Focus on Fractions:

Fraction Equivalence

with Dee Crescitelli
Welcome!

Your host

Dee Crescitelli

Director
Kentucky Center for Mathematics
cresciteld1@nku.edu
Are Fractions Numbers?

Artie said: “Choose a number between 1 and 10.” Kay said: “3/2.” Artie: “That’s not a number! It’s a fraction.” Kay: “But fractions are numbers!”

Are fractions numbers? Take this quiz and decide.

1. Is it possible to add two fractions? ............. Yes  No  Don’t Know
2. Is it possible to add a fraction and a whole number? ........................................ Yes  No  Don’t Know
3. Is it possible to place a fraction on the number line? .............................................. Yes  No  Don’t Know
4. Is it possible for a fraction to equal a whole number? .............................................. Yes  No  Don’t Know
5. Is it possible to add two fractions and have the sum equal a whole number? ........ Yes  No  Don’t Know

Based on your answers, do you agree with Artie or with Kay? Write 1-2 sentences to explain why.
Research

Developing Effective Fractions Instruction for Kindergarten Through 8th Grade

Review of Recommendations

Recommendation 1.
Build on students’ informal understanding of sharing and proportionality to develop initial fraction concepts.
- Use equal-sharing activities to introduce the concept of fractions. Use sharing activities that involve dividing sets of objects as well as single whole objects.
- Extend equal-sharing activities to develop students’ understanding of ordering and equivalence of fractions.
- Build on students’ informal understanding to develop more advanced understanding of proportional reasoning concepts. Begin with activities that involve similar proportions, and progress to activities that involve ordering different proportions.

Recommendation 2.
Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.
- Use measurement activities and number lines to help students understand that fractions are numbers, with all the properties that numbers share.
- Provide opportunities for students to locate and compare fractions on number lines.
- Use number lines to improve students’ understanding of fraction equivalence, fraction density (the concept that there are an infinite number of fractions between any two fractions), and negative fractions.
- Help students understand that fractions can be represented as common fractions, decimals, and percentages, and develop students’ ability to translate among these forms.

Recommendation 3.
Help students understand why procedures for computations with fractions make sense.
- Use area models, number lines, and other visual representations to improve students’ understanding of formal computational procedures.
- Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions.
- Address common misconceptions regarding computational procedures with fractions.
- Present real-world contexts with plausible numbers for problems that involve computing with fractions.

Recommendation 4.
Develop students’ conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.
- Develop students’ understanding of proportional relations before teaching computational procedures.
Recommendation 2.
Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

- Use measurement activities and number lines to help students understand that fractions are numbers, with all the properties that numbers share.
- Provide opportunities for students to locate and compare fractions on number lines.
- Use number lines to improve students’ understanding of fraction equivalence, fraction density (the concept that there are an infinite number of fractions between any two fractions), and negative fractions.
- Help students understand that fractions can be represented as common fractions, decimals, and percentages, and develop students’ ability to translate among these forms.
Number Lines: Foundations in Whole Number

KY.2.MD.6 [Relate addition and subtraction to length.] Represent whole numbers as lengths from 0 on a number line with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line. MP.3, MP.4
Through a Standards Lens

KY.3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line.

a. Represent a fraction $\frac{1}{b}$ (unit fraction) on a number line by defining the interval from 0 to 1 as the whole and partitioning it into $b$ equal parts.

- Recognize each part has size $\frac{1}{b}$.
- A unit fraction, $\frac{1}{b}$, is located $\frac{1}{b}$ of a whole unit from 0 on the number line.
Thirds

Partition the unit interval

\[
\begin{array}{c}
\frac{1}{3} \\
\frac{1}{3} \\
\frac{1}{3}
\end{array}
\]
Thirds
**KY.3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line.

b. Represent a non-unit fraction $\frac{a}{b}$ on a number line by marking off $a$ lengths of $\frac{1}{b}$ (unit fractions) from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the non-unit fraction $\frac{a}{b}$ on the number line.
Thirds

Iterate the unit fraction
Interval of size $\frac{4}{3}$

4 lengths of $\frac{1}{3}$ starting from 0

The number $\frac{4}{3}$ on the number line
Learning Mathematics through Representations

Complete resources with printable materials for

- Positive Integers
- Negative Integers
- Fractions (positive only)

- Develop understanding of numbers on a number line using C-rods
Learning Mathematics through Representations (LMR) is a research-based curriculum unit for the teaching and learning of integers and fractions in the elementary grades, using the number line as the principal representational context. The curriculum builds on two core ideas: mathematical representations are fundamental to mathematical communication and learning, and curriculum units should be designed as well-orchestrated lesson sequences that support insight and understanding of representational forms.

The members of the LMR staff bring expertise in developmental and educational research, curriculum development, pre-service education and professional development, and elementary classroom teaching. The LMR team is led by Geoffrey B. Saxe and includes additional faculty and graduate students in the Graduate School of Education, University of California, Berkeley.

There are 3 units.

