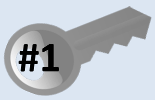
Math Fact Fluency

Three KEYS to Helping Your Child *Learn* Basic Facts for *Life* AND *Like* Math

Learn

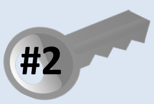


Focus on *Real* Math Fluency… use STRATEGIES
(*rather than just memorize the facts with worksheets or flash cards*)

Why?

- Your child is *much more* likely to remember facts later on
- Your child is *much less* likely to have stress and anxiety
- The strategies will be used with greater numbers, fractions, and higher-level mathematics to support your child as a confident mathematician.

Life



Help your child 'see' the reasoning STRATEGIES that generalize to numbers beyond basic facts.

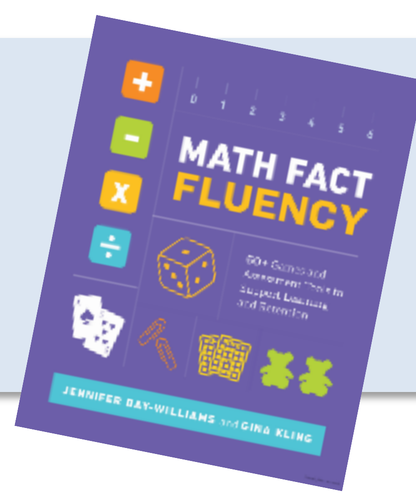
Developing fluency involves children building understandings from within and taking an active part in constructing number sense. Essential to this development is children deriving strategies to approach problems and recognizing that they are capable of reasoning and finding relationships.

To get to that point, though, a child needs multiple opportunities to interact with number sense ideas, use number sense, and discuss number sense ideas and strategies.

Like

Make practice *enjoyable* and *meaningful*.

- Play games.
- Talk through STRATEGIES.
- Focus on strategy selection, not speed. Speed will come with strategy practice.



Based on ***Math Fact Fluency***: *60+ Games and Assessment Tools to Support Learning and Retention* by J. Bay-Williams and G. Kling

Addition Strategies

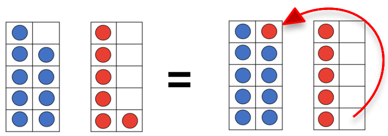
Example
9 + 6

Making 10

Talk

Imagine shifting some counters over to make a full 10, then add.

Ten Frames



Numbers

9 + 6 = *10* + 5

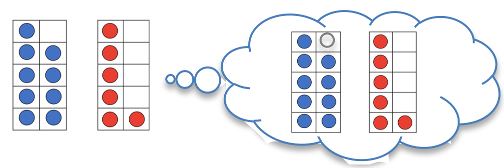
= 15

Pretend-a-10
[Compensation]

Talk

Pretend the biggest number is 10. Add. Adjust your answer to remove the extra you added.

Ten Frames



Numbers

9 + 6 ∘ ∘ ○ *10* + 6 = 16

Subtract the extra one(s) you "pretended," so
9 + 6 = 15

Home Made Ten Frame for Hands-On Learning

Cut off two cups of an egg carton so that you have ten cups. Use any [safe] household objects as counters (e.g., erasers, Lego, coins, game pieces, candies, etc.).

Why Strategies Matter:

Addition Strategies Extended to 3-digit Addition and Subtraction

A child with math fluency looks to see when these strategies can save them from doing the more time-consuming standard algorithms.

"Making 10"

198 + 237 = ☐

+2 ↙ ↘ −2

200 + 235 = 435

"Pretend-a-10"

198 + 237 = ☐

+2

200 + 237 = 437

−2

435

"Pretend-a-10"

504 – 98 = ☐

−2

504 – 100 = 404

+2

406

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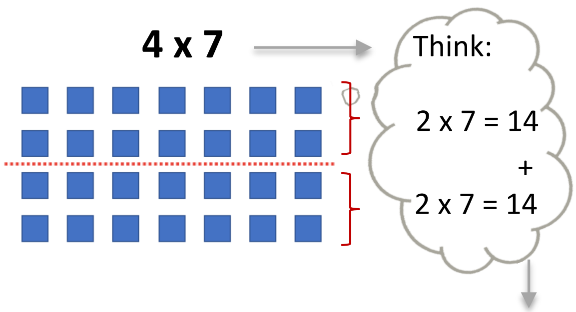
Multiplication Strategies

Doubling
(and Halving)

With any even factor, I can use half that number to multiply and then double my answer.

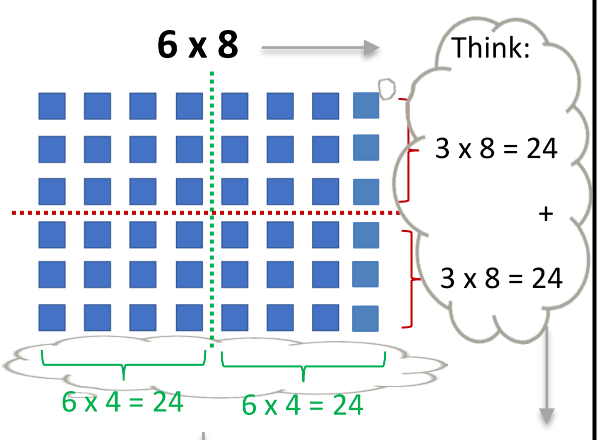
4 x 7



So, 4 x 7 = **28**

If both factors are even – pick either one to halve, then double!

6 x 8



So, 6 x 8 = **48**

Talk

Numbers &

Models

Tip: To help students see *why* these strategies work, use concept language like…

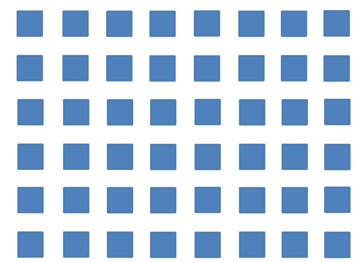
4 ***groups of*** 7,
4 ***sets of*** 7,
4 ***rows of*** 7

to help them understand the meaning of
4 *times* 7.

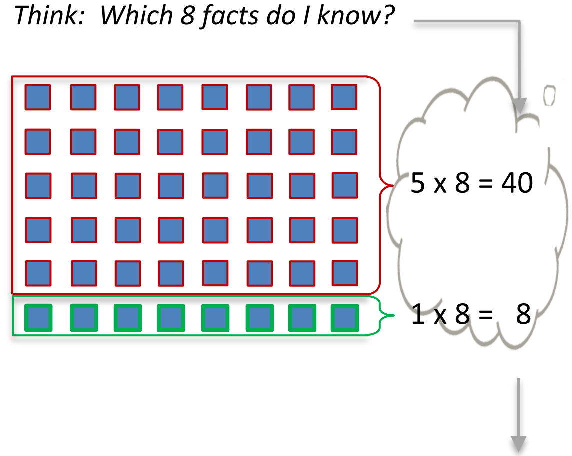
Break Apart
[Distributive Property]

I can take either factor and break it into two 'friendly' numbers, find the product of each part, and then join the parts together to find the product.

6 x 8



Think: Which 8 facts do I know?



So, 6 x 8 = **48**

Home Made Materials for Hands-On Learning

Egg Carton Ten Frames (*see above*). Use small objects (jelly beans) as counters. For 4 × 7, fill 4 cups with 7 beans each.

Cupcake Cups (or Bowls). Start with counters, then eventually place numbers in each cup (using post-its). See how you can arrange the cups to show Doubling.

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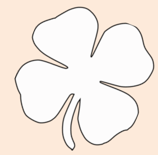
Games

Lucky 13

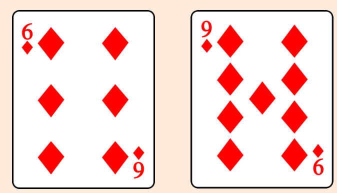


1. Deal 4 cards to each player.
2. Players use 2 of their cards to get a sum as close to 13 as they can.
3. Your score matches how far you are from 13 (e.g., if your two cards add to 15, your score is 2. If you get lucky and get 13, your score is 0!).
4. Play 5 rounds. Lowest score wins!

More ways to play: Deal 5 cards, or play Lucky 10 or Lucky 15.

Factor War

[*Think classic game of war.*]



1. Share the cards equally among the players.
2. Each player flips over2 cards and announces their product.
3. The greatest [correct] product wins those cards.
4. A tie means – War. Repeat steps 2 and 3.
5. Winner has most cards when time is up!

More ways to play: *Fixed Factor War.* You pick a number (e.g., 9 if you are working on your 9 facts). Place it in center for reference. Players only flip one card and multiply their card by the fixed factor. The greatest[correct] product wins. Play *Sum (or Fixed Addend) War* to practice addition facts.

Questions to Ask for Fact Fluency Games

- *How did you solve that fact?*
- *Why did you choose that strategy?*
- *Are there other ways you could solve for that fact?*
- *What other facts might be solved with that* strategy?*
- *When do you like to use that* strategy (when is that strategy a good idea)?*

*you can replace 'that'

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Success with MATH FACT FLUENCY

Teachers + Students + Families

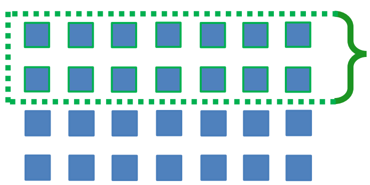
The Doubling Strategy

What does doubling look like? When is doubling useful?

Doubling: × 4

It works great for the 4s facts, and is sometimes called Double and Double Again. Have a look!

Marvin arranged his new set of Legos into four rows. Each row has seven pieces.



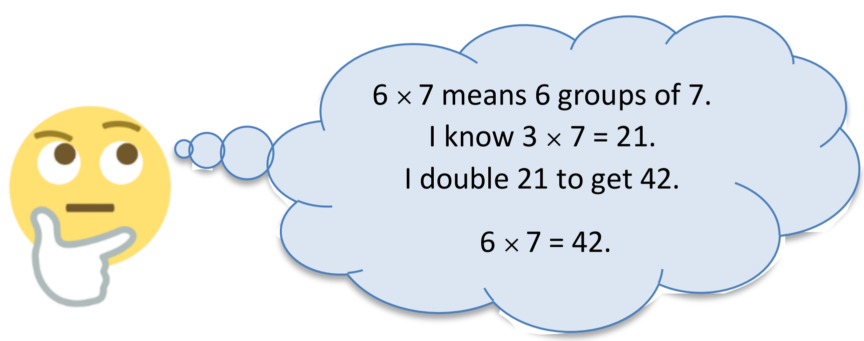
I know…
2 *groups of* 7 is 14:
2 x 7 = 14

Doubling 14 is 28:
14 + 14 = 28

Thinking about 14 + 14 is more efficient than skip counting, 7 + 7 + 7 + 7.

Extending Doubling: × 6 and × 8

Doubling works for other even numbers, like 6 facts and 8 facts. Once your child knows their 3s facts, they double to solve for 6s; once they know their 4s facts, they double to solve 8s. Have a look at a fact that is commonly difficult for children: 7 × 6 or 6 × 7.



Extending Doubling: Beyond Basic Facts

(see "fact #5" above)

Doubling and halving turns problems into ones that can be solved mentally – *very useful*!

Examples:

12 × 15 → (*Half* / *Double*) → 6 × 30 → **180**

5 × 380 (300 + 80 → *Half* → 150 + 40) → *Double* → 10 × 190 → **1,900**

4.5 × 8 → (*Double* / *Half*) → 9 × 4 → **36**

Multiplication Strategy Rationale

Research-based *learning* facts:

1. Students start learning multiplication facts by skip counting. That is natural, but they must progress to more efficient reasoning strategies.
2. Implementing reasoning strategies may initially be slower than counting, but eventually it is faster and will lead to quick recall (automaticity), with the added (critical) benefit of long-term retention (rather than forgetting a fact and having to drop back to skip counting).
3. Visuals and stories help students to understand the reasoning strategy.
4. Mathematical reasoning emerges as children notice patterns and relationships through repeated opportunities. Playing purposeful math games is a great way to do this (see pages 3 and 4)!
5. Reasoning strategies themselves are important to learn because they generalize to larger numbers. Learning the strategies builds stronger math skills!

Thank you for your support in developing fact fluency with your child!

Games for *Doubling* and Learning Facts

Game: *Switch* (2 players)

Materials:
- ✓ One piece of paper, shared by all players
- ✓ 5 or 6 unique game pieces per player (e.g., colored counters or heads/tails of coins)
- ✓ Deck of cards, with Kings and Jacks removed; Queens = 0; Aces = 1

How to play:
1. Write the multiples of **4** (0, 4, 8, 12, 16, 20, 24, 28, 32, 36, and 40) on a piece of paper (mix up the order of products to discourage skip counting as you play - see below).

2. Shuffle the deck of cards and place them face down in a draw pile.

3. Players take turns drawing one card and multiplying the drawn card by **4**. The player places his/her game piece above the resulting product. For example, if a player draws the number 7, the player multiplies 7 by **4** and places a game piece above 28. Players share their strategy aloud (e.g., "I doubled 7 to get 14, and doubled again to get 28.").

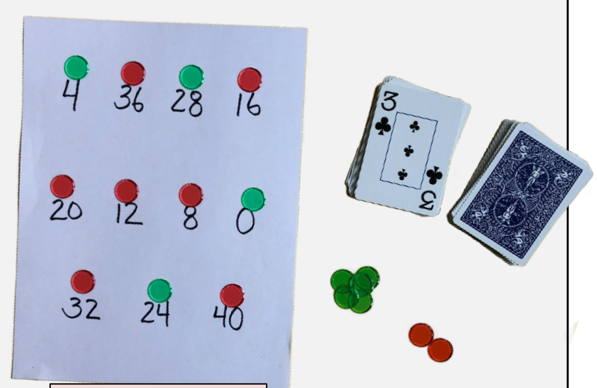
4. If a player's resulting product already has an opponent's game piece on it, that player gets to SWITCH, placing their own game piece above that number, and returning their opponent's game piece. If the player's resulting product is one that they themselves already have, they lose that turn.

5. To win:

   Option 1: First to get 5 game pieces on the board

   Option 2: Have the most game pieces when the board is completely covered (as illustrated here).

   Pick an option and play best 3 out of 5!



Red Wins!

More ways to play:
Play with any fact set by creating a game board with the multiples of that set (e.g., × 7).
Play with three people (use three different colors of counters or three different coins).

Game: ***Fixed Factor War*** (Game 32, p. 88, *Math Fact Fluency*) (2 players)

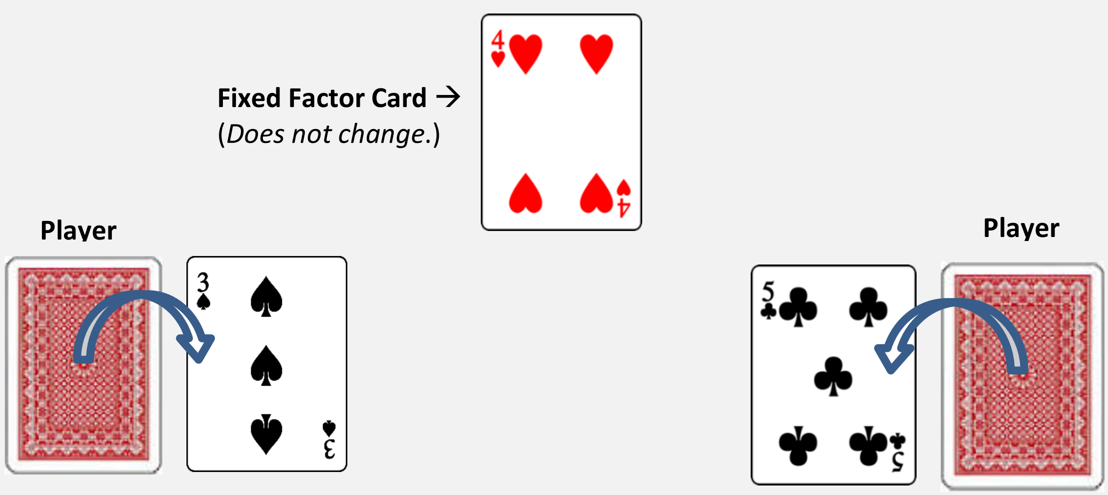
Materials:

- ✓ Deck of cards, with Kings and Jacks removed. Queens = 0; Aces = 1.

How to Play:

1. Find a 4 in the deck and place it between the two players (or 6 or 8) face up. That number is the fixed factor.
2. Deal the rest of the cards equally, face down.
3. Each player takes a turn to flip over the top card of his/her pile of cards. The player must state the product of the "fixed" factor card and the card they flipped, and share how they know (see example below).
4. The player who correctly states the greater product in the round gets both players' cards. (The "middle" fixed factor card stays.)
5. If there is a tie, a "war" is declared, and players repeat the process, with the winner taking all played cards.
6. The player with the most cards wins when time is up.

More ways to play: Use different Fixed Factors (e.g., a 6). Play *Factor War* – No fixed factor, each player draws 2 cards. Play with addition, too! (*Fixed Addend War* or *Addend War*)



Nicolas turns over a 3. He says, "Twelve. I doubled 3 to get 6 and doubled again to get 12."

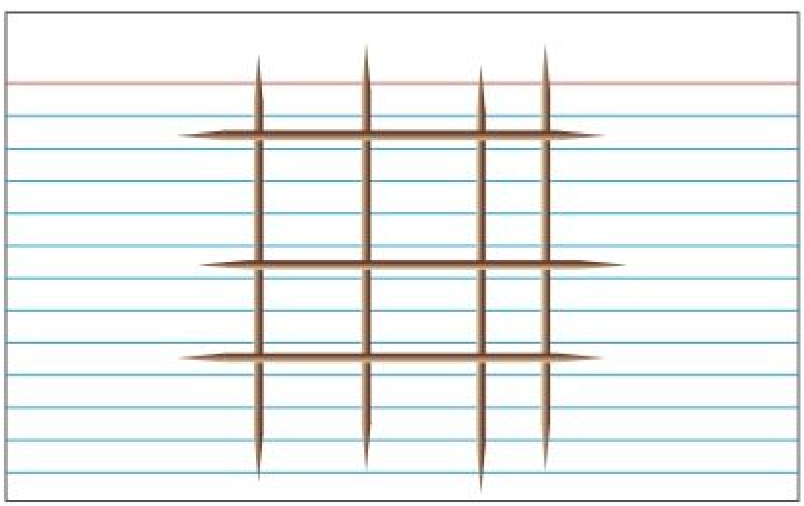
MacKenna turns over a 5. She says, "Twenty. I just know my 5s, 5 times 4 equals 20."

Streets, Avenues and Stoplights

Materials: 6 sided dice, toothpicks

Directions: Student A rolls the dice and places that many toothpicks horizontally. These toothpicks represent the number of streets in the town. Student A rolls the dice again and places that many toothpicks vertically. These toothpicks represent the number of avenues in the town. Student A predicts how many stoplights will be needed for the town (one must go at each intersection of a street and avenue). The player then determines the number of stoplight (points of intersection). The student may partition the array into smaller pieces to figure out the product. The student's partner repeats the process. The student with the larger product is the winner.

Variation: Start with a given number of streets and only roll for the avenues to work on a specific set of facts. Cards can be used instead of dice and the numbers limited (example only use aces-fives). Utilize the attached recording sheet from NCTM.

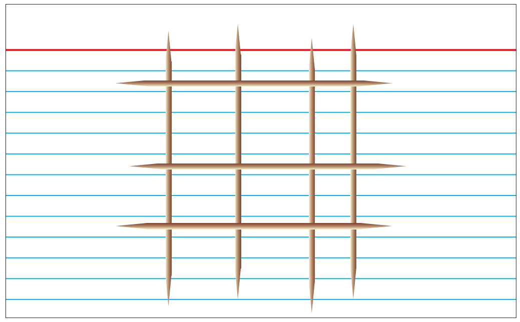


*Adapted from [Modeling Multiplication With Streets and Avenues] from NCTM and Game 34 Crossed Wires Bay-Williams, J., & Kling, G. (2019). *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention*. Alexandria, VA: ASCD.

Number of Streets

NAME ______________________________

Use the table below to record the examples of streets, avenues, and stoplights you created on your index cards.



NUMBER OF STREETS	NUMBER OF AVENUES	NUMBER OF STOPLIGHTS

youcubed at Stanford University

Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts

By Jo Boaler
Professor of Mathematics Education, co-founder youcubed

with the help of Cathy Williams, co-founder youcubed, & Amanda Confer
Stanford University.

Introduction

A few years ago a British politician, Stephen Byers, made a harmless error in an interview. The right honorable minister was asked to give the answer to 7 x 8 and he gave the answer of 54, instead of the correct 56. His error prompted widespread ridicule in the national media, accompanied by calls for a stronger emphasis on 'times table' memorization in schools. This past September the Conservative education minister for England, a man with no education experience, insisted that all students in England memorize all their times tables up to 12 x 12 by the age of 9. This requirement has now been placed into the UK's mathematics curriculum and will result, I predict, in rising levels of math anxiety and students turning away from mathematics in record numbers. The US is moving in the opposite direction, as the new Common Core State Standards (CCSS) de-emphasize the rote memorization of math facts. Unfortunately misinterpretations of the meaning of the word 'fluency' in the CCSS are commonplace and publishers continue to emphasize rote memorization, encouraging the persistence of damaging classroom practices across the United States.

