geometry

1. What are the defining attributes of (name a polygon)? Does it belong to a larger category of shapes? Name other shapes in that category.

2. What is the difference between a ____ and a ____ ?

3. What is area? Show me where the area is for this shape. How can I calculate the area?

4. Explain why area can be found by multiplying length times width. What units are appropriate to measure this area? Why are they called that?

5. What is perimeter? Show me where the perimeter is for this shape (or for something in the room). How can I calculate the perimeter?

6. Prove whether or not a rectangle with a certain perimeter will always look the same.

7. Someone said that division is like finding the missing side of a rectangle. Explain that.

8. Why do we use square units when we talk about area?

9. What are angles and how are they measured?

10. When we find the volume of a figure, what are we finding?

11. Create a model that can be used to determine the volume of a square or right rectangular prism and explain how the model shows how to find the volume without building a concrete model.

12. What is the difference between an attribute and a defining attribute?

Other

1. What's the pattern of growth?
2. How are the numbers changing?
3. What could a table show you?
4. Is there something that is the same every time?
5. What does that tell you?

Neither well-written standards, nor tasks with high cognitive demand, nor questions by themselves guarantee that students will engage in high-level discussions or learn rigorous mathematics, weaving together conceptual understanding, procedural skill and fluency, and appropriate application to the world in which they live. But what happens between students and teachers as they work with tasks can make a huge difference in whether they do. The kinds of questions asked embody a teacher's expectations about what is important to know.

We urge you to keep in mind the findings below. (Henningsen & Stein, 1997; Stein et al., 1996. *Implementing Standards-Based Mathematics Instruction.*)

- Tasks take on a life of their own after being introduced, and the introduction itself sets the teacher's expectations. Are students to think and learn or follow a prescribed path?

- "Teachers can promote sense-making and deeper levels of understanding by consistently asking students to explain how they are thinking about the task. Or, conversely, they may cut off opportunities for sense-making by hurrying student through the tasks, thereby not allowing them the time to grapple with perplexing ideas."

- When high-level tasks were enacted as intended, support factors provided by the teacher included "tasks that built on students' prior knowledge, appropriate scaffolding of student thinking (thought-provoking questions that preserve task complexity), sustained pressure for explanation and meaning, [and] modeling of high-level thinking and reasoning by the teacher and more capable peers."
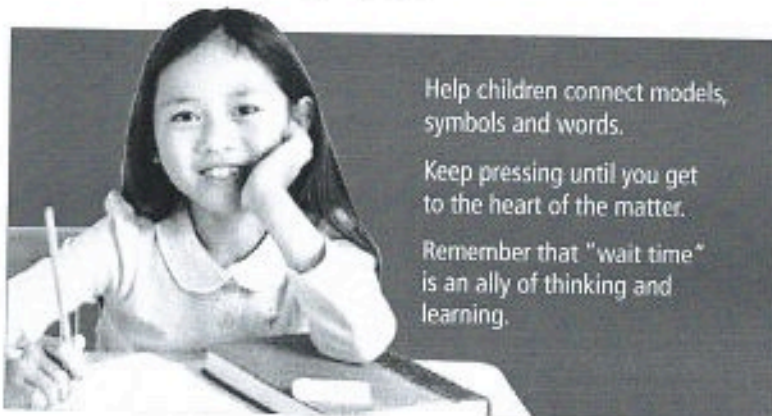
Help children connect models, symbols and words.

Keep pressing until you get to the heart of the matter.

Remember that "wait time" is an ally of thinking and learning.

Observation Tool: Multiplication Foundational Fact Sets

Names	Foundational Facts Sets: Multiplication					
	2s	10s	5s	1s	0s	Squares

Bay-Williams, J. M. & Kling, G. G. (2019). *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention.* Alexandria, VA: ASCD.

Observation Tool: × 2, × 10, × 5

Note: The 'Goals' are to not have students skip counting (N), but to have them recognize commutativity (Y), and to get to automaticity (Y)

Names	× 2			× 10			× 5		
	Skip Counts	Commutativity	Automaticity (within 3 seconds)	Skip Counts	Commutativity	Automaticity (within 3 seconds)	Skip Counts	Commutativity	Automaticity within 3 seconds)
Goals	N	Y	Y	N	Y	Y	N	Y	Y

Bay-Williams, J. M. & Kling, G. G. (2019). Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention. Alexandria, VA: ASCD.

Observation Tool: ✕ 5 (Multiplication)

Names ꜀	✕ 5 (Assuming Commutativity)										Key: • = Correct * = Automatic
	5×0	5×1	5×2	5×3	5×4	5×5	5×6	5×7	5×8	5×9	

Bay-Williams, J. M. & Kling, G. G. (2019). *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention.* Alexandria, VA: ASCD.

Observation Tool: Multiplication Strategy

Names	Multiplication Strategy Selected						
	Foundational Fact (known)	Doubling	Adding a Group	Subtracting a Group	Near Square	Break Apart Strategy	Other (e.g., skip counting)

Bay-Williams, J. M. & Kling, G. G. (2019). *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention.* Alexandria, VA: ASCD.

Observation Tool: Multiplication Derived Facts

Names	Multiplication Facts that are Not Foundational Facts (the "toughies")														
	4 ×3	6 ×3	7 ×3	8 ×3	9 ×3	6 ×4	7 ×4	8 ×4	9 ×4	7 ×6	8 ×6	9 ×6	8 ×7	9 ×7	9 ×8

Code: I = Inefficient method used (e.g., counting when it is not + 1, +2 situation)
S = (efficient) Strategy used (e.g., adding a group)
K = Just knew (recall)
A = Automatic (strategy applied/just knew in 3 seconds or less)

Bay-Williams, J. M. & Kling, G. G. (2019). *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention*. Alexandria, VA: ASCD.