- **Positive Integers** - 8 lessons
- **Fractions** - 14 lessons
- **Negative Integers** - 4 lessons

Resources for each unit include a Lesson Guide, Transparencies (can also be used with a document, designed to support full group discussions), Worksheets (for in-class use), Homework, Classroom Drills, Student Drills (with blanks to be filled in by the student), and a Completed.
Worksheet Example

Fractions
Lesson 1
Worksheet 3

Some fraction of this circle is shaded. Which number line shows the same amount?

A.

B.

C.

Explain why your answer is correct and why the other two answer choices are incorrect.
Example of Student work

Worksheet 3

Some fraction of this circle is shaded. Which number line shows the same amount?

A. 

B. 

C. 

Explain why your answer is correct and why the other two answer choices are incorrect.
Through a Standards Lens

KY.3.NF.3 Explain equivalence of fractions in special cases and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or same point on a number line.

b. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent through writing or drawing.

c. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers.
Through a Standards Lens

KY.4.NF.1 [Extend understanding of fraction equivalence and ordering.] Understand and generate equivalent fractions.

a. Use visual fraction models to recognize and generate equivalent fractions that have different numerators/denominators even though they are the same size.

b. Explain why a fraction \( \frac{a}{b} \) is equivalent to a fraction \( \frac{(n \times a)}{(n \times b)} \).

MP.4, MP.7, MP.8
Equivalent Fractions on a Number Line

\[ \frac{1}{12}, \frac{\color{red}{9}}{12}, \frac{3}{4} \]

0 \quad \frac{1}{4} \quad \frac{2}{4} \quad \frac{3}{4} \quad 1 \quad 2
Lesson 10 – Worksheet 1

Fractions Lesson 10: Equivalent Fractions - More Strategies

Worksheet 1

1. Circle the answer that shows another name for $\frac{1}{5}$.
   You can add tickmarks and numbers to help you.

   $\frac{2}{10}$  $\frac{1}{10}$  $\frac{5}{5}$

2. Circle the answer that shows another name for $\frac{1}{5}$.
   You can add tickmarks and numbers to help you.

   $\frac{1}{15}$  $\frac{3}{15}$  $\frac{3}{7}$

3. Write different fraction names for $\frac{1}{5}$.
   You can add tickmarks and numbers to help you.
Worksheet 1

1. Circle the answer that shows another name for $\frac{1}{5}$.
   You can add tickmarks and numbers to help you.

   ![Number line with tickmarks and fractions]
   - $\frac{2}{10}$
   - $\frac{1}{10}$
   - $\frac{5}{10}$

2. Circle the answer that shows another name for $\frac{1}{5}$.
   You can add tickmarks and numbers to help you.

   ![Number line with tickmarks and fractions]
   - $\frac{1}{15}$
   - $\frac{3}{15}$
   - $\frac{3}{7}$
Equivalence as Renaming

Students who have been composing and decomposing whole numbers and seeing different ways of renaming specific quantities have a sense of equivalence as renaming.

\[ 27 = 20 + 7 \]
\[ 27 = 17 + 10 \]
\[ 27 = 30 - 3 \]

\[ \frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \]
\[ \frac{3}{4} = 3 \times \frac{1}{4} \]
Connections to Multiplication

Students learn to work with multiunits…

How do we ask kids to think about $7 \times 6$?

- Skip counts by 6s
- Number line – 7 hops of 6
- Area & arrays – 7 “rows” of 6
- Think about 5 groups of 6 and 2 more groups of 6

These should be the SAME WAYS we ask kids to think about fractions

- Numbers built out of subunits
Equivalent Fractions on a Number Line
Place the arrows where they belong on the number line.
After you have placed all the numbers as best you can, reveal the number line below to check your answers.

Move this box down to check your work.
Activities in Handouts

7704 Number Lines - Level 2

Preparing materials: This master will create two double number lines. Cut along the heavy black lines. Fold each along the light line so that both number lines face outward. Place paperclips on the folded edge.

Number Lines - Fraction Wall

7704 Number Lines - Level 3

Preparing materials: This master will create two double number lines. Cut along the heavy black lines. Fold each along the light line so that both number lines face outward. Place paperclips on the folded edge.
Upcoming This Week

Focus on Fractions!

MAY 4 - 8
2:00-2:30 PM EST

Monday, May 4 - Third Grade Fraction Concepts

Tuesday, May 5 - Fraction Equivalence

Wednesday, May 6 - Fraction Operations: + - with Common Denominators

Thursday, May 7 - Fraction Operations: +/- with Unlike Denominators

Friday, May 8 - Fraction Operations- Multiplication and Division
Good News!
The KCM is hard at work to ensure Kentucky teachers have access to innovative professional development from home.

Through the newly launched KCM Virtual site, mathematics teachers from all grade levels will have access to live zoom meetings, video records and corresponding materials. Read more.

Focus on Fractions - May 4 - May 8

And the math continues with these sessions under development:

Focus on Geometry - May 11 - May 15

More Multiplicative Thinking - May 18 - May 22

Focus on Measurement & Data - May 26 - May 29

Follow Us!
www.kentuckymathematics.org

@KyMath
@KyCenterforMath
KCM is here to support you!

Your host

Dee Crescitelli
Director
Kentucky Center for Mathematics
cresciteld1@nku.edu