Mathematics facts are important but the memorization of math facts through times table repetition, practice and timed testing is unnecessary and damaging. The English minister's mistake when he was asked 7 x 8 prompted calls for more memorization. This was ironic as his mistake revealed the limitations of memorization without 'number sense'. People with number sense are those who can use numbers flexibly. When asked to solve 7 x 8 someone with number sense may have memorized 56 but they would also be able to work out that 7 x 7 is 49 and then add 7 to make 56, or they may work out ten 7's and subtract two 7's (70-14). They would not have to rely on a distant memory. Math facts, themselves, are a small part of mathematics and they are best learned through the use of numbers in different ways and situations. Unfortunately many classrooms focus on math facts in unproductive ways, giving students the impression that math facts are the essence of mathematics, and, even worse that the fast recall of math facts is what it means to be a strong mathematics student. Both of these ideas are wrong and it is critical that we remove them from classrooms, as they play a large role in the production of math anxious and disaffected students.

It is useful to hold some math facts in memory. I don't stop and think about the answer to 8 plus 4, because I know that math fact. But I learned math facts through using them in different mathematical situations, not by practicing them and being tested on them. I grew up in the progressive era of England, when primary schools focused on the 'whole child' and I was not presented with tables of addition, subtraction or multiplication facts to memorize in school. This has never held me back at any time or place in my life, even though I am a mathematics education professor. That is because I have number sense, something that is much more important for students to learn, and that includes learning of math facts along with deep understanding of numbers and the ways they relate to each other.

Number Sense

In a critical research project researchers studied students as they solved number problems (Gray & Tall, 1994). The students, aged 7 to 13, had been nominated by their teachers as being low, middle or high achieving. The researchers found an important difference between the low and high achieving students - the high achieving students used number sense, the low achieving students did not. The high achievers approached problems such as 19 + 7 by changing the problem into, for example, 20 + 6. No students who had been nominated as low achieving used number sense. When the low achieving students were given subtraction problems such as 21-16 they counted backwards, starting at 21 and counting down, which is extremely difficult to do. The high achieving students used strategies such as changing the numbers into 20 -15 which is much easier to do. The researchers concluded that low achievers are often low achievers not because they know less but because they don't use numbers flexibly – they have been set on the wrong path, often from an early age, of trying to memorize methods instead of interacting with numbers flexibly (Boaler, 2009). This incorrect pathway means that they are often learning a harder mathematics and sadly, they often face a lifetime of mathematics problems.

Number sense is the foundation for all higher-level mathematics (Feikes & Schwingendorf, 2008). When students fail algebra it is often because they don't have number sense. When students work on rich mathematics problems – such as those we provide at the end of this paper – they develop number sense and they also learn and can remember math facts. When students focus on memorizing times tables they often memorize facts without number sense, which means they are very limited in what they can do and are prone to making errors –such as the one that led to nationwide ridicule for the British politician. Lack of number sense has led to more catastrophic errors, such as the Hubble Telescope missing the stars it was intended to photograph in space. The telescope was looking for stars in a certain cluster but failed due to someone making an arithmetic error in the programming of the telescope (LA Times, 1990). Number sense, critically important to students' mathematical development, is inhibited by over-emphasis on the memorization of math facts in classrooms and homes. The more we emphasize memorization to students the less willing they become to think about numbers and their relations and to use and develop number sense (Boaler, 2009).

The Brain and Number Sense

Some students are not as good at memorizing math facts as others. That is something to be celebrated, it is part of the wonderful diversity of life and people. Imagine how dull and uninspiring it would be if teachers gave tests of math facts and everyone answered them in the same way and at the same speed as though they were all robots. In a recent brain study scientists examined students' brains as they were taught to memorize math facts. They saw that some students memorized them much more easily than others. This will be no surprise to readers and many of us would probably assume that those who memorized better were higher achieving or "more intelligent" students. But the researchers found that the students who memorized more easily were not higher achieving, they did not have what the researchers described as more "math ability", nor did they have higher IQ scores (Supekar et al, 2013). The only differences the researchers found were in a brain region called the hippocampus, which is the area of the brain that is responsible for memorized facts (Supekar et al, 2013). Some students will be slower when memorizing but they still have exceptional mathematics potential. Math facts are a very small part of mathematics but unfortunately students who don't memorize math facts well often come to believe that they can never be successful with math and turn away from the subject.

Teachers across the US and the UK ask students to memorize multiplication facts, and sometimes addition and subtraction facts too, usually because curriculum standards have specified that students need to be "fluent with numbers". Parish, drawing from Fosnot and Dolk (2001) defines fluency as 'knowing how a number can be composed and decomposed and using that information to be flexible and efficient with solving problems.' (Parish 2014, p 159). Whether or not we believe that fluency requires more than the recall of math facts, research evidence points in one direction: The best way to develop fluency with numbers is to develop number sense and to work with numbers in different ways, not to blindly memorize without number sense.

When teachers emphasize the memorization of facts, and give tests to measure number facts students suffer in two important ways. For about one third of students the onset of timed testing is the beginning of math anxiety (Boaler, 2014). Sian Beilock and her colleagues have studied people's brains through MRI imaging and found that math facts are held in the working memory section of the brain. But when students are stressed, such as when they are taking math questions under time pressure, the working memory becomes blocked and students cannot access math facts they know (Beilock, 2011; Ramirez, et al, 2013). As students realize they cannot perform well on timed tests they start to develop anxiety and their mathematical confidence erodes. The blocking of the working memory and associated anxiety particularly occurs among higher achieving students and girls. Conservative estimates suggest that at least a third of students experience extreme stress around timed tests, and these are not the students who are of a particular achievement group, or economic background. When we put students through this anxiety provoking experience we lose students from mathematics.

Math anxiety has now been recorded in students as young as 5 years old (Ramirez, et al, 2013) and timed tests are a major cause of this debilitating, often life-long condition. But there is a second equally important reason that timed tests should not be used – they prompt many students to turn away from mathematics. In my classes at Stanford University, I experience many math traumatized undergraduates, even though they are among the highest achieving students in the country. When I ask them what has happened to lead to their math aversion many of the students talk about timed tests in second or third grade as a major turning point for them when they decided that math was not for them. Some of the students, especially women, talk about the need to understand deeply, which is a very worthwhile goal, and being made to feel that deep understanding was not valued or offered when timed tests became a part of math class. They may have been doing other more valuable work in their mathematics classes, focusing on sense making and understanding, but timed tests evoke such strong emotions that students can come to believe that being fast with math facts is the essence of mathematics. This is extremely unfortunate. We see the outcome of the misguided school emphasis on memorization and testing in the numbers dropping out of mathematics and the math crisis we currently face (see youcubed.stanford.edu). When my own daughter started times table memorization and testing at age 5 in England she started to come home and cry about maths. This is not the emotion we want students to associate with mathematics and as long as we keep putting students under pressure to recall facts at speed we will not erase the widespread anxiety and dislike of mathematics that pervades the US and UK (Silva & White, 2013; National Numeracy, 2014).

In recent years brain researchers have found that the students who are most successful with number problems are those who are using different brain pathways – one that is numerical and symbolic and the other that involves more intuitive and spatial reasoning (Park & Brannon, 2013). At the end of this paper we give many activities that encourage visual understanding of number facts, to enable important brain connections. Additionally brain researchers have studied students learning math facts in two ways – through strategies or memorization. They found that the two approaches (strategies or memorization) involve two distinct pathways in the brain and that both pathways are perfectly good for life long use. Importantly the study also found that those who learned through strategies achieved 'superior performance' over those who memorized, they solved problems at the same speed, and showed better transfer to new problems. The brain researchers concluded that automaticity should be reached through understanding of numerical relations, achieved through thinking about number strategies (Delazer et al, 2005).

Why is Mathematics Treated Differently?

In order to learn to be a good English student, to read and understand novels, or poetry, students need to have memorized the meanings of many words. But no English student would say or think that learning about English is about the fast memorization and fast recall of words. This is because we learn words by using them in many different situations – talking, reading, and writing. English teachers do not give students hundreds of words to memorize and then test them under timed conditions. All subjects require the memorization of some facts, but mathematics is the only subject in which teachers believe they should be tested under timed conditions. Why do we treat mathematics in this way?

Mathematics already has a huge image problem. Students rarely cry about other subjects, nor do they believe that other subjects are all about memorization or speed. The use of teaching and parenting practices that emphasize the memorization of math facts is a large part of the reason that students disconnect from math. Many people will argue that math is different from other subjects and it just has to be that way – that math is all about getting correct answers, not interpretation or meaning. This is another misconception. The core of mathematics is reasoning - thinking through why methods make sense and talking about reasons for the use of different methods (Boaler, 2013). Math facts are a small part of mathematics and probably the least interesting part at that. Conrad Wolfram, of Wolfram-Alpha, one of the world's leading mathematics companies, speaks publically about the breadth of mathematics and the need to stop seeing mathematics as calculating. Neither Wolfram nor I are arguing that schools should not teach calculating, but the balance needs to change, and students need to learn calculating through number sense, as well as spend more time on the under-developed but critical parts of mathematics such as problem solving and reasoning.

It is important when teaching students number sense and number facts never to emphasize speed. In fact this is true for all mathematics. There is a common and damaging misconception in mathematics – the idea that strong math students are fast math students. I work with a lot of mathematicians and one thing I notice about them is that they are not particularly fast with numbers, in fact some of them are rather slow. This is not a bad thing, they are slow because they think deeply and carefully about mathematics. Laurent Schwartz, a top mathematician, wrote an autobiography about his school days and how he was made to feel "stupid" because he was one of the slowest math thinkers in his class (Schwartz, 2001). It took him many years of feeling inadequate to come to the conclusion that: 'rapidity doesn't have a precise relation to intelligence. What is important is to deeply understand things and their relations to each other. This is where intelligence lies. The fact of being quick or slow isn't really relevant.' (Schwartz, 2001) Sadly speed and test driven math classrooms lead many students who are slow and deep thinkers, like Schwartz, to believe that they cannot be good at math.

Math 'Fluency' and the Curriculum

In the US the new Common Core curriculum includes 'fluency' as a goal. Fluency comes about when students develop number sense, when they are mathematically confident because they understand numbers. Unfortunately the word fluency is often misinterpreted. 'Engage New York' is a curriculum that is becoming increasingly popular in the US that has incorrectly interpreted fluency in the following ways:

> Fluency: Students are expected to have speed and accuracy with simple calculations; teachers structure class time and/or homework time for students to memorize, through repetition, core functions such as multiplication tables so that they are more able to understand and manipulate more complex functions. (Engage New York)

There are many problems with this directive. Speed and memorization are two directions that we urgently need to move away from, not towards. Just as problematically 'Engage New York' links the memorization of number facts to students' understanding of more complex functions, which is not supported by research evidence. What research tells us is that students understand more complex functions when they have number sense and deep understanding of numerical principles, not blind memorization or fast recall (Boaler, 2009). I am currently working with PISA analysts at the OECD. The PISA team not only issues international mathematics tests every 4 years they collect data on students' mathematical strategies. Their data from 13 million 15-year olds across the world show that the lowest achieving students are those who focus on memorization and who believe that memorizing is important when studying for mathematics (Boaler & Zoido, in press). This idea starts early in classrooms and is one we need to eradicate. The highest achievers in the world are those who focus on big ideas in mathematics, and connections between ideas. Students develop a connected view of mathematics when they work on mathematics conceptually and blind memorization is replaced by sense making.

In the UK directives have similar potential for harm. The new national curriculum states that all students should have 'memorised their multiplication tables up to and including the 12 multiplication table' by the age of 9 and whilst students can memorize multiplication facts to 12 x 12 through rich engaging activities this directive is leading teachers to give multiplication tables to students to memorize and then be tested on. A leading group in the UK, led by children's author and poet Michael Rosen, has formed to highlight the damage of current policies in schools and the numbers of primary age children who now walk to school crying from the stress they are under, caused by over-testing (Garner, The Independent, 2014). Mathematics is the leading cause of students' anxiety and fear and the unnecessary focus on memorized math facts in the early years is one of the main reasons for this.

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Activities to Develop Number Facts and Number Sense

Teachers should help students develop math facts, not by emphasizing facts for the sake of facts or using 'timed tests' but by encouraging students to use, work with and explore numbers. As students work on meaningful number activities they will commit math facts to heart at the same time as understanding numbers and math. They will enjoy and learn important mathematics rather than memorize, dread and fear mathematics.

Number Talks

One of the best methods for teaching number sense and math facts at the same time is a teaching strategy called 'number talks', developed by Ruth Parker and Kathy Richardson. This is an ideal short teaching activity that teachers can start lessons with or parents can do at home. It involves posing an abstract math problem such as 18 x 5 and asking students to solve the problem mentally. The teacher then collects the different methods and looks at why they work. For example a teacher may pose 18 x 5 and find that students solve the problem in these different ways:

20 x 5 = 100 2 x 5 = 10 100 - 10 = 90	10 x 5 = 50 8 x 5 = 40 50 + 40 = 90	18 x 5 = 9 x 10 9 x 10 = 90	18 x 2 = 36 2 x 36 = 72 18 + 72 = 90	9 x 5 = 45 45 x 2 = 90

Students love to give their different strategies and are usually completely engaged and fascinated by the different methods that emerge. Students learn mental math, they have opportunities to memorize math facts and they also develop conceptual understanding of numbers and of the arithmetic properties that are critical to success in algebra and beyond. Parents can use a similar strategy by asking for their children's methods and discussing the different methods that can be used. Two books, one by Cathy Humphreys and Ruth Parker (in press) and another by Sherry Parish (2014) illustrate many different number talks to work on with secondary and elementary students, respectively.

Research tells us that the best mathematics classrooms are those in which students learn number facts and number sense through engaging activities that focus on mathematical understanding rather than rote memorization.
The following five activities have been chosen to illustrate this principle; the appendix to this document provides a greater range of activities and links to other useful resources that will help students develop number sense.

Addition Fact Activities

Snap It: This is an activity that children can work on in groups. Each child makes a train of connecting cubes of a specified number. On the signal "Snap," children break their trains into two parts and hold one hand behind their back. Children take turns going around the circle showing their remaining cubes. The other children work out the full number combination.

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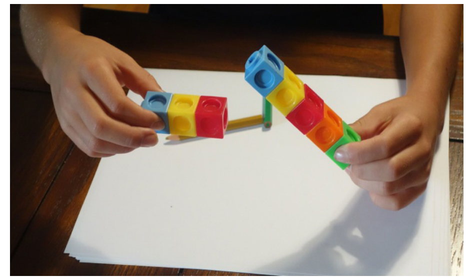
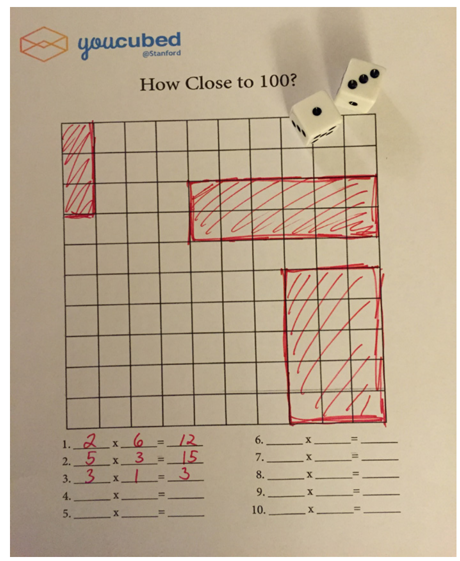


For example, if I have 8 cubes in my number train I could snap it and put 3 behind my back. I would show my group the remaining 5 cubes and they should be able to say that three are missing and that 5 and 3 make 8.

How Many Are Hiding? In this activity each child has the same number of cubes and a cup. They take turns hiding some of their cubes in the cup and showing the leftovers. Other children work out the answer to the question "How many are hiding," and say the full number combination.

Example: I have 10 cubes and I decide to hide 4 in my cup. My group can see that I only have 6 cubes. Students should be able to say that I'm hiding 4 cubes and that 6 and 4 make 10.

Multiplication Fact Activities

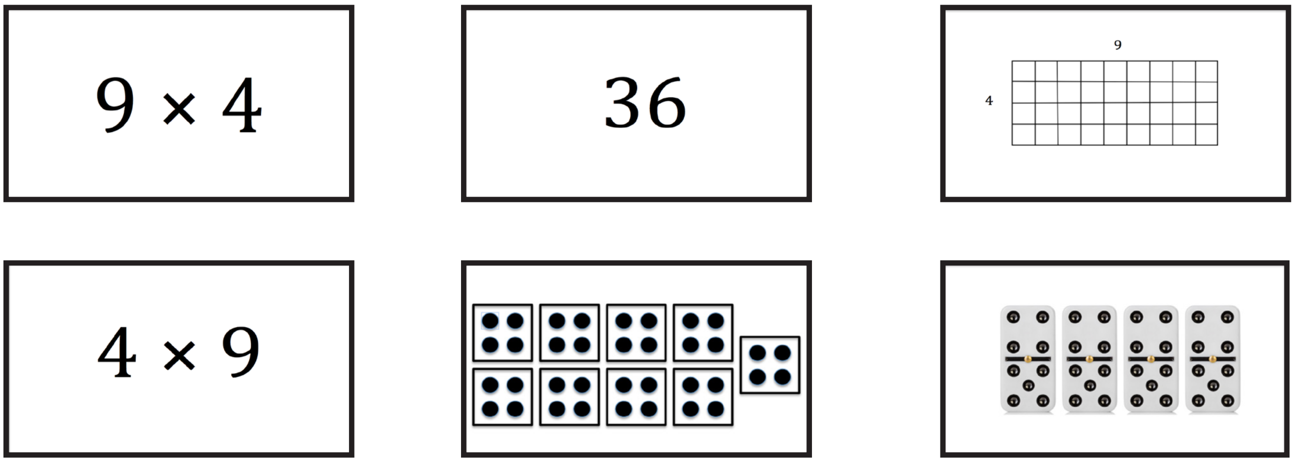


How Close to 100? This game is played in partners. Two children share a blank 100 grid. The first partner rolls two number dice. The numbers that come up are the numbers the child uses to make an array on the 100 grid. They can put the array anywhere on the grid, but the goal is to fill up the grid to get it as full as possible. After the player draws the array on the grid, she writes in the number sentence that describes the grid. The game ends when both players have rolled the dice and cannot put any more arrays on the grid. How close to 100 can you get?

Pepperoni Pizza: In this game, children roll a dice twice. The first roll tells them how many pizzas to draw. The second roll tells them how many pepperonis to put on EACH pizza. Then they write the number sentence that will help them answer the question, "How many pepperonis in all?"
For example, I roll a dice and get 4 so I draw 4 big pizzas. I roll again and I get 3 so I put three pepperonis on each pizza. Then I write 4 x 3 = 12 and that tells me that there are 12 pepperonis in all.

Math Cards

Many parents use 'flash cards' as a way of encouraging the learning of math facts. These usually include 2 unhelpful practices – memorization without understanding and time pressure. In our Math Cards activity we have used the structure of cards, which children like, but we have moved the emphasis to number sense and the understanding of multiplication. The aim of the activity is to match cards with the same numerical answer, shown through different representations. Lay all the cards down on a table and ask children to take turns picking them; pick as many as they find with the same answer (shown through any representation). For example 9 and 4 can be shown with an area model, sets of objects such as dominoes, and the number sentence. When students match the cards they should explain how they know that the different cards are equivalent. This activity encourages an understanding of multiplication as well as rehearsal of math facts. A full set of cards is given in Appendix A.



Conclusion: Knowledge is Power

The activities given above are illustrations of games and tasks in which students learn math facts at the same time as working on something they enjoy, rather than something they fear. The different activities also focus on the understanding of addition and multiplication, rather than blind memorization and this is critically important. Appendix A presents other suggested activities and references.

As educators we all share the goal of encouraging powerful mathematics learners who think carefully about mathematics as well as use numbers with fluency. But teachers and curriculum writers are often unable to access important research and this has meant that unproductive and counter-productive classroom practices continue. This short paper illustrates both the damage that is caused by the practices that often accompany the teaching of math facts – speed pressure, timed testing and blind memorization – as well as summarizes the research evidence of something very different – number sense. High achieving students use number sense and it is critical that lower achieving students, instead of working on drill and memorization, also learn to use numbers flexibly and conceptually. Memorization and timed testing stand in the way of number sense, giving students the impression that sense making is not important. We need to urgently reorient our teaching of early number and number sense in our mathematics teaching in the UK and the US. If we do not, then failure and drop out rates - already at record highs in both countries (National Numeracy, 2014; Silva & White, 2013) - will escalate. When we emphasize memorization and testing in the name of fluency we are harming children, we are risking the future of our ever-quantitative society and we are threatening the discipline of mathematics. We have the research knowledge we need to change this and to enable all children to be powerful mathematics learners. Now is the time to use it.

