Focus on Fractions

Multiplication and Division
Welcome!

Your host

Kelly DeLong

Executive Director
Kentucky Center for Mathematics
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Good News!
The KCM is hard at work to ensure Kentucky teachers have access to innovative professional development from home.

Through the newly launched KCM Virtual site, mathematics teachers from all grade levels will have access to live Zoom meetings, video recordings and corresponding materials. Read more.

Focus on Fractions - May 4 - May 8
Focus on Geometry - May 11 - May 15
More Multiplicative Thinking - May 18 - May 22
Agenda

● Standards
● Research
  ○ Neagoy
  ○ Lamon
  ○ Van de Walle
● Multiplication Progression
  ○ Fractions
● Division of Fractions
● Resources
As we learn together today, write down 3 words that capture your learning experience today. I will ask you to share in the chat box at the end of our session.
## Standards

### Progression - Building from 4th grade

<table>
<thead>
<tr>
<th>Standard</th>
<th>Clarification</th>
</tr>
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</table>
| KY.4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.  
  a. Understand a fraction \( \frac{a}{b} \) as a multiple of \( \frac{1}{b} \).  
  b. Understand a multiple of \( \frac{a}{b} \) as a multiple of \( \frac{1}{b} \) and use this understanding to multiply a fraction by a whole number.  
  c. Solve word problems involving multiplication of a fraction by a whole number.  
  MP.5, MP.8 | Students refer this standard to \( n \) groups of a fraction (where \( n \) is a whole number) for example 3 groups of \( \frac{1}{4} \), which can be seen as repeated addition. In grade 5 students will multiply a fraction by a whole number.  
  a. Students use visual fraction models to represent \( \frac{7}{5} = 7 \times \frac{1}{5} \)  
  b. Students use the same thinking to see \( 3 \times \frac{2}{5} \) as \( \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = 3 \times \frac{2}{5} = \frac{6}{5} \)  
  KY.4.OA.2  
  Coherence KY.3.NF.1 → KY.4.NF.4 → KY.5.NF.4 |
Standards

Progression - 5th grade

<table>
<thead>
<tr>
<th>Standard</th>
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</tr>
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<tbody>
<tr>
<td>KY.5.NF.4 Apply and extend previous understanding of multiplication to</td>
<td>a. Students use a visual fraction model to show $\left(\frac{2}{3}\right) \times 4 = \frac{8}{3}$ and create a story context for this equation. Do the same with $\left(\frac{2}{3}\right) \times \left(\frac{4}{5}\right) = \frac{8}{15}$. (In general, $\left(\frac{a}{b}\right) \times \left(\frac{c}{d}\right) = \frac{ac}{bd}$.)</td>
</tr>
<tr>
<td>multiply a fraction or whole number by a fraction.</td>
<td>b. For example the shaded portion shows the rectangle with the appropriate unit fraction side lengths.</td>
</tr>
<tr>
<td>a. Interpret the product $\left(\frac{a}{b}\right) \times q$ as $a$ parts of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.</td>
<td>Coherence KY.4.NF.4 → KY.5.NF.4 → KY.6.G.1</td>
</tr>
<tr>
<td>b. Find the area of a rectangle with fractional side lengths by tiling</td>
<td></td>
</tr>
<tr>
<td>it with squares of the appropriate unit fraction side lengths and</td>
<td></td>
</tr>
<tr>
<td>show that the area is the same as would be found by multiplying the side</td>
<td></td>
</tr>
<tr>
<td>lengths. Multiply fractional side lengths to find areas of rectangles</td>
<td></td>
</tr>
<tr>
<td>and represent fraction products as rectangular areas.</td>
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</tr>
</tbody>
</table>

MP.1
## Standards

### Progression - 5th grade

<table>
<thead>
<tr>
<th>Standard</th>
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</tr>
</thead>
<tbody>
<tr>
<td>KY.5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers.</td>
<td>KY.5.MD.2 Coherence KY.4.NF.4 → KY.5.NF.6</td>
</tr>
</tbody>
</table>
## Standards