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Appendix A: Further Activities and Resources

Table of Contents

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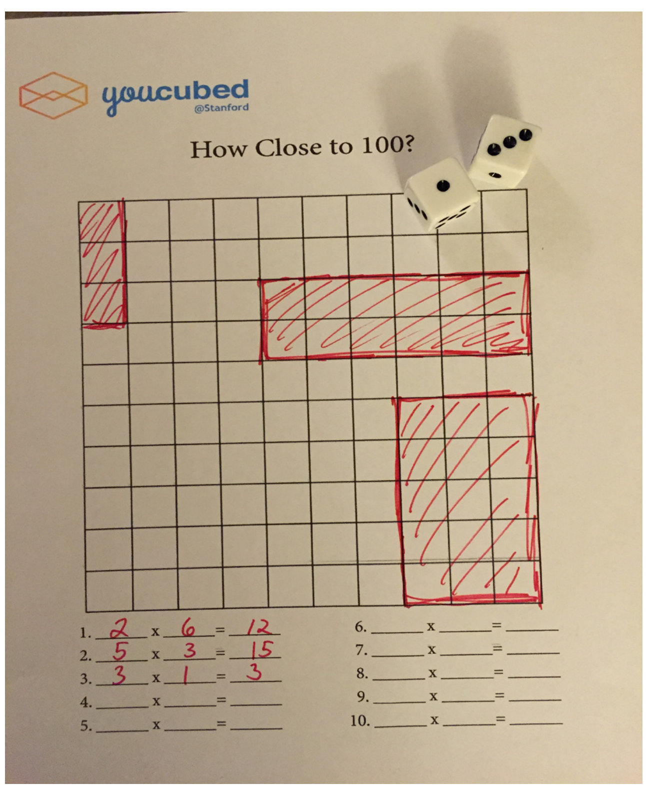
How Close to 100?

You need
- two players
- two dice
- recording sheet (see next page)

This game is played in partners. Two children share a blank 100 grid. The first partner rolls two number dice. The numbers that come up are the numbers the child uses to make an array on the 100 grid. They can put the array anywhere on the grid, but the goal is to fill up the grid to get it as full as possible. After the player draws the array on the grid, she writes in the number sentence that describes the grid. The second player then rolls the dice, draws the number grid and records their number sentence. The game ends when both players have rolled the dice and cannot put any more arrays on the grid. How close to 100 can you get?

Variation
Each child can have their own number grid. Play moves forward to see who can get closest to 100.



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How Close to 100?

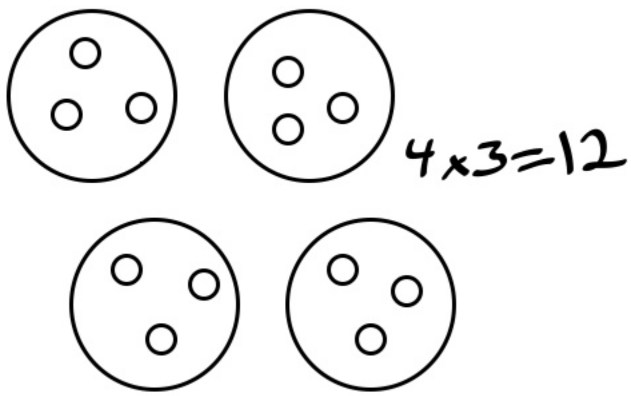
1. _______ x _______= _______
2. _______ x _______= _______
3. _______ x _______= _______
4. _______ x _______= _______
5. _______ x _______= _______
6. _______ x _______= _______
7. _______ x _______= _______
8. _______ x _______= _______
9. _______ x _______= _______
10. _______ x _______= _______

Pepperoni Pizza

You will need
- one or more players
- 2 dice per player
- 10 or more snap cubes per player

In this game, children roll a dice twice. The first roll tells them how many pizzas to draw. The second roll tells them how many pepperonis to put on EACH pizza. Then they write the number sentence that will help them answer the question, "How many pepperonis in all?"

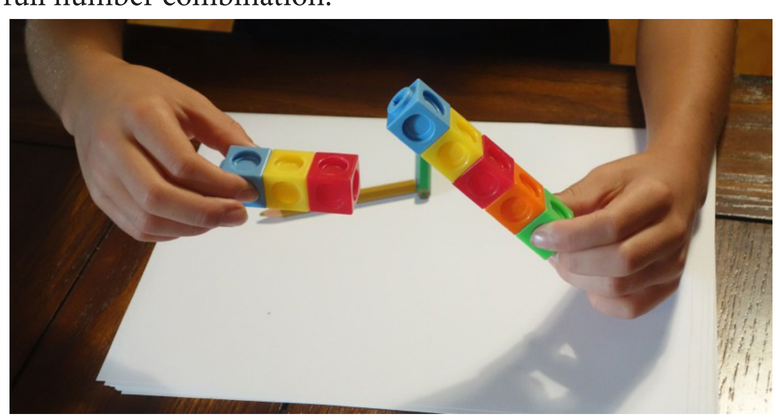
For example, I roll a dice and get 4 so I draw 4 big pizzas. I roll again and I get 3 so I put three pepperonis on each pizza. Then I write 4 x 3 = 12 and that tells me that there are 12 pepperonis in all.



Snap It

You will need
- one or more players
- 10 or more snap cubes per player

This is an activity that children can work on in groups. Each child makes a train of connecting cubes of a specified number. On the signal "Snap," children break their trains into two parts and hold one hand behind their back. Children take turns going around the circle showing their remaining cubes. The other children work out the full number combination.



How Many Are Hiding

You will need
- one or more players
- 10 or more snap cubes /objects per player
- a cup for each player

In this activity each child has the same number of cubes and a cup. They take turns hiding some of their cubes in the cup and showing the leftovers. Other children work out the answer to the question "How many are hiding," and say the full number combination.

Example: I have 10 cubes and I decide to hide 4 in my cup. My group can see that I only have 6 cubes. Students should be able to say that I'm hiding 4 cubes and that 6 and 4 make 10.

Shut the Box

You will need
- one or more players
- 2 dice
- paper and pencil

Write the numbers 1 through 9 in a horizontal row on the paper. Player 1 rolls the dice and calculates the sum of the two numbers. Player 1 then chooses to cross out numbers that have the same sum as what was calculated from the dice roll. If the numbers 7, 8 and 9 are all covered, player 1 may choose to roll one or two dice. If any of these numbers are still uncovered, the player must use both dice. Player 1 continues rolling dice, calculating the sum and crossing out numbers until they can no longer continue. If all numbers are crossed out the player say's "shut the box". If not all numbers are crossed out player 1 determines the sum of the numbers that are not crossed out and that is their score. If "shut the box" is achieved, player 1 records a score of "0".

Player two writes the numbers 1 through 9 and follows the same rules as player 1. The player with the lowest score wins.

Variation
Player 1 and 2 can choose to play 5 rounds, totaling their score at the end of each round. The player with the lowest total score wins the game.

Math Cards

You will need
- one or more players
- 1 deck of cards (see next pages)

Many parents use ‘flash cards’ as a way of encouraging the learning of math facts. These usually include 2 unhelpful practices – memorization without understanding and time pressure. In our Math Cards activity we have used the structure of cards, which children like, but we have moved the emphasis to number sense and the understanding of multiplication. The aim of the activity is to match cards with the same numerical answer, shown through different representations. Lay all the cards down on a table and ask children to take turns picking them; pick as many as they find with the same answer (shown through any representation). For example 9 and 4 can be shown with an area model, sets of objects such as dominoes, and the number sentence. When students match the cards they should explain how they know that the different cards are equivalent. This activity encourages an understanding of multiplication as well as rehearsal of math facts.

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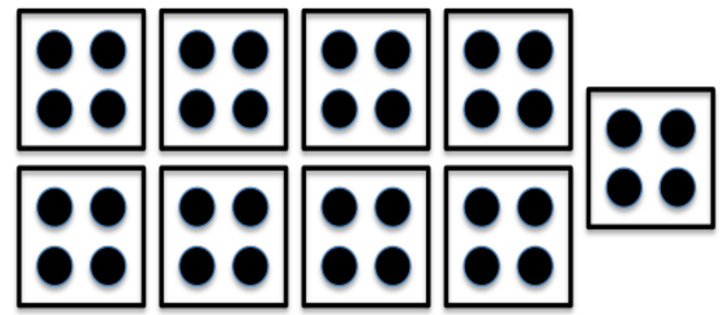

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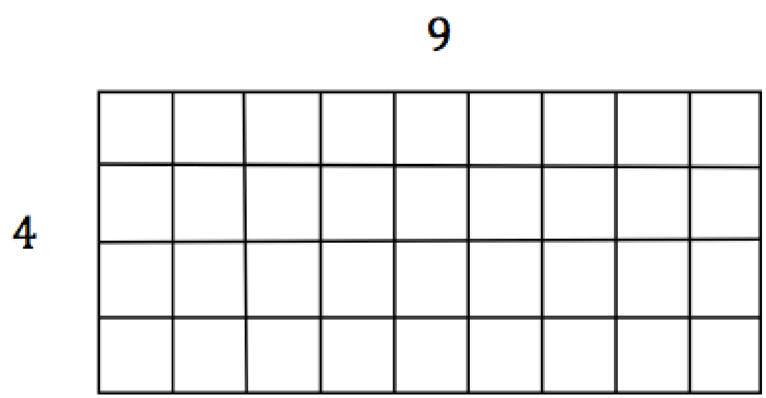
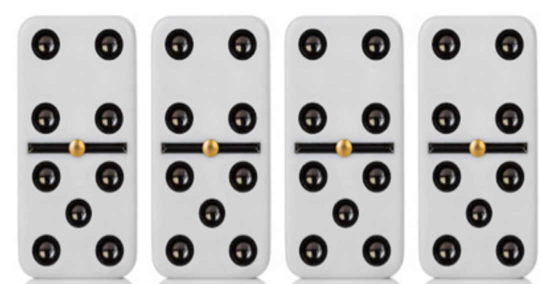
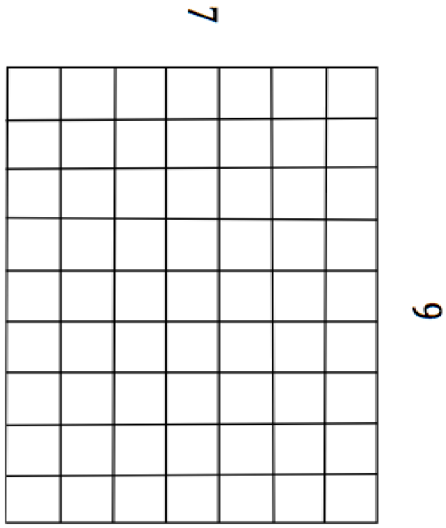
7×9

9×7







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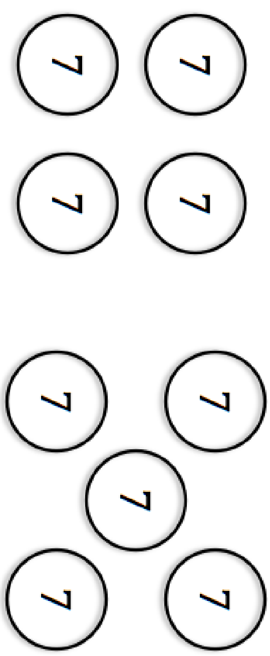

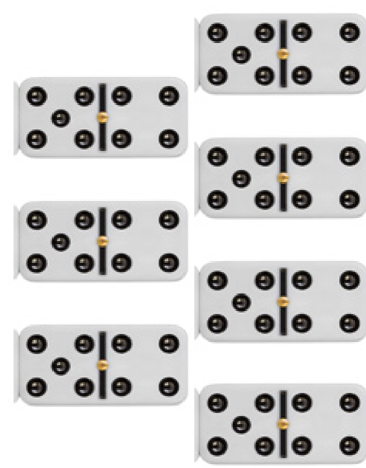

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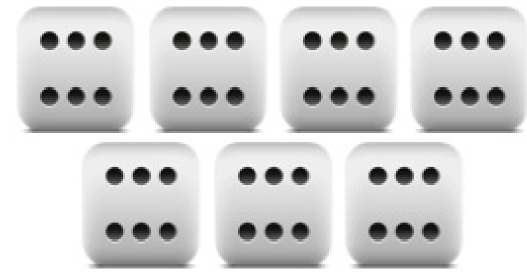

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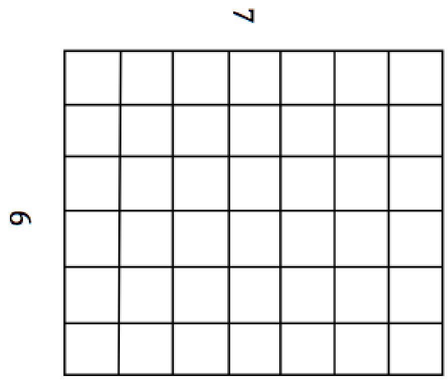

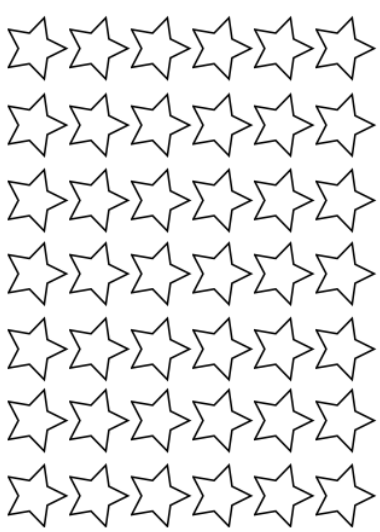

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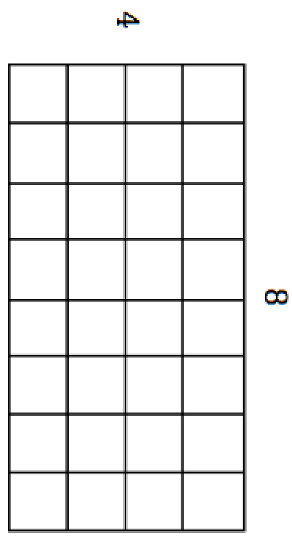

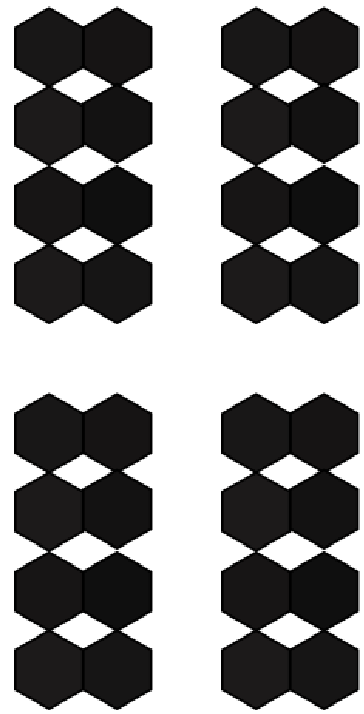

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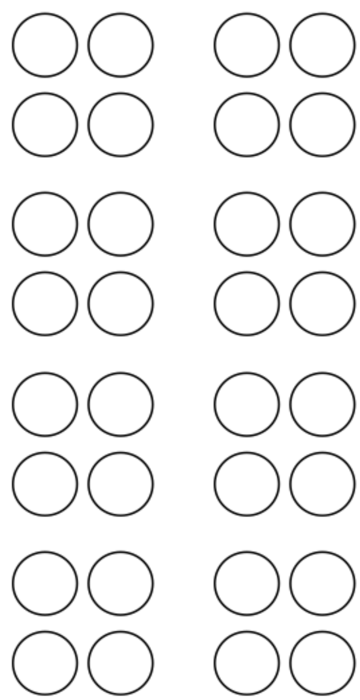

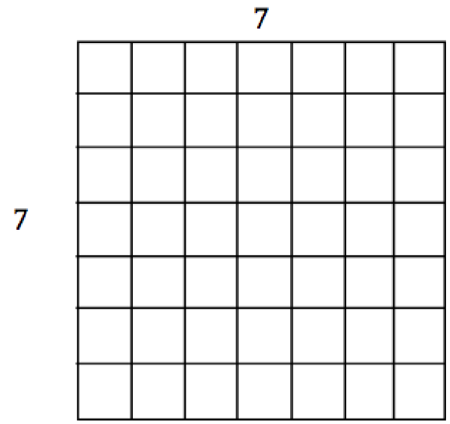

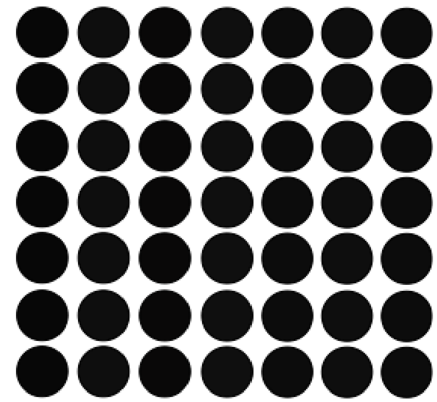

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49

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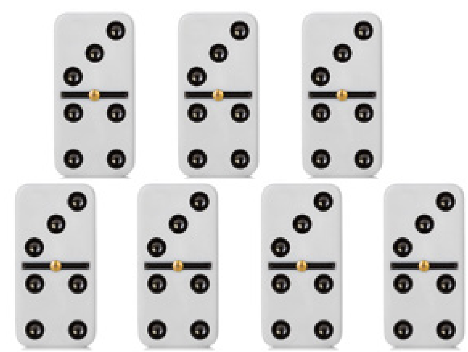
7^2	

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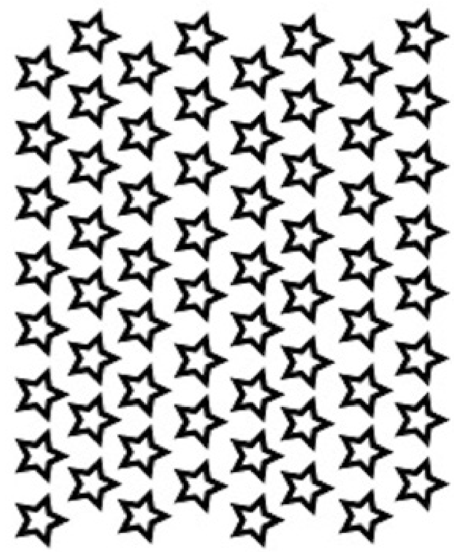

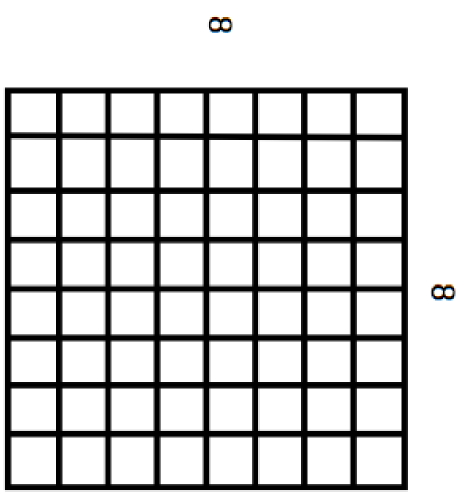

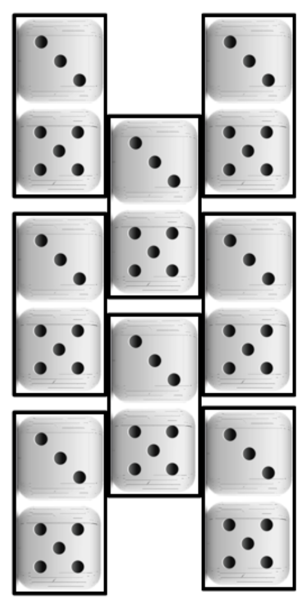

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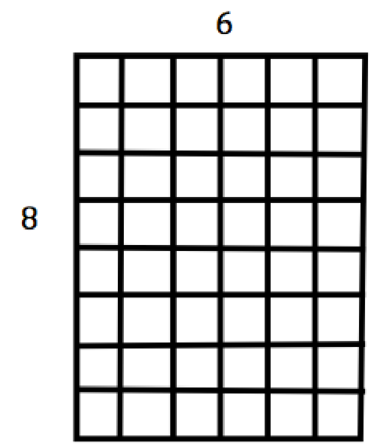

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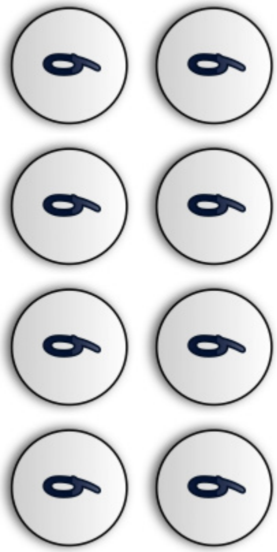

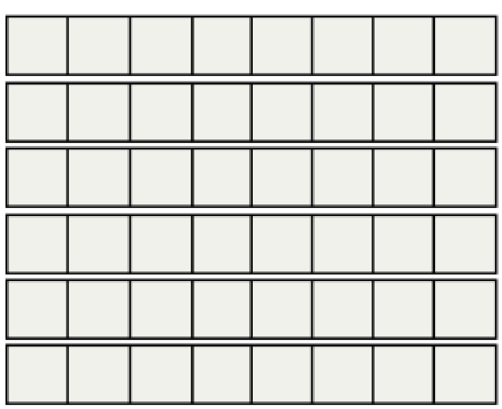

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Books:

By Jo Boaler

Boaler, J. (2015). What's Math Got To Do With It? How Teachers and Parents Can Transform Mathematics Learning and Inspire Success. New York: Penguin.

By Jo Boaler and Cathy Humphreys

Boaler, J., & Humphreys, C. (2005). Connecting Mathematical ideas: Middle school video cases to support teaching and learning. Portsmouth, NH: Heinemann.

Math Solutions - http://mathsolutions.com/

Math Solutions is a publishing company that has a range of excellent books to help parents and teachers with number sense

for example:

Burns, Marilyn (2007), About Teaching Mathematics: A K–8 Resource, Third Edition

By Sherry Parrish

Parish, S. (2014). Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K-5, Updated with Common Core Connections. Math Solutions.

By Kathy Richardson

Richardson, K. (1998). Developing Number Concepts: Counting, Comparing, and Pattern. Dale Seymour Publications
Richardson, K. (1998). Developing Number Concepts: Addition and Subtraction Dale Seymour Publications
Richardson, K. (1998). Developing Number Concepts: Place Value, Multiplication and Division. Dale Seymour Publications
Dale Seymour Publications. Understanding Geometry (1999) Lummi Bay Publishing

By Cathy Fosnot and Maarten Dolk

Fosnot, C., Dolk, M. (2001). Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction: Heinemann
Fosnot, C., Dolk, M. (2001). Young Mathematicians at Work: Constructing Multiplication and Division:

Heinemann
Fosnot, C., Dolk, M. (2001). Young Mathematicians at Work: Constructing Fraction, Decimals and Percent (2002: Heinemann

By John Van De Walle and Lou Ann Lovin

Van de Walle, J. , Lovin, L.A. (2006). Teaching Student Centered Mathematics, grades K – 3: Pearson
Van de Walle, J. , Lovin, L.A. (2006). Teaching Student Centered Mathematics, grades 5 – 8: Pearson

By Heibert, Carpenter, Fennema, Fuson, Wearne and Murray

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H. (1997). Making Sense: teaching and learning mathematics with understanding. Portsmouth, NH: Heinemann.

Additional Games:

Set	http://www.setgame.com/set
Muggins!	http://www.mugginsmath.com/store.asp
Mancala	

Games & Apps:

Mathbreakers	https://www.mathbreakers.com
Motion Math	http://motionmathgames.com/
Dragon Box	http://www.dragonboxapp.com/
Refraction	http://play.centerforgamescience.org/refraction/site/
Wuzzit Trouble	http://innertubegames.net
Mancala	http://www.coolmath-games.com/0-mancala/

Multiplication Version 1

(No Distractor Cards)

Grade 3

Formative Assessment Lesson

Designed and revised by the Kentucky Department of Education
Field-tested by Kentucky Mathematics Leadership Network Teachers

If you encounter errors or other issues with this file, please contact the KDE math team at: kdemath@education.ky.gov

Revised 2019

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should be used to inform the instruction that will take place for the remainder of your unit.

Mathematical Goals

This lesson is intended to evaluate the understanding of your students in multiplication and how well students are able to interpret various representations of multiplication facts. It will help you to identify students who have the following difficulties:

- lack of conceptual understanding of the properties of numbers
- do not see connections from addition to multiplication
- have multiplication facts memorized, but lack conceptual understanding

Kentucky Academic Standards

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

Grade 3 Operations and Algebraic Thinking
Cluster: Represent and solve problems involving multiplication and division.
Cluster: Multiply and divide within 100.

This lesson involves a range of *Standards for Mathematical Practices,* with emphasis on:

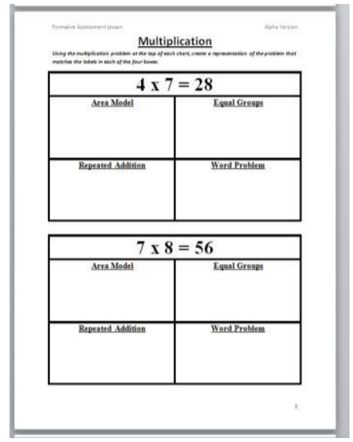
MP1. Make sense of problems and persevere in solving them.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Introduction

This lesson is structured in the following way:

- A day or two before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work and create questions for students to answer in order to improve their solutions.
- A whole class introduction provides students with guidance on how to engage with the content of the task.
- Students work with a partner on a collaborative discussion task to match representations of multiplication problems. As they do this, they interpret the cards' meanings and begin to link them together. Throughout their work, students justify and explain their decisions to their peers and teacher(s).
- In a final whole class discussion, students synthesize and reflect on the learning to make connections within the content of the lesson.
- Finally, students revisit their original work or a similar task, and try to improve their individual responses.

Materials required



Multiplication

4 x 7 = 28	
Area Model	Equal Groups
Repeated Addition	Word Problem

7 x 8 = 56	
Area Model	Equal Groups
Repeated Addition	Word Problem

Each individual student will need:

- Two copies of the assessment task *Multiplication.* (One for the initial task and one for the revision.)

Each student will need the following resources:

- Card Set A
- Have Card Set B ready, but do not pass out.
- Have Card Set C ready, but do not pass out.
- Have Card Set D ready, but do not pass out.
- Have Card Set E ready, but do not pass out.
- Have Card Set F ready, but do not pass out.
- Have Card Set G ready, but do not pass out.

Time needed

Approximately 15 minutes a day or two before the lesson (for the individual assessment task), one 40 minute lesson (30 minutes for group task and 10 minutes for whole class discussion), and 15 minutes for a follow-up lesson (for students to revisit individual assessment task). Timings given are only approximate. Exact timings will depend on the needs of the class.

Before the Lesson

Assessment task: Multiplication (15 minutes)

Students should have already been exposed to lessons that required them to build, draw, and write story problems for multiplication. Work should be focused around repeated addition, area models, use of arrays, equal groups, distributive property and story problems.

Have students do this task individually in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with conceptual understanding of multiplication. You will be able to target your help more effectively in the follow-up lesson. Depending on your class you can have them do it all at once or in small groups (they should still work individually).

Give each student a copy of the assessment task *Multiplication.*

Teacher says: *Using the multiplication problem at the top of each chart, create a model/representation of the problem that matches the labels in each of the four boxes.*

It is important that the students are allowed to answer the questions without your assistance.

Students should not worry too much if they do not understand or cannot do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

Assessing students' responses

Collect students' responses to the task. Make some notes about what their work reveals about their current levels of understanding, and their different approaches.

We suggest that you do not score student's work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some questions on the following page may serve as examples. These questions have been drawn from commonly identified student misconceptions.

We suggest that you write a list of your own questions, based on your students' work, using the ideas that follow. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the end of the lesson before students revisit initial task.

Below is a list of common issues and questions/prompts that may be written on individual initial tasks or during the collaborative activity to help students clarify and extend their thinking.

Common issues:	**Suggested questions and prompts:**
A group has trouble getting started.	• What information do you already know? • What do you need to find out?
Students have trouble drawing correct representations. Students may draw 3 groups of 6 instead of drawing 6 groups of 3.	• Use an example of cards that have already been matched and discuss. For example, the 3 groups of 4 card. Talk about how the picture and context of the problem would change if the card was 4 groups of 3.
Students are not attentive to the orientation of the array models.	• How is this model different if we turn it on its side? • Is the orientation of the model important?
Students misinterpret correct representation of each multiple in the problem. Although arranging the problem either way gives you the same answer, mathematically, it does not represent the same image.	• How does the order of the factors affect the model? • Can you think of a situation that changing the order of the problem would not work? • For example, if I have a party and I prepare 3 bags with 9 cookies in each bag. I will have 9 people at the party and each should get 3 cookies. Did the order matter?
Students are not attentive to details and structure of the word problems.	• Provide manipulatives for students. • Can you draw a model? • Can you act out what the problem is telling us?
	•

Suggested lesson outline

Whole Class Introduction (10 minutes)

Materials- whiteboard, dry erase markers, and an eraser

Teacher says: *Today we are continuing our work with multiplication. I want you to use the next three minutes to brainstorm models for this multiplication problem. Draw your models on your whiteboard.*

Display the problem 6 x 4 = 24. Set the timer.

Draw models for the following problem:

6 X 4 = 24

Ready-Set-Think Start the timer.
Walk around and monitor students as they are working.
When the timer sounds, ask students to hold their boards up.
Ask students to share their models with a shoulder partner.
Point out student models and call on a few students to explain their model. Briefly compare different and similar models.

Collaborative Activity (30 minutes)

Strategically partner students based on pre assessment data. Partner students with others who display similar errors/misconceptions on the preassessment task. While this may seem counterintuitive, this will allow each student to more confidently share their thinking. This may result in partnering students who were very successful together, those who did fairly well together, and those who did not do very well together.

Introduce the lesson carefully.

Teacher says: *I want you to work together with your partner. You are going to work on matching cards that represent the same math idea.*

Have cards ready to show the students a model of two cards that would match and two cards that would not match. Have a student tell why the cards do/do not match.

Every pair of students may not work through all card sets. Partners should work at their own pace.

During the partner work, the teachers' tasks are to:

- question but not interfere with student work;
- make notes of student approaches to the task;
- support student problem solving through guided questions.

Make notes of student approaches to the task

You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. For example, students may not consider or understand the concept of multiplication. Students may be able to recall the facts, but have difficulty attaching the meaning to a model. This lack of understanding prevents students from applying multiplication in real life situations.

Support student problem solving

Try not to make suggestions that move students toward a particular approach to the task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

When a student creates a match, challenge their partner to provide an explanation.

If you find students have difficulty articulating their decisions, then you may want to use the questions from the *Common Issues* table to support your questioning.

If the whole class is struggling on the same issue, then you may want to write a couple of questions on the board and organize a brief whole class discussion.

As you monitor the work, listen to the discussion and help students to look for patterns and generalizations.

Card Set A

*Note: There will be one expression that does not have a match. However, there is a ***blank card*** provided for the students to draw their own representation to complete the set.

Card Set B

As students finish with Card Set A and are able to explain their reasoning give them Card Set B. Do not take up the previous sets of cards. Students may use these for guidance in making further decisions. Card Set B will consist of area model representations.

Card Set C

As students finish with Activity B and are able to explain their reasoning give them Card Set C. Card Set C will consist of repeated addition representations. *Note: There will be one blank card for students to fill in to complete this level. Do not tell students; allow them to develop this idea on their own.

Card Set D

As students finish with Activity C and are able to explain their reasoning give them Card Set D.

Card Set D is the “sets of” cards. *Note: There will be one blank card in this set in which the students will need to fill in to complete the set. Do not tell students; allow them to develop this idea on their own.

Card Set E

As students finish with Card Set D and are able to explain their reasoning give them Card Set E. Card Set E will consist of interpreting word problems.

Card Set F

As students finish with Card Set E and are able to explain their reasoning give them Card Set F. This is an opportunity for students to stretch their thinking of how the multiplication problem can also be represented through the distributive property of multiplication.

Card Set G

As students finish with Card Set F and are able to explain their reasoning give them Card Set G. Card Set G will consist of finding the product.

Sharing Work (10 minutes)

When students get as far as they can with the task, ask one student from each group to visit another pair's work. Students remaining at their desk should explain their reasoning for the way they worked the problem at their own desk.

> **Teacher says:** *If you are staying at your desk, be ready to explain the reasons for your pair's work. If you are visiting another pair, check to see which answers or explanations are different from your own. If their matches are different than your groups, ask for an explanation. If you still don't agree, explain your own thinking. When you return to your own desk, you need to consider, as a pair, whether to make any changes to your work.*

Provide time for groups to discuss and make changes to their original work.

Extension activities

Students may create their own set of cards with different multiplication facts.
If time allows, they may challenge each other to sort their sets.

Whole-class discussion (10 minutes)

Conduct a whole-class discussion about what has been learned and highlight misconceptions and strategies you want to be revealed. Select students or pairs who demonstrated strategies and misconceptions you want to share with the class. Be intentional about the order of student sharing from least complex to most complex thinking. As each pair shares, highlight the connections between strategies.

Choose students to share how they figured out which model matched the expressions. Discuss which model they liked best and why.

Possible questions to ask:

- *Which cards were easiest/hardest to match? Why?*
- *What might be a different way to explain?*
- *Did anyone do the same or something different?*
- *How would you explain in words your model?*

Conclude the lesson by discussing and generalizing what has been learned. The generalization involves extending what has been learned to new examples.

Improving individual solutions to the assessment task (10 minutes)

Return to the students their original assessment, *Multiplication*, as well as a second blank copy of the task.

> **Teacher says:** *Look at your original responses and think about what you have learned during this lesson. Using what you have learned, try to improve your work.*

If you (teacher) have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only the questions appropriate to their own work.

*Card sorting tasks adapted from *www.makingmathmagic.com*

This lesson format was designed from the Classroom Challenge Lessons intended for students in grades 6 through 12 from the Math Assessment Project.

Name ____________________ Student Materials

Multiplication

Using the multiplication problem at the top of each chart, create a representation of the problem that matches the labels in each of the four boxes.

4 x 7 = 28	
Area Model	**Equal Groups**
Repeated Addition	**Word Problem**

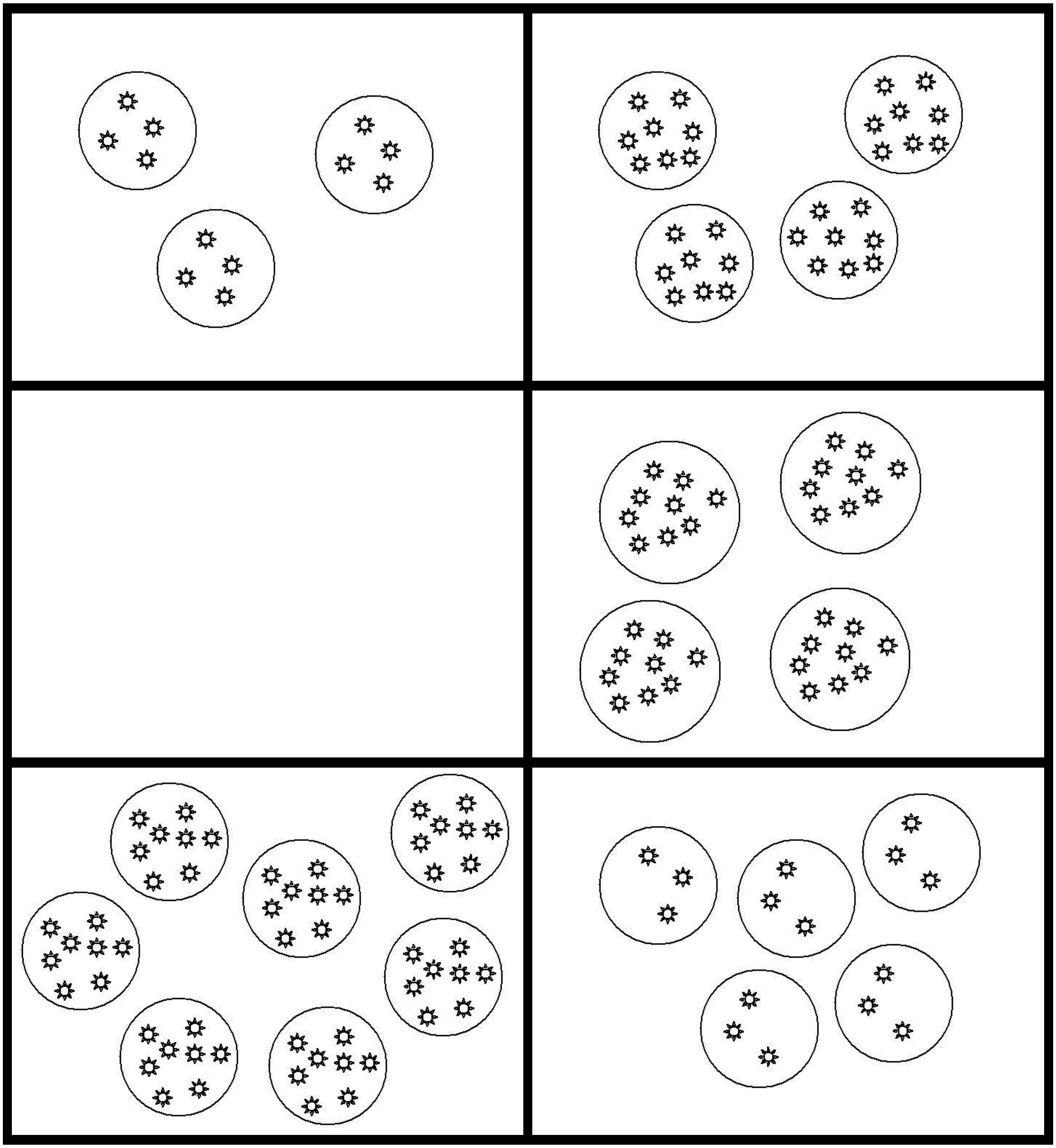
7 x 8 = 56

Area Model	**Equal Groups**
Repeated Addition	**Word Problem**

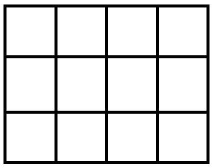
Card Set A

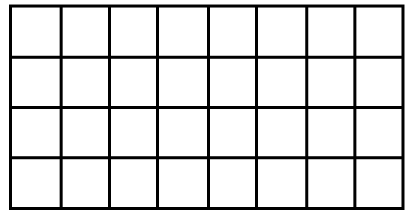
3 x 4	4 x 8
6 x 3	4 x 9
7 x 8	5 x 3

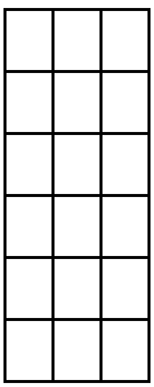
Card Set A

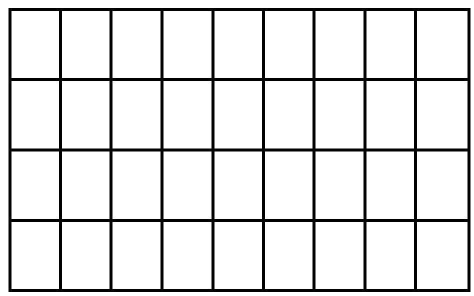


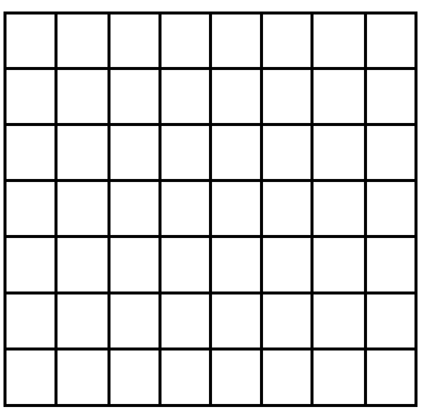
Card Set B

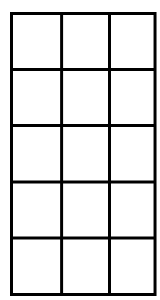












Card Set C

4 + 4 + 4	8 + 8 + 8 + 8
3 + 3 + 3 + 3 + 3 + 3	
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Card Set D

3 sets of 4	
6 sets of 3	4 sets of 9
7 sets of 8	5 sets of 3

Card Set E

Connlee walked for 3 hours. He walked at the rate of 4 miles per hour. How many miles did Connlee walk?	**Sarah has 8 bracelets. Madison has 4 times as many bracelets as Sarah. How many bracelets does Madison have?**
Noah made 6 shots in his basketball game. Each shot was worth 3 points. How many points did Noah score?	
Joanna planted 7 rows of trees. She planted 8 trees in each row. How many trees did Joanna plant?	**Katrina has 3 oranges. Renee' has five times as many oranges as Katrina. How many oranges does Renee' have?**

Card Set F

	(2 x 8) + (2 x 8)
(2 x 3) + (4 x 3)	(2 x 9) + (2 x 9)
(5 x 8) + (2 x 8)	(2 x 3) + (3 x 3)

Card Set G

12	32
18	36
56	15

Lesson Plan

Teacher:	Class/Group:	Date:

KNPIG ID #: M 4448.4 (Tiling with Number)	**Task Group Name:** Tiling Rectangles	
AVMR Strand: Multiplication & Division	**AVMR Construct Level/Color:** 3 to 4 Purple	

Fluency Benchmark for RTI:
3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 / 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products.

KAS(s):	KAS Domain and Cluster:
1) 3.OA.5 Apply properties of operations as strategies to multiply and divide. Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56. (Distributive property.) **2)** 3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 / 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **3)** 3.MD.7 Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with wholenumber side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a x b and a x c. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	Operations and Algebraic Thinking , **1)** Understand properties of multiplication and the relationship between multiplication and division. **2)** Multiply and divide within 100. **3)** Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Learning Target: I can combine two rectangles to make a larger rectangle and find the total tiles needed to cover each part and the larger rectangle.

Setting/Materials:
Rectangle Cards, 1 recording sheet per player

Activity:
Shuffle rectangles and create a face down pile. On a player's turn, the player will draw 1 rectangle and lay it face down in front of him or her. If possible, the player will pair exactly two of his/her rectangles to make a larger rectangle and record the information on a recording sheet. If no larger rectangle can be made, the player must pass. Play ends when all rectangles have been drawn. The player who has created the most larger rectangles wins the game.

Evidence of Learning (Diagnostic Assessment of Progress):
Ask student to find two rectangles that can be paired to make a larger rectangle. Ask student to determine the area of the larger rectangle and explain how that relates to the area of the smaller rectangles.

Teacher Notes:
This activity is meant to support students in developing an understanding of the distributive property and relationships between addition and multiplication. Through discussion, bring out the idea that the areas of the smaller rectangles are added together to find the whole. If students are struggling to "see" the number of tiles needed to cover an individual rectangle, either move to a lower level activity in this task group or use the printable files from a lower level.