### Progression - 5th grade

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>KY.5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.</td>
<td>Students build upon the knowledge of division they gained in grades 3 and 4. Students connect previous understanding of division of whole numbers to divide whole numbers by unit fractions and unit fractions by whole numbers. Division of a fraction by a fraction is not a requirement at grade 5.</td>
</tr>
<tr>
<td>a. Interpret division of a unit fraction by a non-zero whole number and compute such quotients.</td>
<td>a. Create a story context for ( \frac{1}{3} \div 4 ) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that ( \frac{1}{3} \div 4 = \frac{1}{12} ) because ( \frac{1}{12} \times 4 = \frac{1}{3} ).</td>
</tr>
<tr>
<td>b. Interpret division of a whole number by a unit fraction and compute such quotients.</td>
<td>b. Create a story context for ( 4 \div \frac{1}{3} ) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that ( 4 \div \frac{1}{3} = 12 ), because ( 20 \times \frac{1}{3} = 4 ).</td>
</tr>
<tr>
<td>c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions.</td>
<td>c. By using visual fraction models and equations to represent the problem.</td>
</tr>
</tbody>
</table>

MP.1, MP.4, MP.8
Standards

*Apply and extend* previous understandings of…

- multiplication
  - whole numbers to fractions
- division
  - whole numbers to fractions
Help students understand why procedures for computations with fractions make sense.

Recommendation 3

Summary of evidence: Moderate Evidence

The panel based this recommendation in large part on three well-designed studies that demonstrated the effectiveness of teaching conceptual understanding when developing students’ computational skill with fractions. These studies focused on decimals and were relatively small in scale; however, the panel believes that their results, together with extensive evidence showing that meaningful information is remembered much better than meaningless information, provide persuasive evidence for this recommendation. Additional support for the recommendation comes from four studies that showed a positive relation between conceptual and computational knowledge of fractions. The studies that contributed to the evidence base for this recommendation used computer-based interventions to examine the link.

Dr. Monica Neagoy

- Teach meaning first, algorithms last.
- I have learned important lessons from practice and research about the long-standing rule-based approach to teaching fractions.
- Students need time to understand fractions, and teachers must be patient and give them time.
- In particular, student need ample experience with constructing new meaning for fractional symbols, working with concrete and visual representations of fractions, connecting these representations, and developing a good sense of the kinds of quantities the symbols may denote.

http://www.ascd.org/publications/books/115071.aspx
### Meaning of Multiplication and Division

<table>
<thead>
<tr>
<th>Multiplication</th>
<th>Division</th>
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</thead>
<tbody>
<tr>
<td>1. Adding equal groups.</td>
<td>1a. Equal or fair sharing (partitive concept)</td>
</tr>
<tr>
<td></td>
<td>1b. Equal grouping or segmenting (quotative, measurement, or repeated subtraction).</td>
</tr>
<tr>
<td>2. Increasing or reducing quantities</td>
<td></td>
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<tr>
<td>3. Moving from factors to product or</td>
<td></td>
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<tr>
<td>product to factor</td>
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</table>

*Unpacking Fractions, Neagoy, pg 201*
Extending From Whole Number

Skip Counting

\[
\frac{1}{4} + \frac{1}{4} + \frac{1}{4}
\]
Extending From Whole Number Multiplication

Skip Counting Activity

What evidence do you have that students are making sense the skip counting sequence?
Extending From Whole Number Multiplication

Pattern blocks make an excellent tool for teaching multiplication of fractions.


www.tools4ncteachers.com
Extending From Whole Number Multiplication

Pattern blocks make an excellent tool for teaching multiplication of fractions.

Exit Ticket

1. If a trapezoid equals one whole, write an addition equation and multiplication equation for 4 green triangles.

2. Write a multiplication sentence to match \( \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \).


www.tools4ncteachers.com
Extending From Whole Number Multiplication

Using context to build understanding of fraction multiplication.

On a field trip, a teacher brought some large sandwiches to for her nine students. Each student got \( \frac{2}{3} \) of a sandwich. How many sandwiches did the teacher bring?

Neagoy, pg 195

Extending From Whole Number Multiplication

On a field trip, a teacher brought some large sandwiches to for her nine students. Each student got \( \frac{2}{3} \) of a sandwich. How many sandwiches did the teacher bring?

Neagoy, pg 195

Extending From Whole Number Multiplication

On a field trip, a teacher brought some large sandwiches to for her nine students. Each student got $\frac{2}{3}$ of a sandwich. How many sandwiches did the teacher bring?

$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{18}{3}$

Neagoy, pg 195

On a field trip, a teacher brought some large sandwiches to for her nine students. Each student got \( \frac{2}{3} \) of a sandwich. How many sandwiches did the teacher bring?

\[
\frac{2}{3} \times 9 = \frac{18}{3}
\]


Neagoy, pg 195
Extending From Whole Number Multiplication

How does linking a real world context with the visual image help kids construct meaning of multiplication of fractions?