Printables Link:
http://knp.kentuckymathematics.org/knp/uploads/printables_4448.4M.pdf

Student Instructions Link:

Printables for “Tiling with Number”

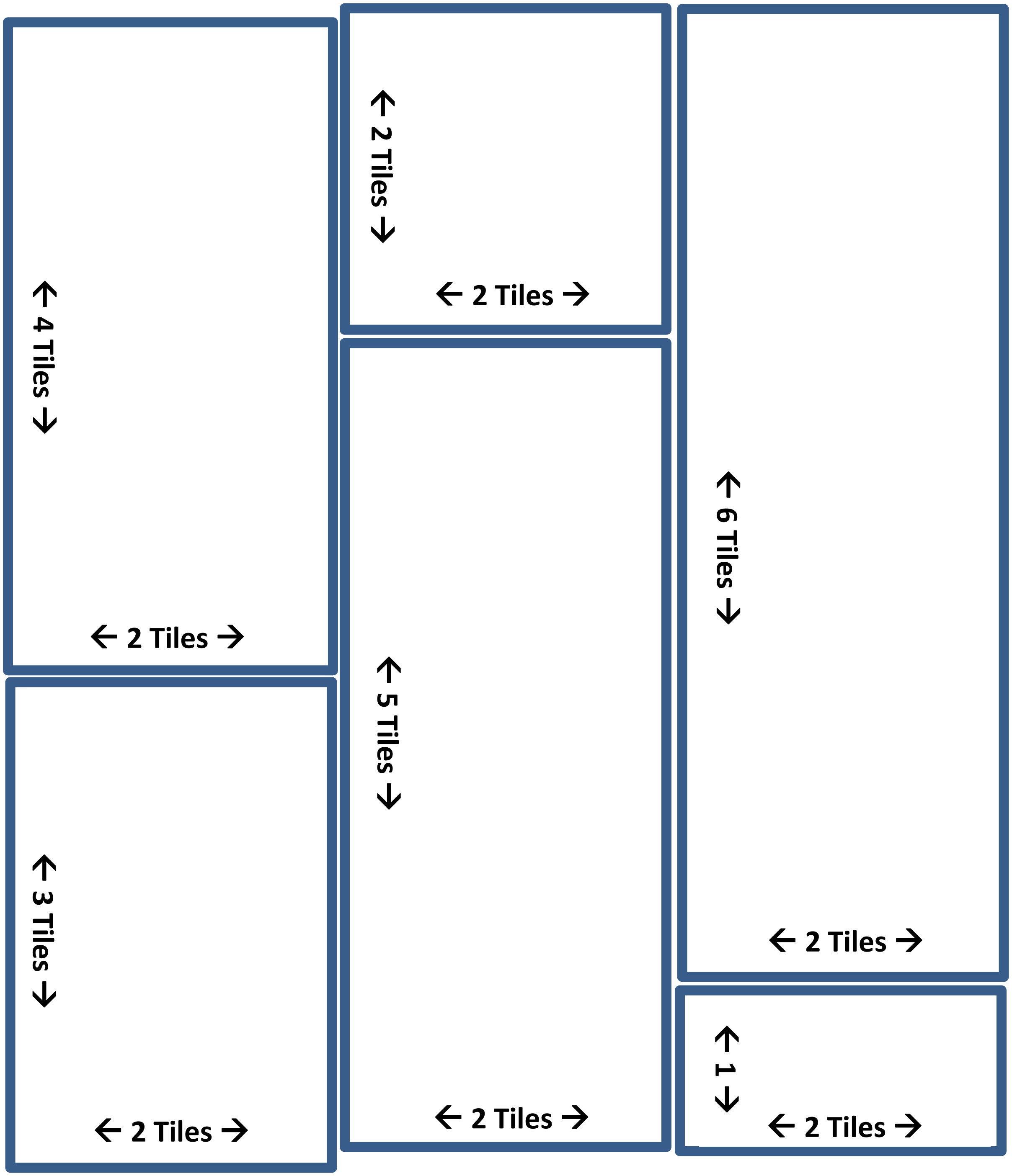
KNPIG # M 4448.4 – PURPLE

This file contains printables for two students.

For each additional pair of students print rectangle cards and one recording sheet per student.

- Rectangle Cards – 22 Pages
- Recording sheet

Teacher Note: This activity is meant to support students in developing an understanding of the distributive property and relationships between addition and multiplication. Through discussion, bring out the idea that the areas of the smaller rectangles are added together to find the whole. If students are struggling to "see" the number of tiles needed to cover an individual rectangle, either move to a lower level activity in this task group or use the printable files from a lower level.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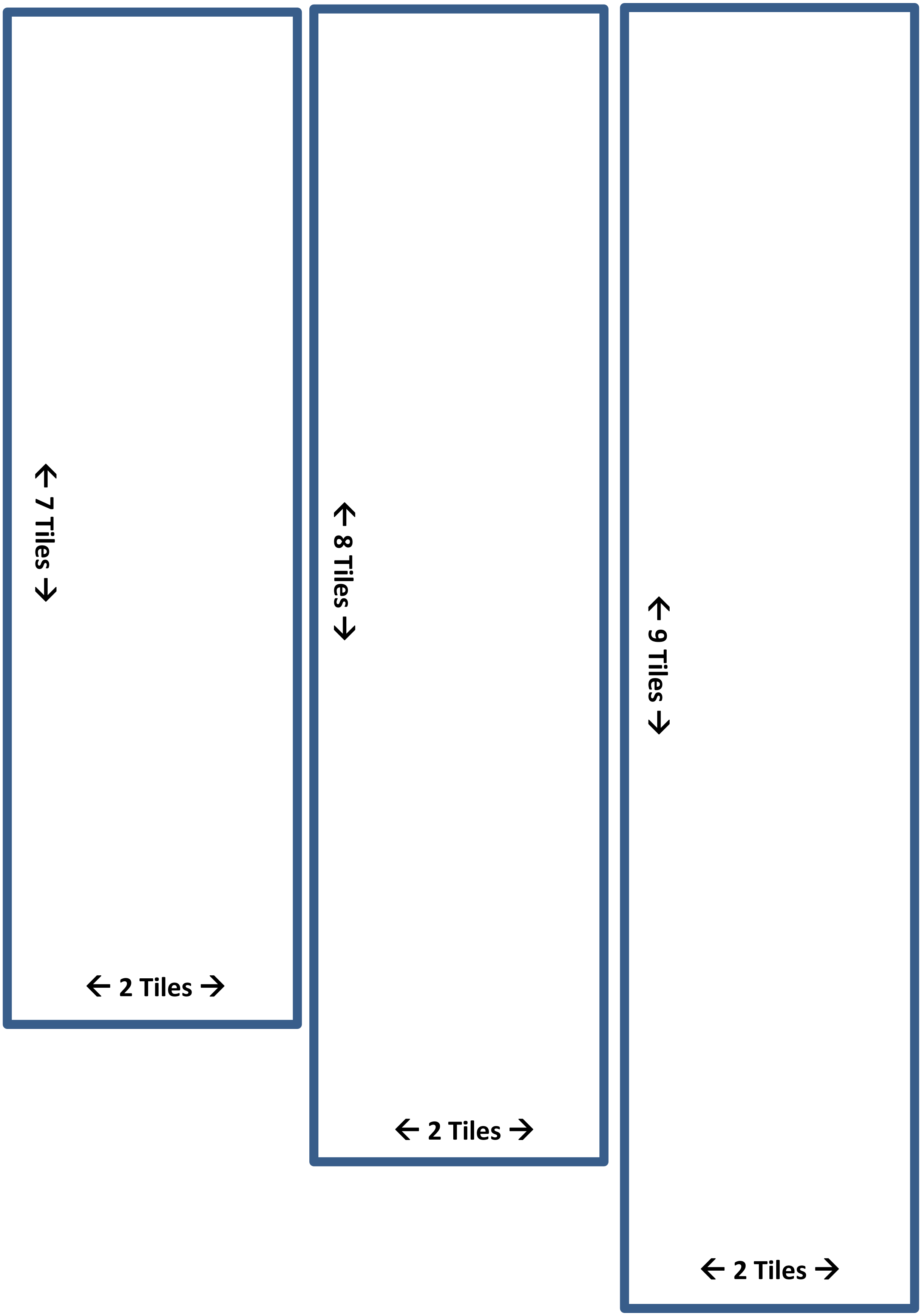
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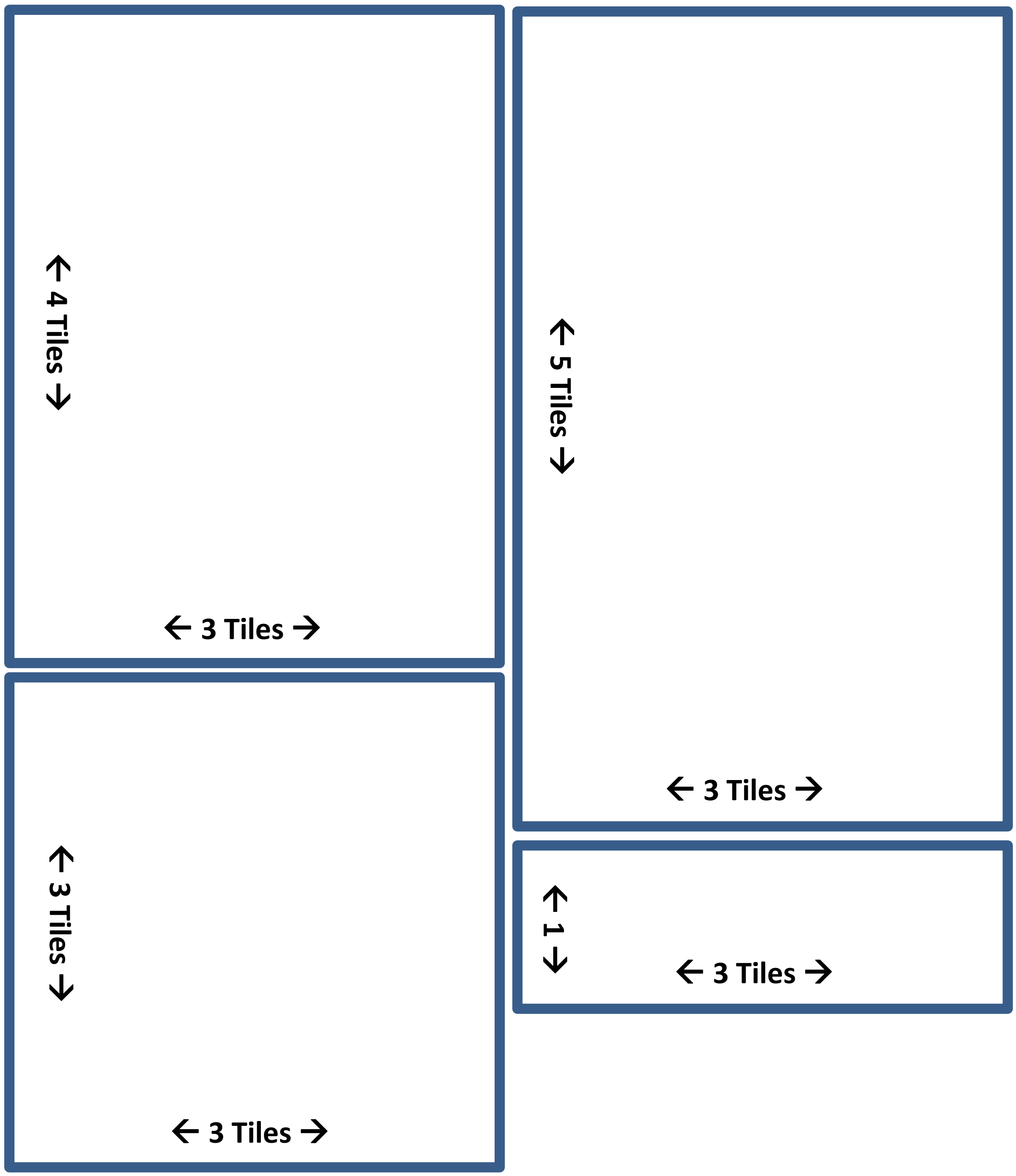
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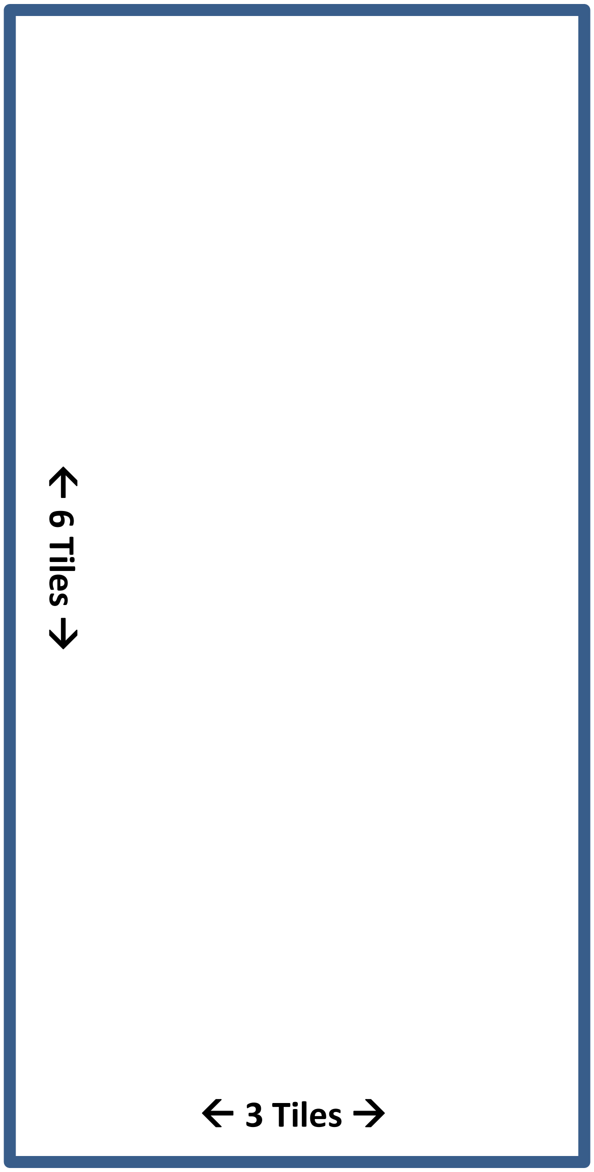
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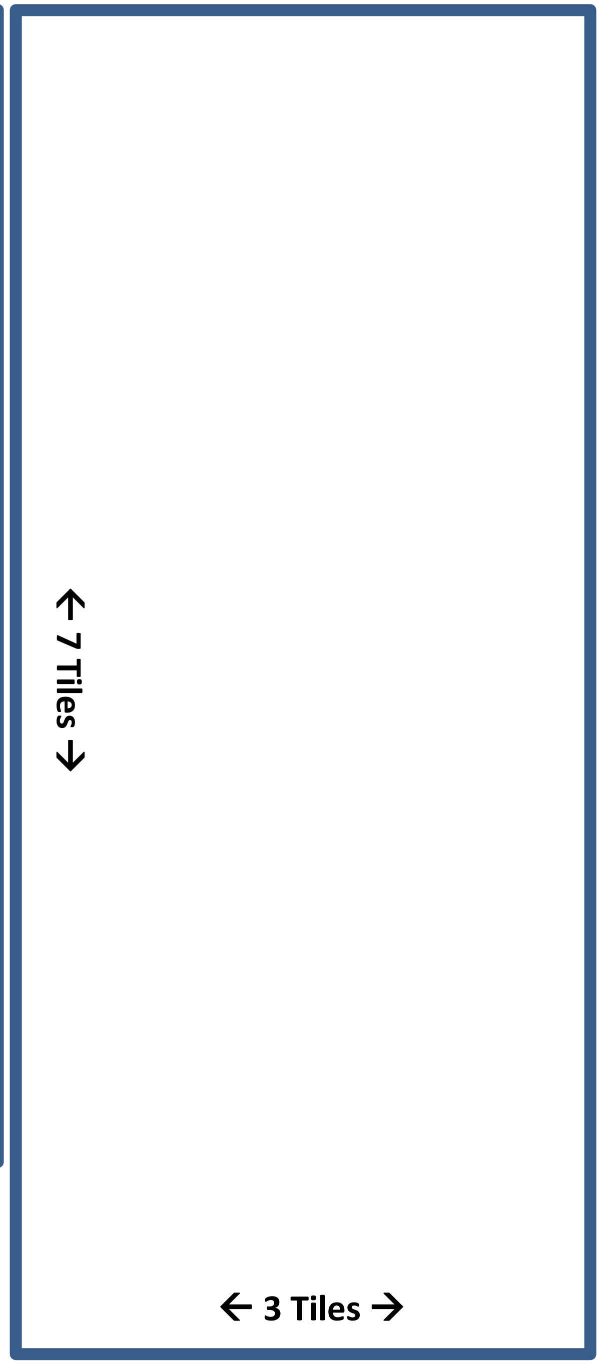
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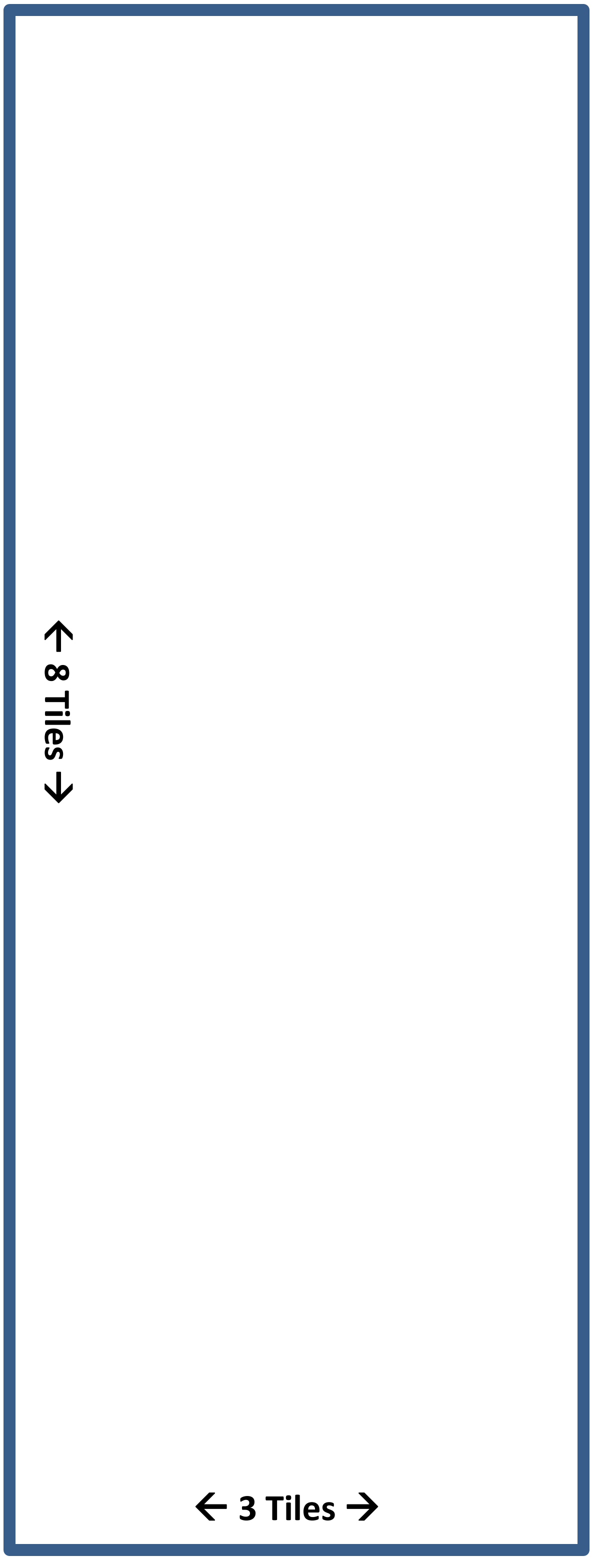

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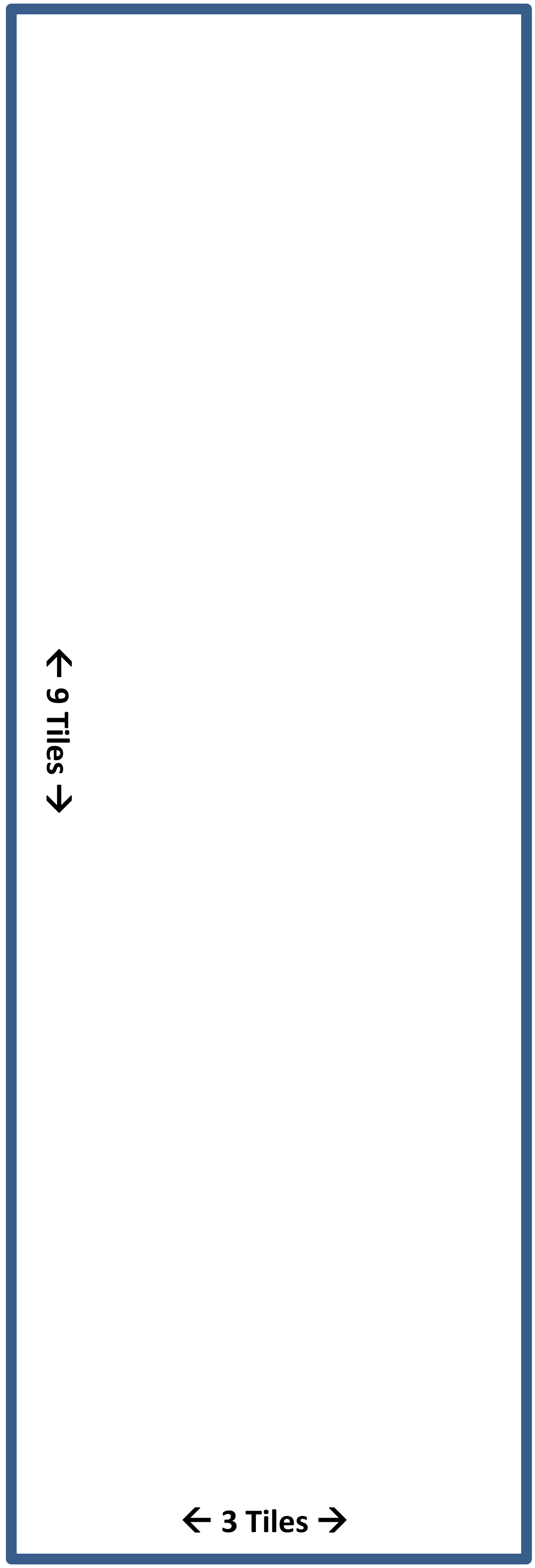

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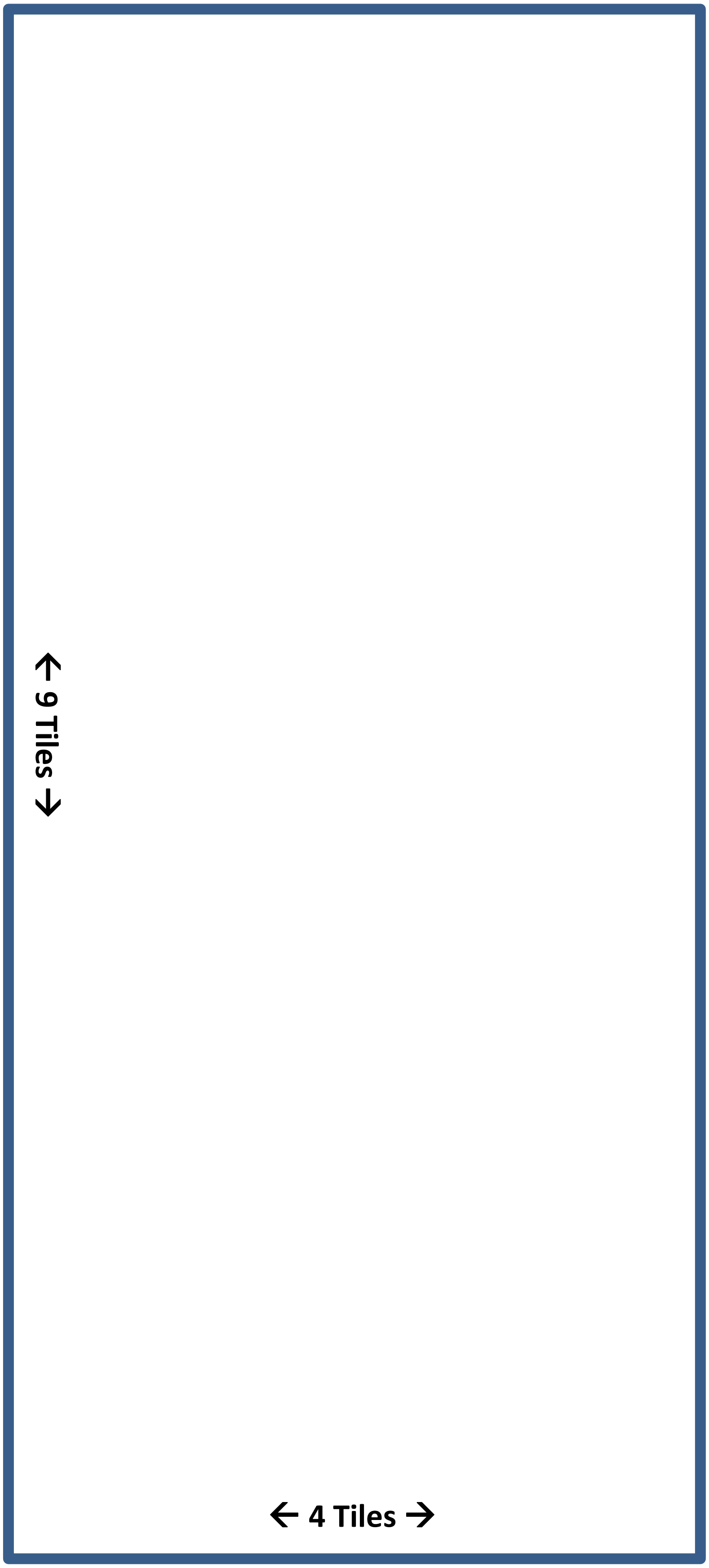
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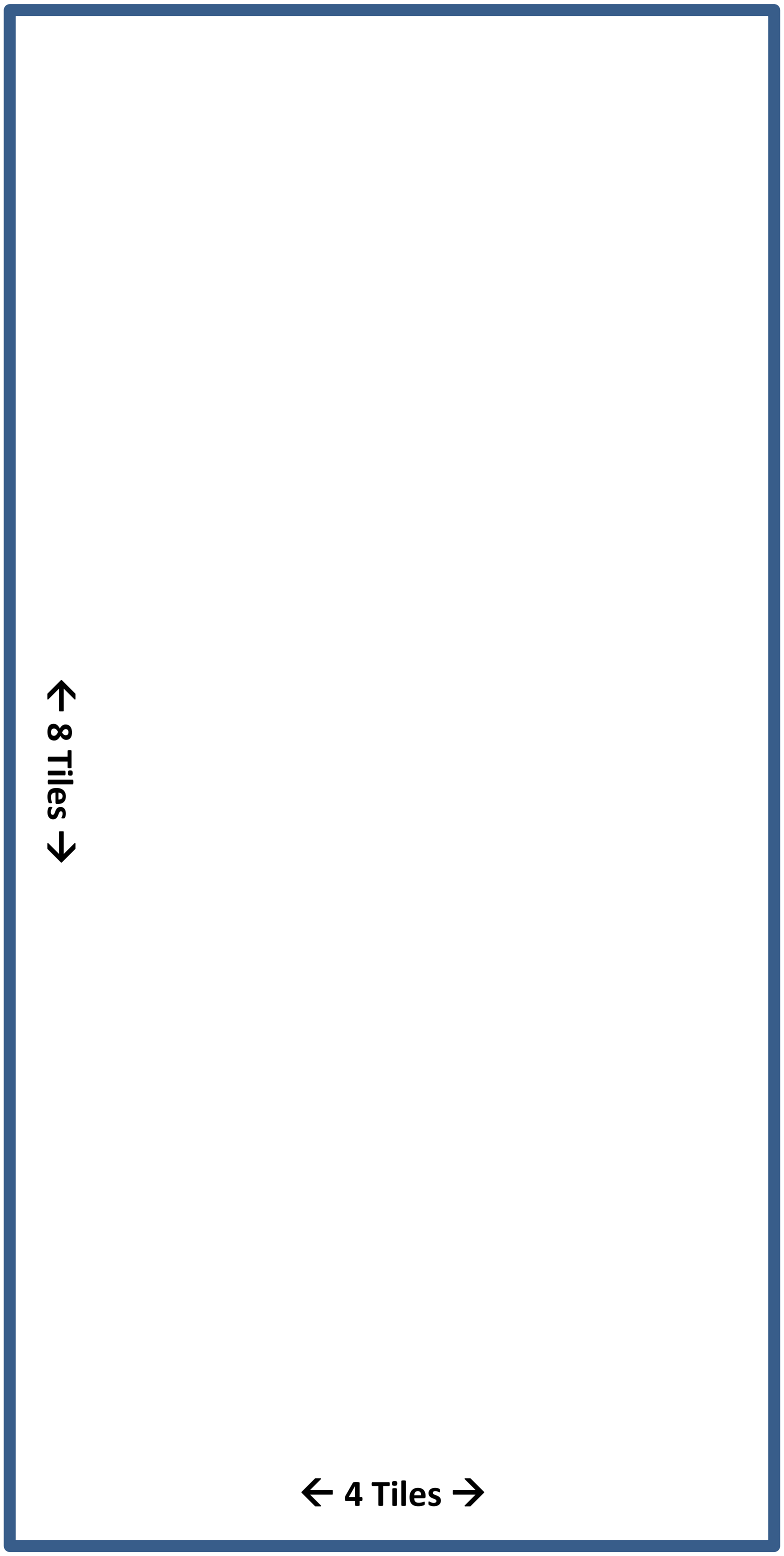
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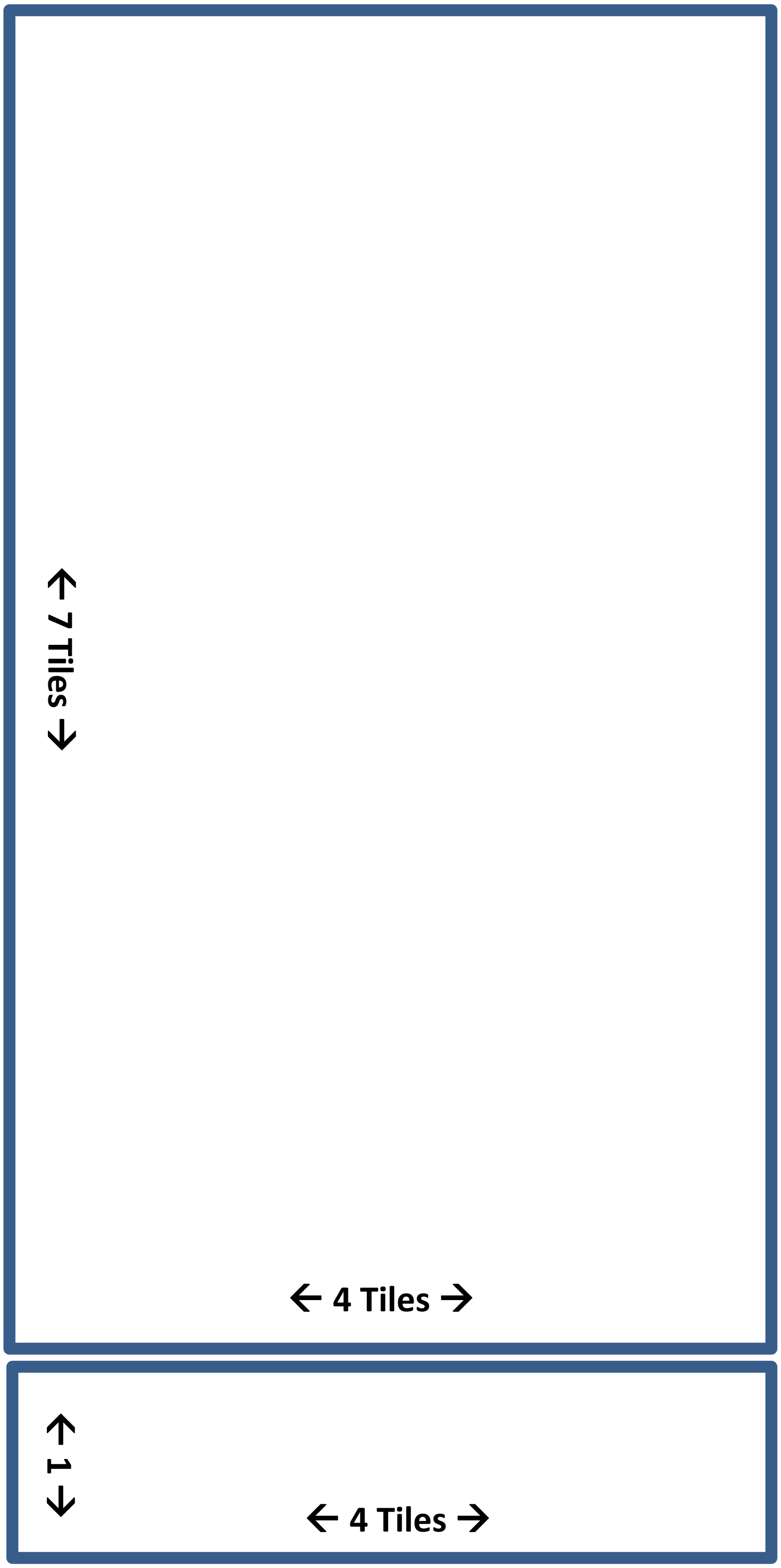
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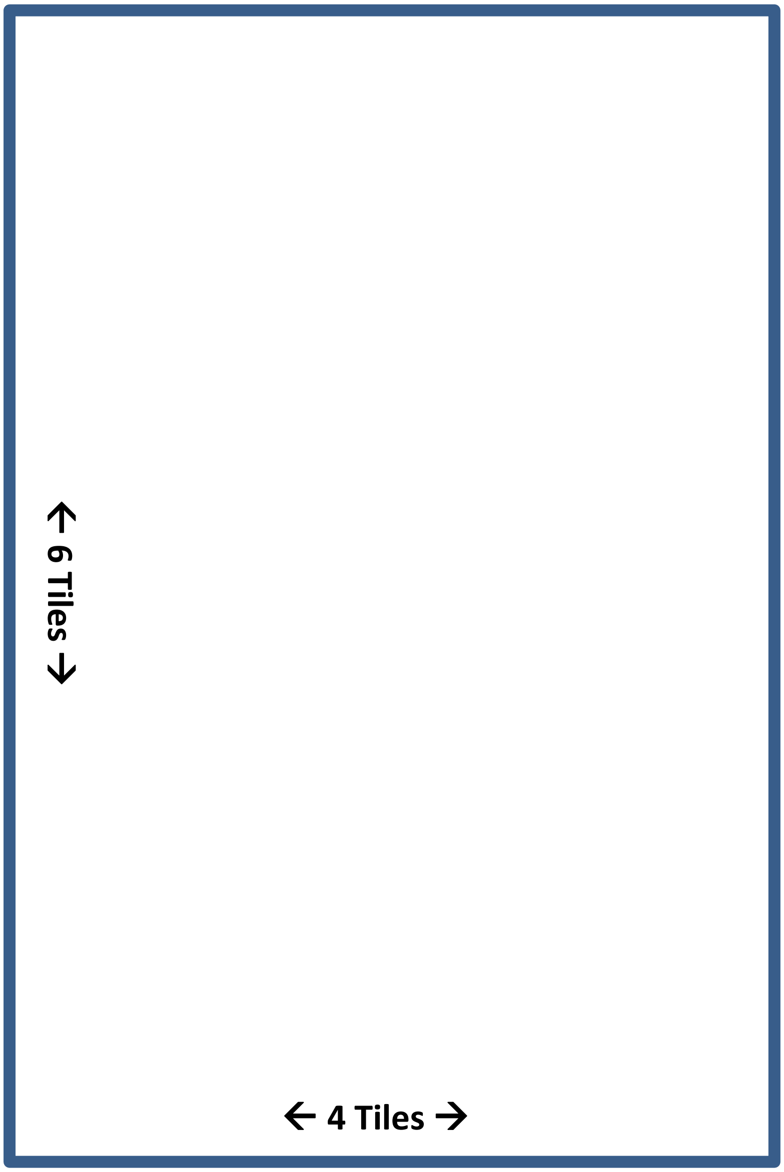