Type your answer in the chat or unmute to share your thinking.
Multiplication of Fractions

Multiply a Fraction by a Whole Number using An Area Model
5.NF.4

\[ \frac{1}{8} \times 3 \quad \frac{3}{5} \times 3 \]

ES 5 Math Multiply Fraction X Whole with Area Models

https://www.youtube.com/watch?v=Cy7I-wT49Y4
Multiplication of Fractions

Multiply a Fraction by a Whole Number using A Number Line

\[ \frac{1}{8} \times 3 \]

https://www.youtube.com/watch?v=5Tx15t8jGZQ&t=86s
Multiplication of Fractions

Blueberry Pancake Party
Activity Sheet

Todd is famous for his homemade blueberry pancakes. His recipe makes enough to serve 6 people. He has invited 24 people to enjoy his pancakes.


- 1 1/2 cups all-purpose flour
- 1/2 teaspoon of salt
- 1 tablespoon of baking powder
- 1 1/4 tsp of sugar
- 1 egg
- 1 cup milk
- 1/2 tablespoon of butter melted
- 1/2 cup frozen blueberries (thawed)

How much of each ingredient would he need to give everyone a serving of pancakes?

www.tools4ncteachers.com
Multiplication of Fractions

Blueberry Pancake Party
Activity Sheet

Todd is famous for his homemade blueberry pancakes. His recipe makes enough to serve 6 people. He has invited 24 people to enjoy his pancakes.

Recipe (http://allrecipes.com/recipe/20177/todds-famous-blueberry-pancakes/):
1 1/2 cup all-purpose flour
3/2 teaspoon of salt
1 tablespoon of baking powder
1 1/4 tsp of sugar
1 egg
1 cup milk
1/2 tablespoon of butter melted
1/2 cup frozen blueberries (thawed)

How much of each ingredient would he need to give everyone a serving of pancakes?

www.tools4ncteachers.com
Multiplication of Fractions

Zach's Zoo Adventure

Building Fluency: multiply whole number by a fraction
Materials: die, gameboard, game markers, calculator (optional)
Number of Players: 2-4

Directions: Zach is visiting the Asheboro Zoo for the day. He needs your help to navigate his way through the zoo.
1. All players begin on "Start." Player 1 rolls the die and multiplies the digit on the die by the fraction their game piece is on.
2. If it is correct (may use calculator to check your work) Player 1 moves forward the number of spaces shown on the die.
3. Players take turns rolling the die and multiplying the digit on the die by the fraction their game piece is on.
4. The first player to cross the finish line wins. Play until every player crosses the finish line. You've helped Zach visit the entire zoo!

Variation/Extension: Students can create their own gameboard and/or use a die with larger numbers. Student can record their work on a piece of paper in math notebook.

https://tools4ncteachers.com/resources/district-leaders/documents/4thgrade-GAMES.pdf
Multiplication of Fractions

Parts of a Whole

Building Fluency: multiplication of whole number by a fraction
Materials: whole number die (1-6), fraction circle, fraction cards or fraction die or spinner
Number of Players: 2

Directions:
1. Player rolls a standard whole number die, and spins the spinner.
2. The standard die represents the number of groups, and the spinner represents the fraction in each group.
   Example: A roll of 3 on the standard die, and spin 2 on the spinner would be represented as 3 groups with \( \frac{2}{3} \).
3. Use fraction circles to help determine the product for each round.
4. If your result is 1 or more, you receive a star.
5. Play several rounds and count the stars you have collected.
6. The player with the most stars collected is the winner.

Variation/Extension: Students may want to modify fractions on spinner or use a die 1-6. A blank spinner and fraction circles are added for your convenience. Teacher may also want students to add the products. Students may want to write coordinating problems to fit each equation.

<table>
<thead>
<tr>
<th>PLAYER 1</th>
<th></th>
<th>PLAYER 2</th>
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</thead>
<tbody>
<tr>
<td>ROLL</td>
<td>SPIN</td>
<td>EQUATION</td>
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Multiplication of Fractions

How Big is the Banner?

6 ft

1 ft

whole number by a fraction

Start with the known.
Numbers that are familiar.

Multiplication of Fractions

How Big is the Banner?

6 ft

\[
\begin{array}{c}
\frac{1}{2} \text{ ft} \\
\frac{1}{2} \text{ ft}
\end{array}
\]

6 x \frac{1}{2}

Start with the known. Numbers that are familiar.

whole number by a fraction

Multiplication of Fractions

Formative Instructional and Assessment Tasks

Training for a 5K

Molly is training for a 5K. Her goal is to run 10 miles by the end of this week. On Monday, the distance she runs is $\frac{3}{4}$ of her goal.

Part 1: How far did Molly run on Monday? Draw a model to show your work.