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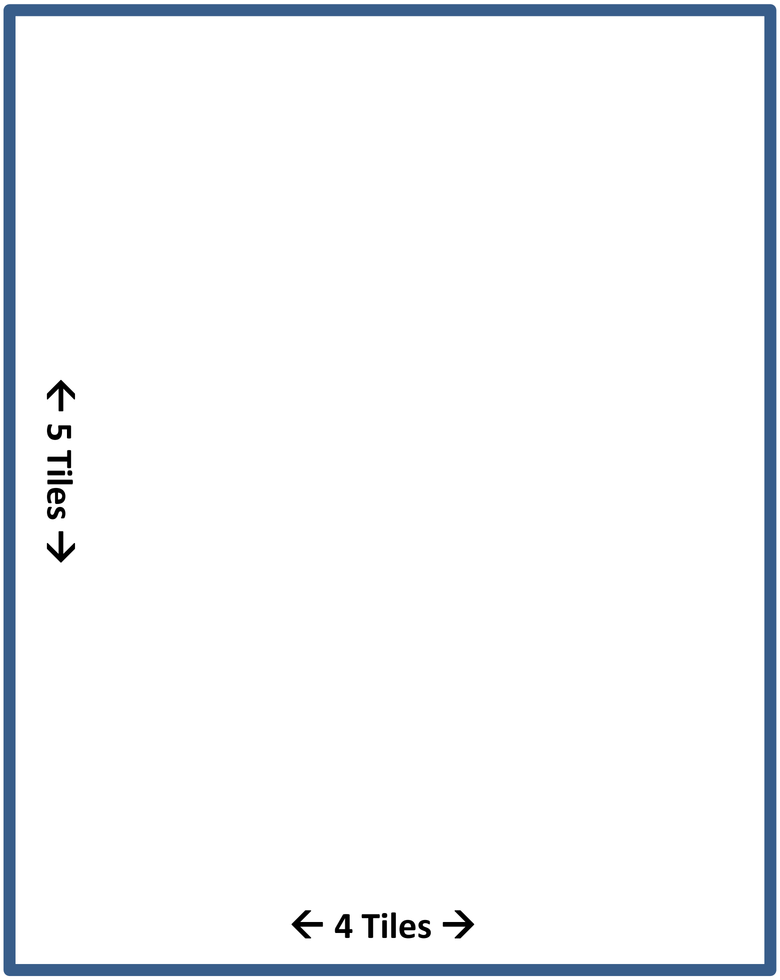
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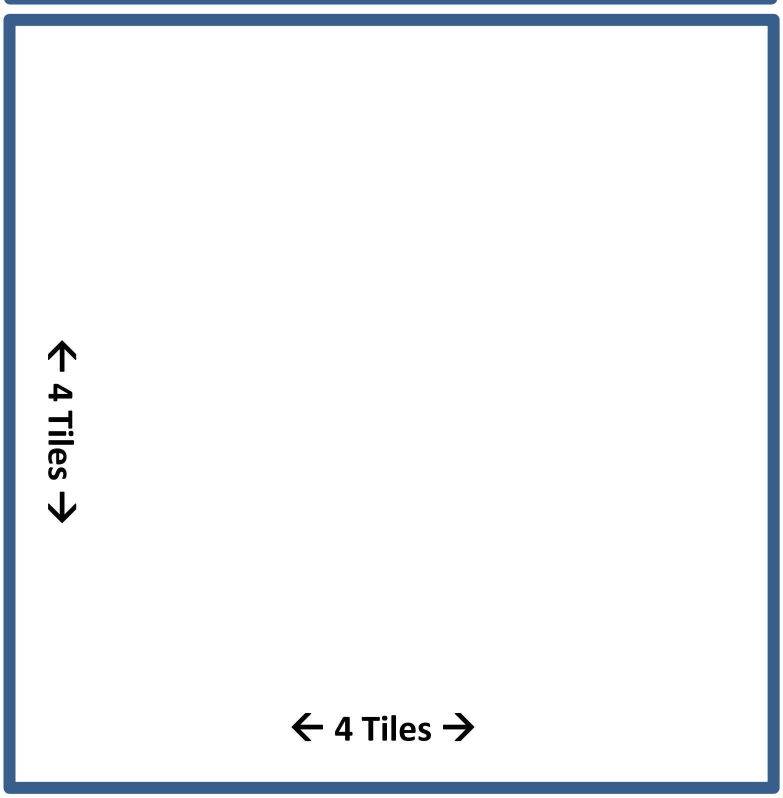
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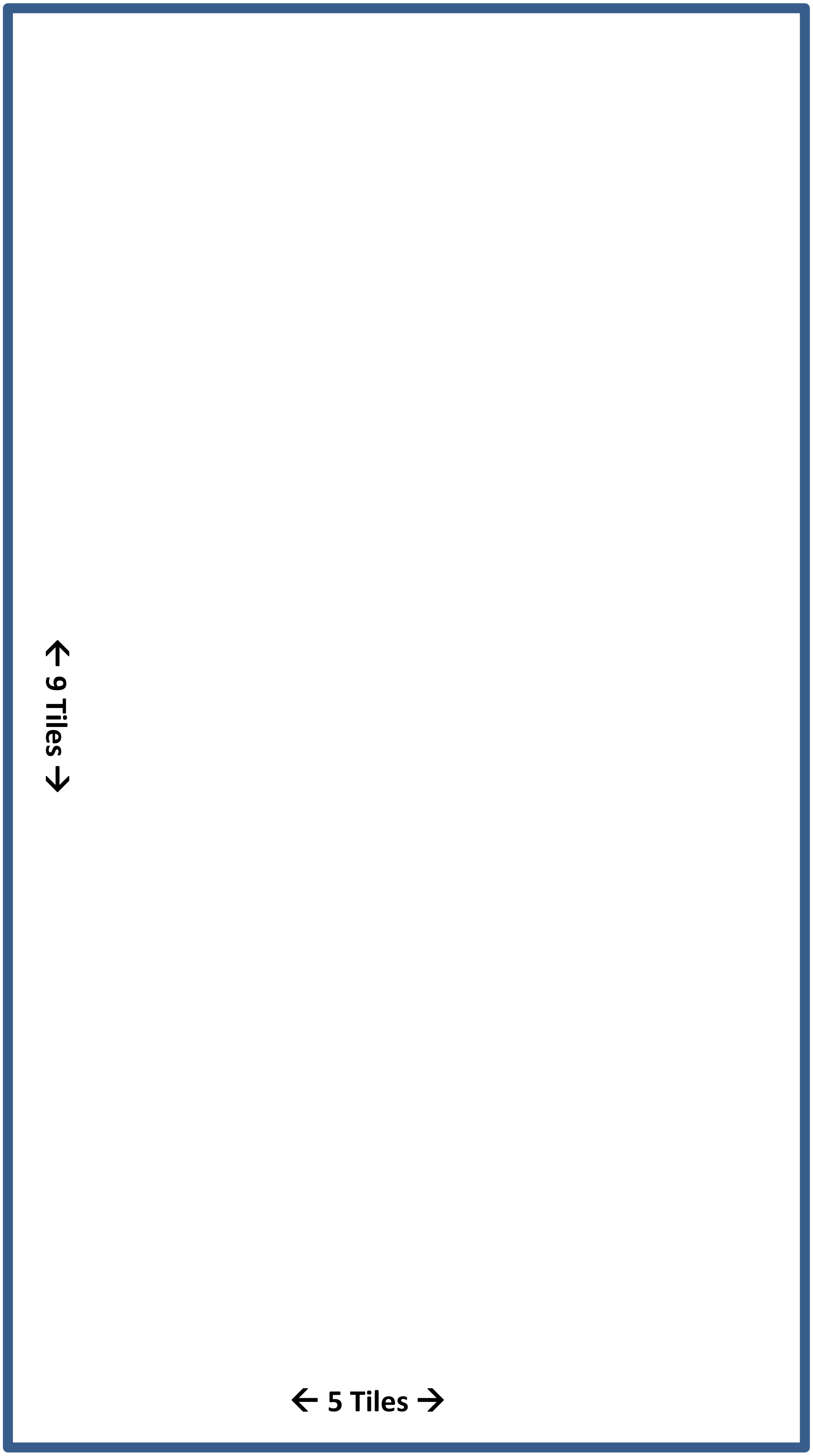
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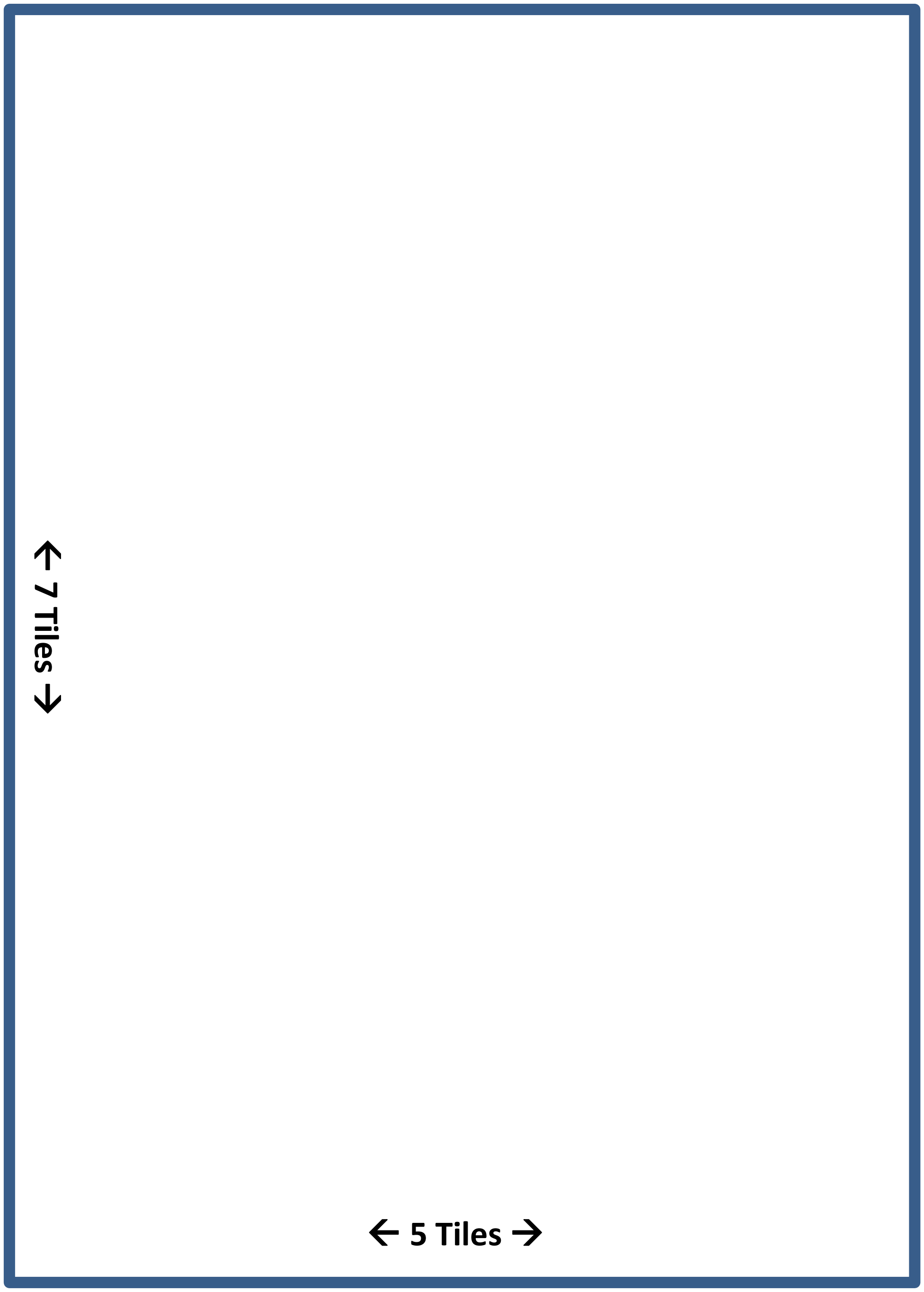
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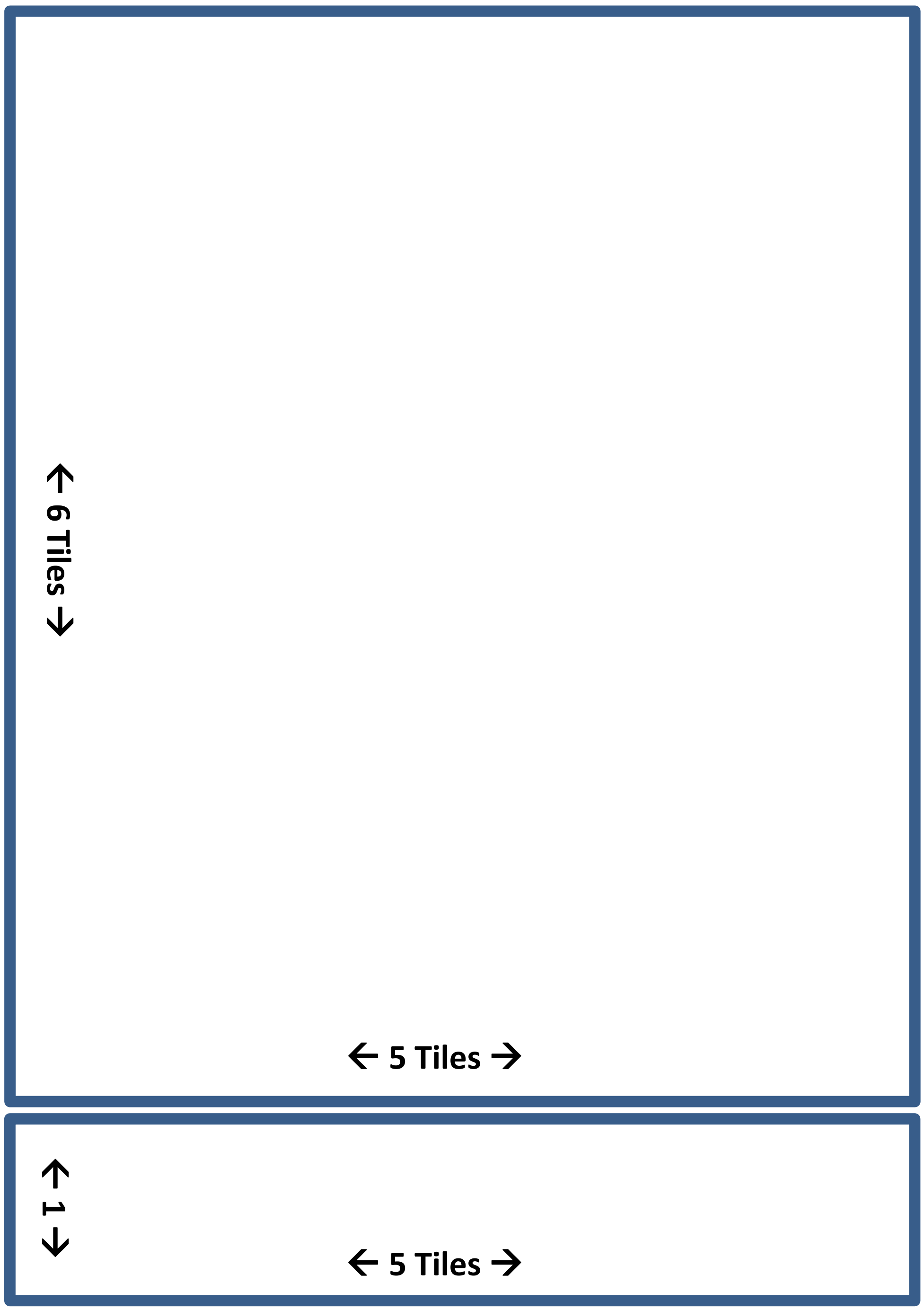
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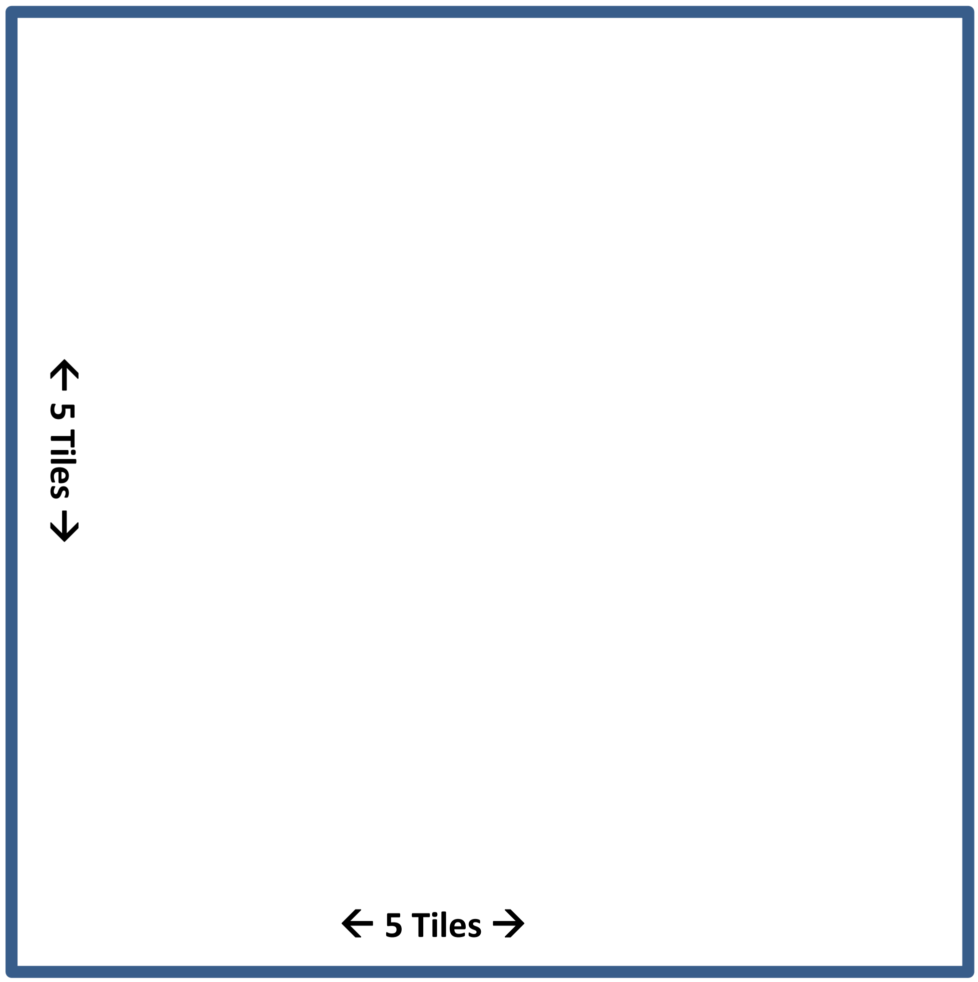
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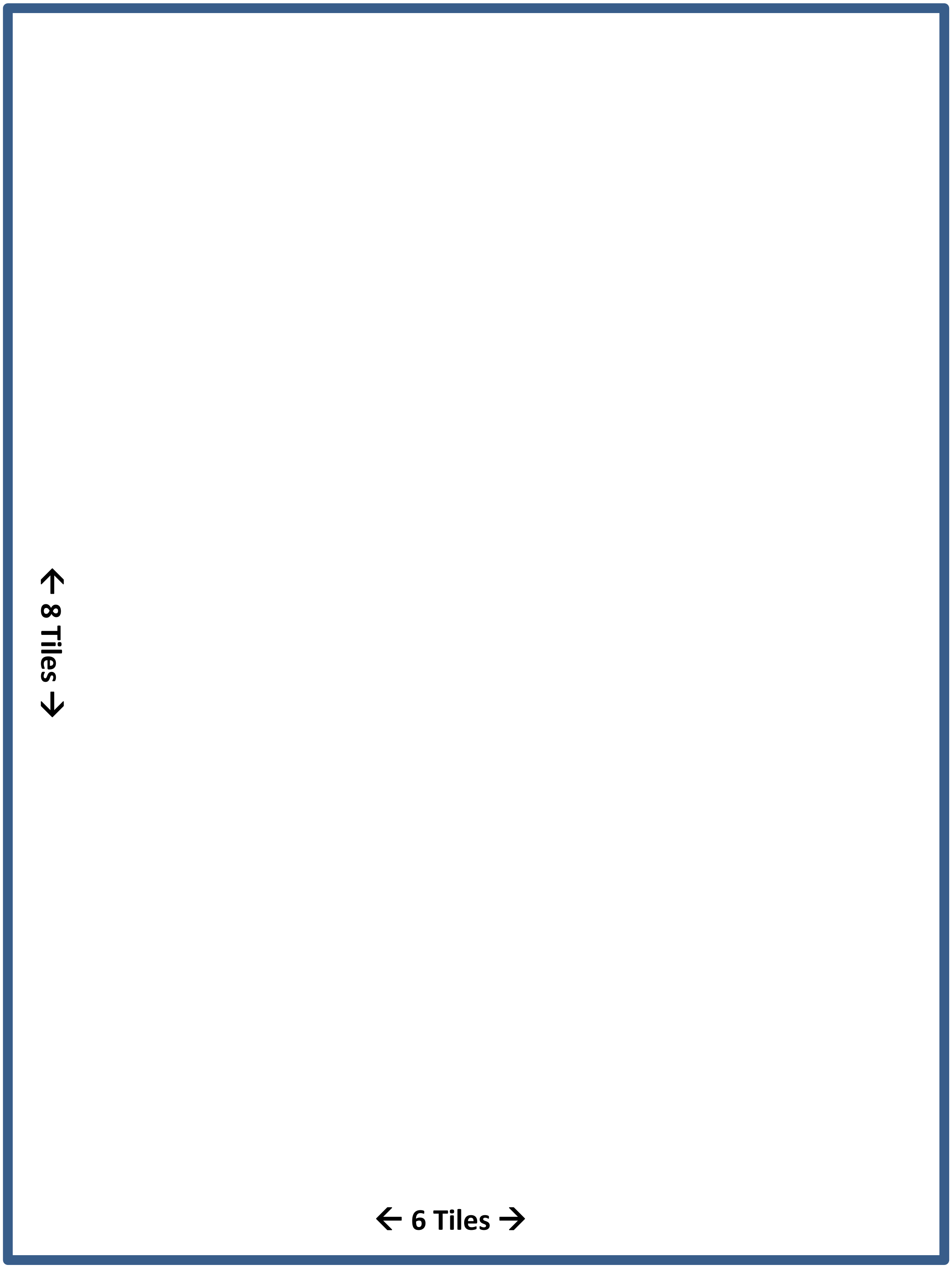
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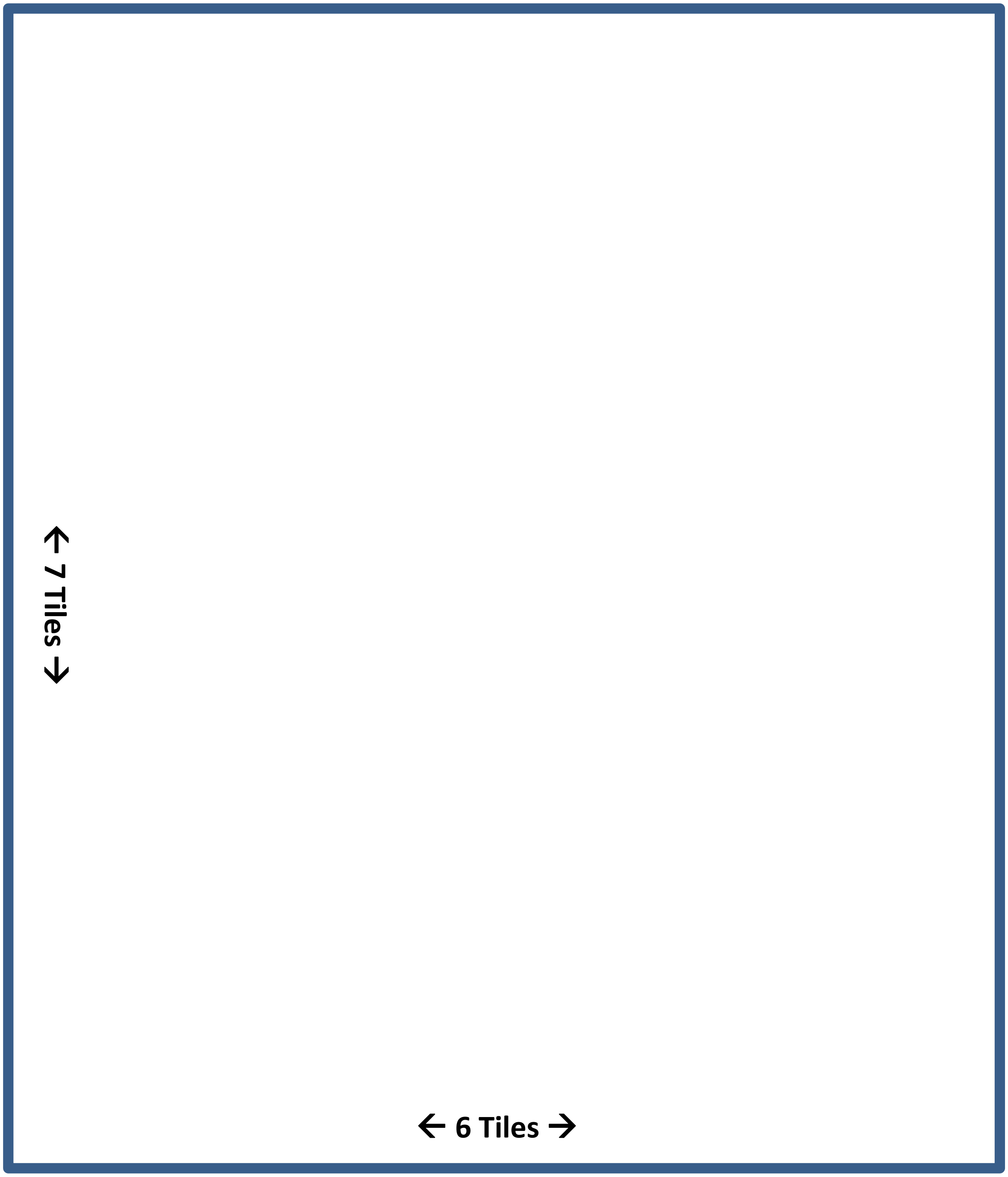
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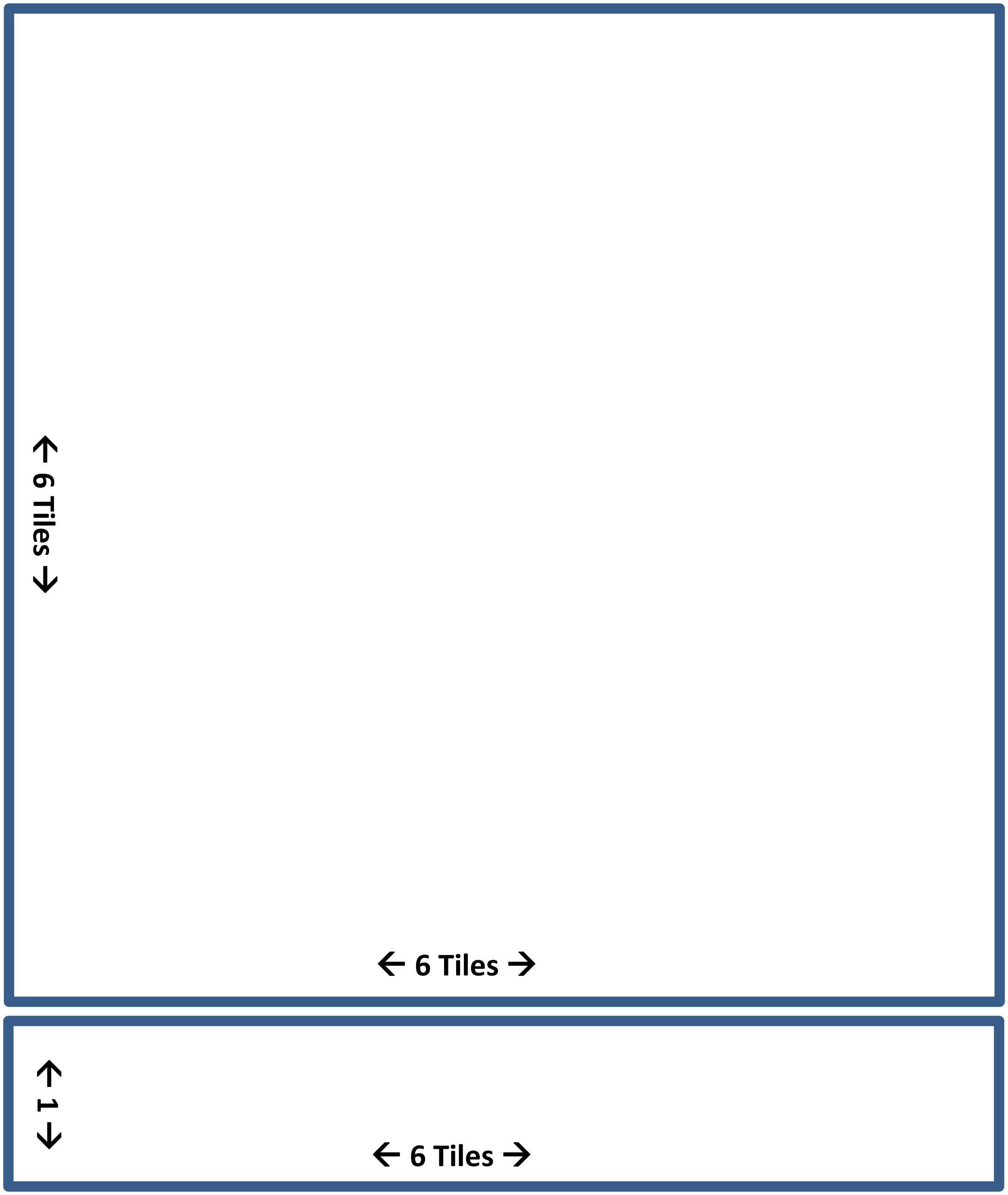
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Tiling with Number **Write the multiplication fact for each rectangle and the total tiles.**					
Name:					
Rectangle 1		**Rectangle 2**		**Large Rectangle**	**Total Tiles**
1 x 3	**and**	**4 x 3**	**makes**	**5 x 3**	**15**
	and		**makes**		
	and		**makes**		
	and		**makes**		
	and		**makes**		
	and		**makes**		
	and		**makes**		
	and		**makes**		
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	and		**makes**		
	and		**makes**		
	and		**makes**		
	and		**makes**		

Tiling with Number

I can combine two rectangles to make a larger rectangle and find the total tiles needed to cover each part and the larger rectangle.

KNP # M 4448.4 - Tiling with Number, Purple
Fluency Standard: 3.OA.7
Standard: 3.OA.5, 3.OA.7, 3.MD.7,

Materials: Rectangle Cards, 1 recording sheet per player

Directions:

1. Place all rectangle cards face down in a pile.

2. With a partner take turns selecting a rectangle card without looking. Each player should place their rectangle cards face up in front of them.

3. When you have two cards that can go together to make a larger rectangle, place them together and find the total number of tiles needed to cover both parts.

4. Write the facts for each match on your recording sheet. The “larger rectangle” is not a separate piece – just a composite of the two smaller rectangles.

5. When all the cards are gone, the player with the most 2-piece rectangles wins.

Squares/ Near Squares Kaboom

Materials: Popsicle sticks with an individual fact from the list below on each stick. Write Kaboom on three or more additional popsicle sticks. Optional: calculator/ multiplication chart to check answers.

Directions: Put all the popsicle sticks face down so students cannot see what is written on them. Here is how to play:

1. First student pulls out a popsicle stick.

2. The student identifies the answer and explains his/her thinking to the group/partner. If the answer is correct, the student gets to keep the popsicle stick. If the student answers it incorrectly, the stick must go back in the cup.

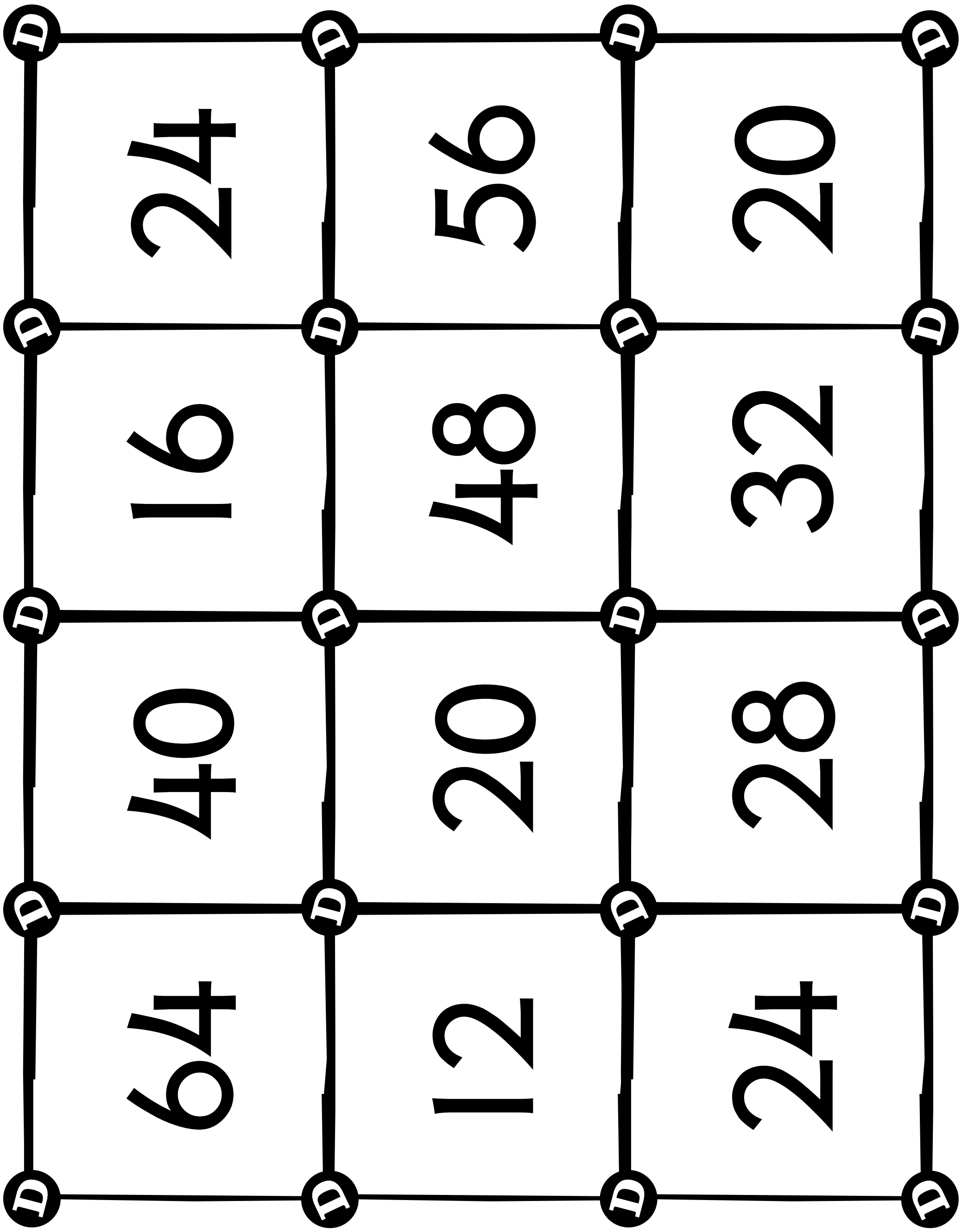
3. The students continue around the circle, or with a partner, selecting one popsicle stick at a time and answering their question.

4. Any student who pulls a KABOOM! stick must place all the popsicle sticks they have accumulated back into the cup, leaving them with zero.

5. The student with the most sticks after 5 rounds, time is up, etc. is the winner.

1 x 1		
2 x 2	2 x 1	2 x 3
3 x 3	3 x 4	3 x 2
4 x 4	4 x 5	4 x 3
5 x 5	5 x 6	5 x 4
6 x 6	6 x 7	6 x 5
7 x 7	7 x 8	7 x 6
8 x 8	8 x 9	8 x 7
9 x 9	9 x 10	9 x 8

Do the Ds



Materials:

Each group of students will need

- *Do the Ds* game board
- 2 blank cubes, marked as follows:
  - o Write "double double" or "DD" on three faces, write "double double double" or "DDD" on the remaining three faces on one cube
  - o Write 3, 4, 5, 6, 7, 8 on the other cube.

Each player will need

- 4 transparent counters (a different color for each player)

Directions (2–4 players):

- The first player rolls the two cubes.
- The player follows the instruction, doubling the number, two or three times. Example: *Lily rolls "4" and "DDD". She thinks double 4 is 8, double 8 is 16, double 16 is 32. 4 multiplied by 8 is 32.*
- The player claims the answer on the game board by covering it with a counter. If an answer is unavailable, the player misses a turn.
- Each of the other players has a turn.
- The first player to place all four counters on the game board is the winner.

Multiples of ___

Players 2
Game can be adapted to play individually

Materials

- Choose one Multiples of ___ game board (game board masters for factors 2-9 are provided)
- 13 colored counters for each player
- Paper clip and pencil to use as a “spinner”

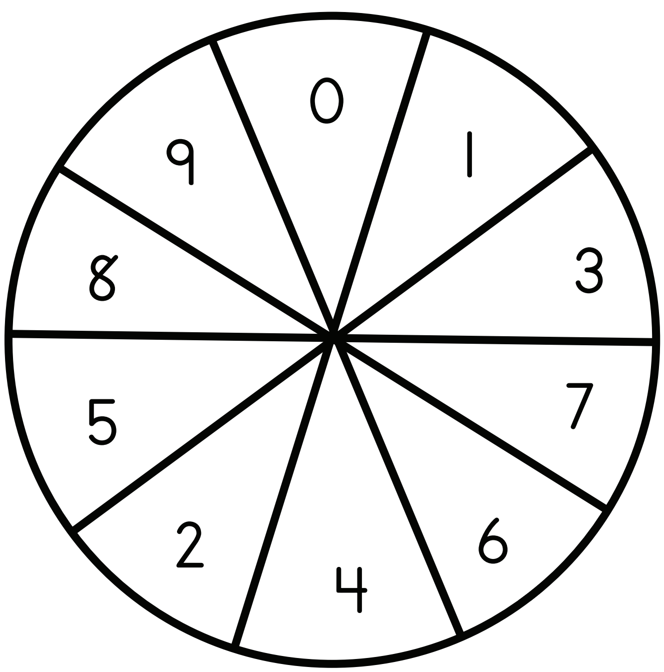
How to play

1. Player one spins to create a multiplication problem. For example, if the Multiples of 4 game board is used and player one spins a 5, then player one creates the problem 4 x 5 or 5 x 4.
2. Player one states the number sentence and places a counter on the product.
3. The next player spins, states the problem, and places a counter on the product.
4. Players take turns until one player has 3 counters in a row horizontally, vertically, or diagonally.

US Math Recovery Council® Original Source: Wright, R.J., Ellemor-Collins, D., & Tabor, P.D. (2012). *Developing Number Knowledge: Assessment, Teaching & Intervention with 7-11 year-olds*. Pages 169-171. London: Sage.

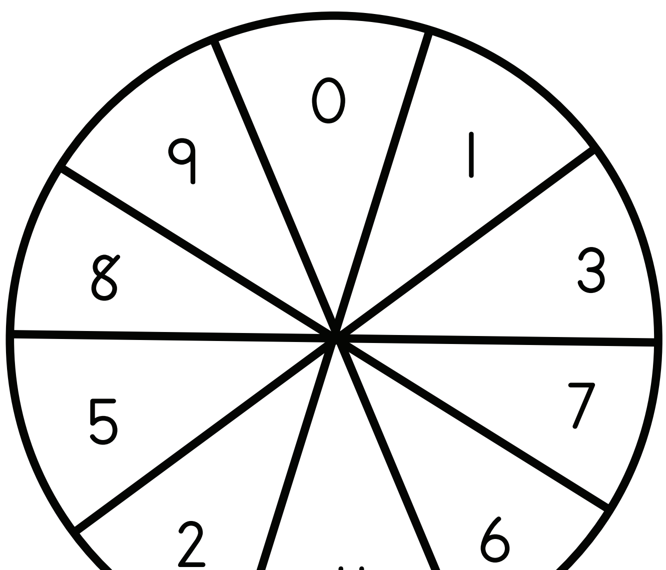
Multiples of 2

10	14	8	6	12
18	2	16	12	18
12	8	4	10	14
4	18	6	0	16
2	16	14	8	6



Multiples of 3

12	21	15	24	9
27	0	6	18	27
21	24	9	15	12
9	18	27	3	24
15	6	12	21	18



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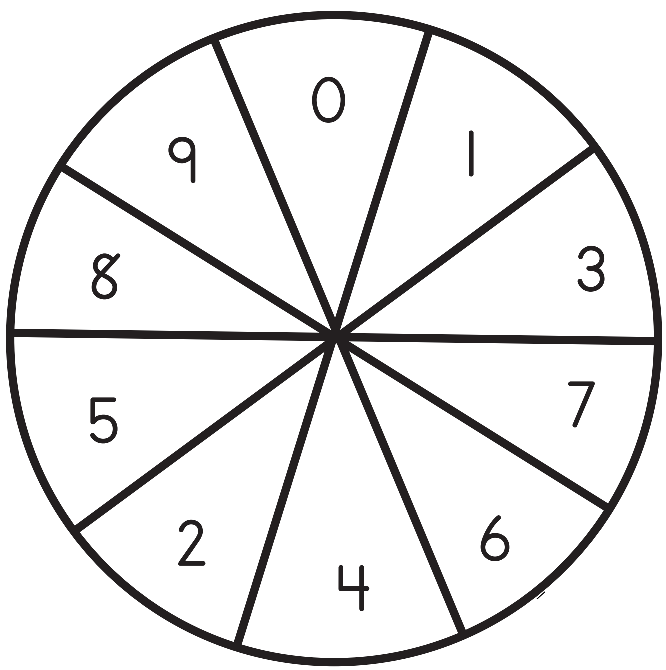
Multiples of 4

24	16	12	32	20
32	4	36	24	28
20	28	16	8	12
36	8	32	0	24
12	28	20	36	16



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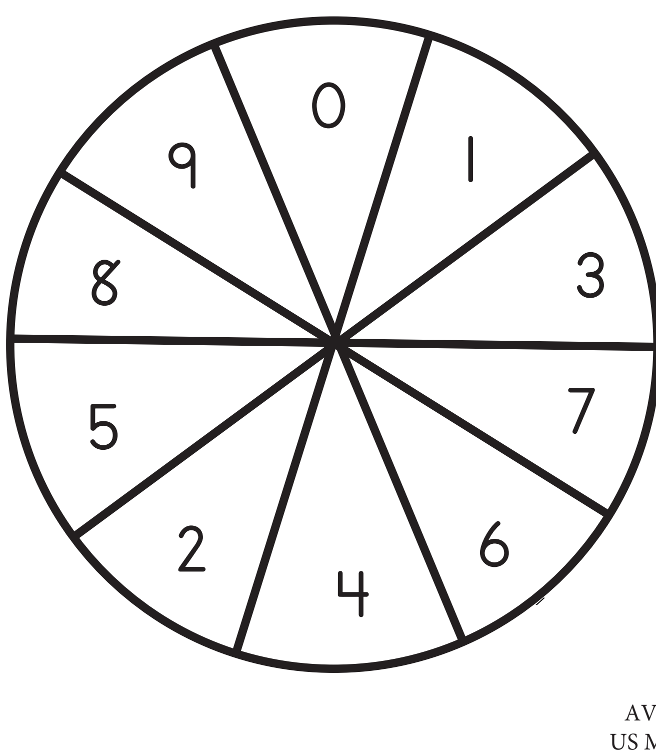
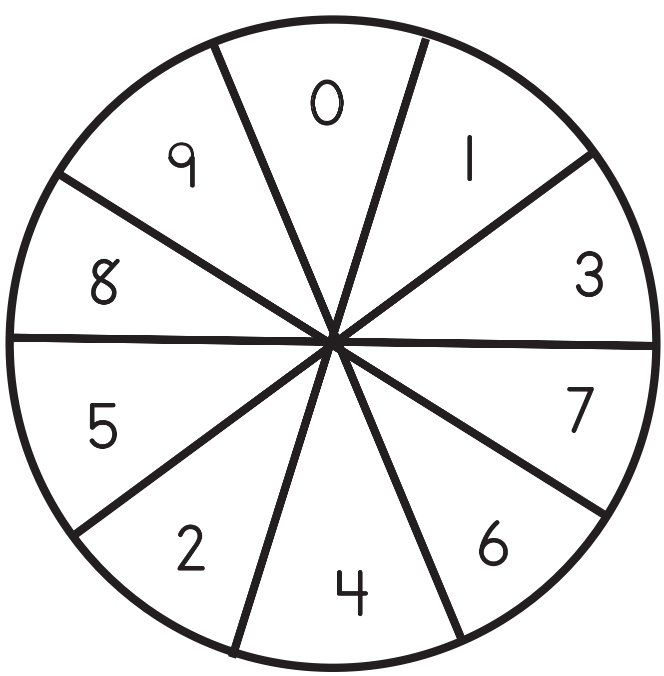
Multiples of 5

35	15	30	0	25
10	40	5	35	45
25	45	20	40	15
30	10	45	20	40
20	35	15	30	25



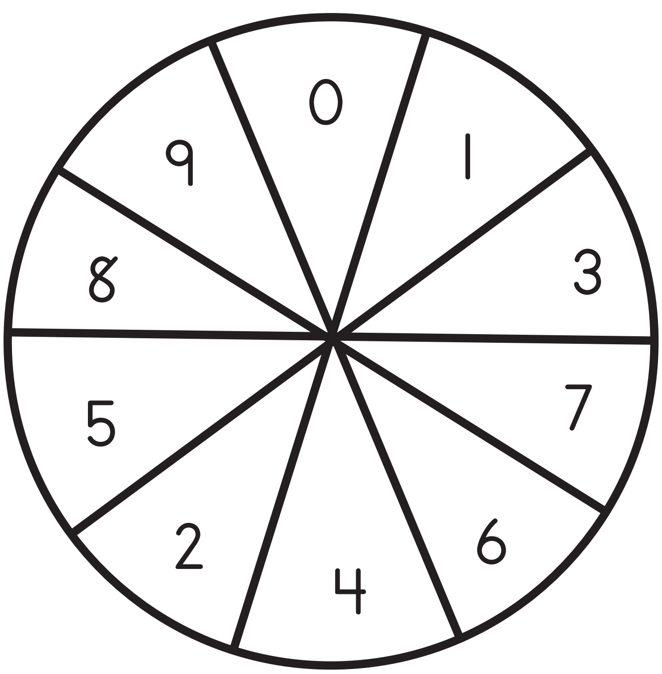
Multiples of 6

24	36	48	18	30
54	0	12	36	42
48	24	30	54	18
36	54	6	42	24
18	42	48	30	12



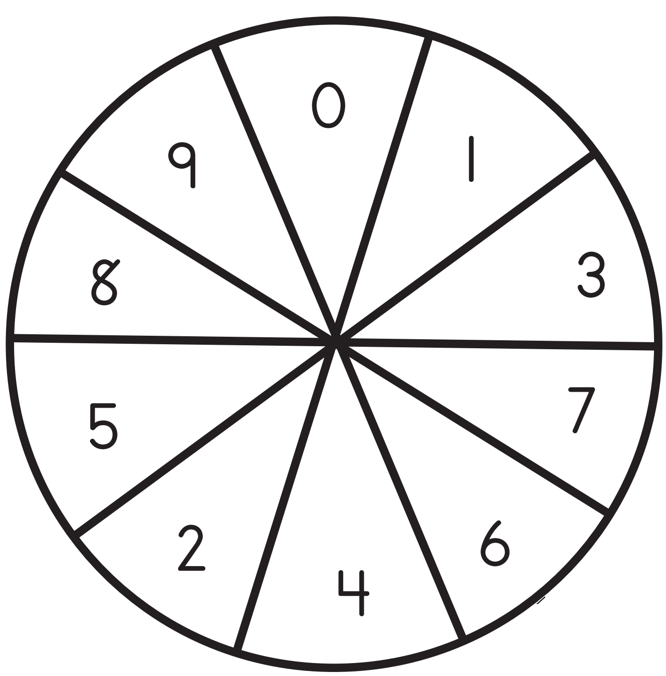
Multiples of 7

42	14	21	35	63
0	56	7	28	49
28	21	49	14	35
35	42	56	63	28
63	49	21	56	42



Multiples of 8

40	8	64	48	32
56	72	24	64	40
48	32	24	0	56
72	56	16	40	64
16	48	32	72	24



Multiples of 9

27	54	72	18	36
63	72	18	45	81
54	36	45	72	0
9	81	63	36	54
45	27	81	18	63