Part 2: After Monday’s run, how far is Molly away from her goal?

Part 3: If Molly runs this same distance each day, when will she reach her goal?

www.tools4ncteachers.com
Multiplication of Fractions

Formative Instructional and Assessment Tasks

Meets Expectation: All parts of the tasks are correct and modeled clearly.

Part 1: How far did Molly run on Monday? Use a number line to show your work.

\[ \frac{4}{10} \times \frac{2}{3} = \frac{8}{15} \]

Part 2: After Monday’s run, how far is Molly away from her goal?

\[ 10 - 2 = 8 - 1.5 = 7.5 \]

Part 3: If Molly runs this same distance each day, when will she reach her goal?

\[ \frac{2+2+2+2}{4} = \frac{8}{4} = 2 \]

\[ 1 + 1 = 2 \]

www.tools4ncteachers.com
Formative Instructional and Assessment Tasks

Chris’s Cookies

On Saturday, Chris bakes a batch of 24 cookies. He uses \( \frac{3}{4} \) of the batch of cookies in treat bags for his birthday party.

Part 1:
How many cookies did Chris use in treat bags? Draw a picture and write a sentence to explain your strategy.

www.tools4ncteachers.com
Multiplication of Fractions

Formative Instructional and Assessment Tasks

Meets Expectation: The student correctly solves all aspects of the task, and explanation is clear and accurate.

On Saturday, Chris bakes a batch of 24 cookies. He uses \( \frac{3}{4} \) of the batch of cookies in treat bags for his birthday party.

Part 1:
How many cookies did Chris use in treat bags? Draw a picture and write a sentence to explain your strategy.

\[ \frac{3}{4} \times 24 = \text{treat bags} \]

18 cookies for the treat bags

www.tools4ncteachers.com
Multiplication of Fractions

Lamon calls this.. fractions as operators.

Operators are transformers that
• lengthen or shorten line segments.
• increase or decrease the number of items in a discrete set of objects
• take a figure in the geometric plane, such as a rectangle, and maps it onto a larger or smaller figure of the same shape.

Lamon, pg 201
Multiplication of Fractions

Troy has $1 \frac{3}{6}$ as many baseball cards as I have. I have 55 cards. How many does Troy have?

I canned 40 pounds of tomatoes last year. Jan did $\frac{5}{6}$ as many. How many pounds did Jan can?

How did you use fractions as an operator?
Multiplication of Fractions

Multiply a Fraction by a Fraction using A Number Line

\[
\frac{2}{3} \times \frac{3}{4}
\]

ES 5 Math Multiply Fraction X Fraction using Number Lines

https://www.youtube.com/watch?v=2uDMVsAtvAw&t=92s
Multiplication of Fractions

Area Model of Multiplication

How much is \( \frac{3}{4} \) of \( \frac{2}{3} \)?

Start with \( \frac{2}{3} \)

Partition the two-thirds into fourths

Find \( \frac{3}{4} \) of the \( \frac{2}{3} \)

What part of the whole is \( \frac{3}{4} \) of \( \frac{2}{3} \)? \( \frac{3}{6} \) or \( \frac{1}{2} \).

Multiplication of Fractions

Area Model of Multiplication

Multiplication of Fractions

Area Model of Multiplication

Multiplication of Fractions

Area Model of Multiplication

Teaching Tip

Some textbooks make this sliced-rectangle approach mechanical, such that it actually becomes a meaningless algorithm in itself. Students are told to shade horizontally to show the first factor and shade vertically for the second factor. Without a rationale, they are told that the product is the region that is double-shaded. Such strategies are without meaning and the same as giving students rules to memorize.

Division of Fractions
Graham Fletcher

https://gfletchy.com/2016/08/02/making-sense-of-invert-and-multiply/
Division of Fractions
Graham Fletcher

https://vimeo.com/335263104
As we learn together today, write down 3 words that capture your learning experience today. I will ask you to share in the chat box at the end of our session.
Developing Geometric Thinking

MAY 11 - 15
2:00-2:30 PM EST

Developing Geometric Thinking
w/ KY Math Leaders

Monday, May 11 - Geometric Primary Focus

Tuesday, May 12 - Geometric Intermediate Focus

Wednesday, May 13 - Conceptual Understanding of Geometric Measurement

Thursday, May 14 - More Conceptual Understanding of Geometric Measurement

Friday, May 15 - High School Geometry with Technology Tools
Follow Us!

https://www.kentuckymathematics.org/kcm_virtural.php

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@KyMath @KyCenterforMath
KCM is here to support you!

Air Hugs!

Contact me

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