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FOR MATHEMATICS

Developing Multiplicative Thinking

Extending Multiplication and Division to Multi-Digit Factors with Belle Rush

Welcome!

Your host:

Belle Rush

Regional Consultant
Kentucky Center for Mathematics
bsrush180@yahoo.com

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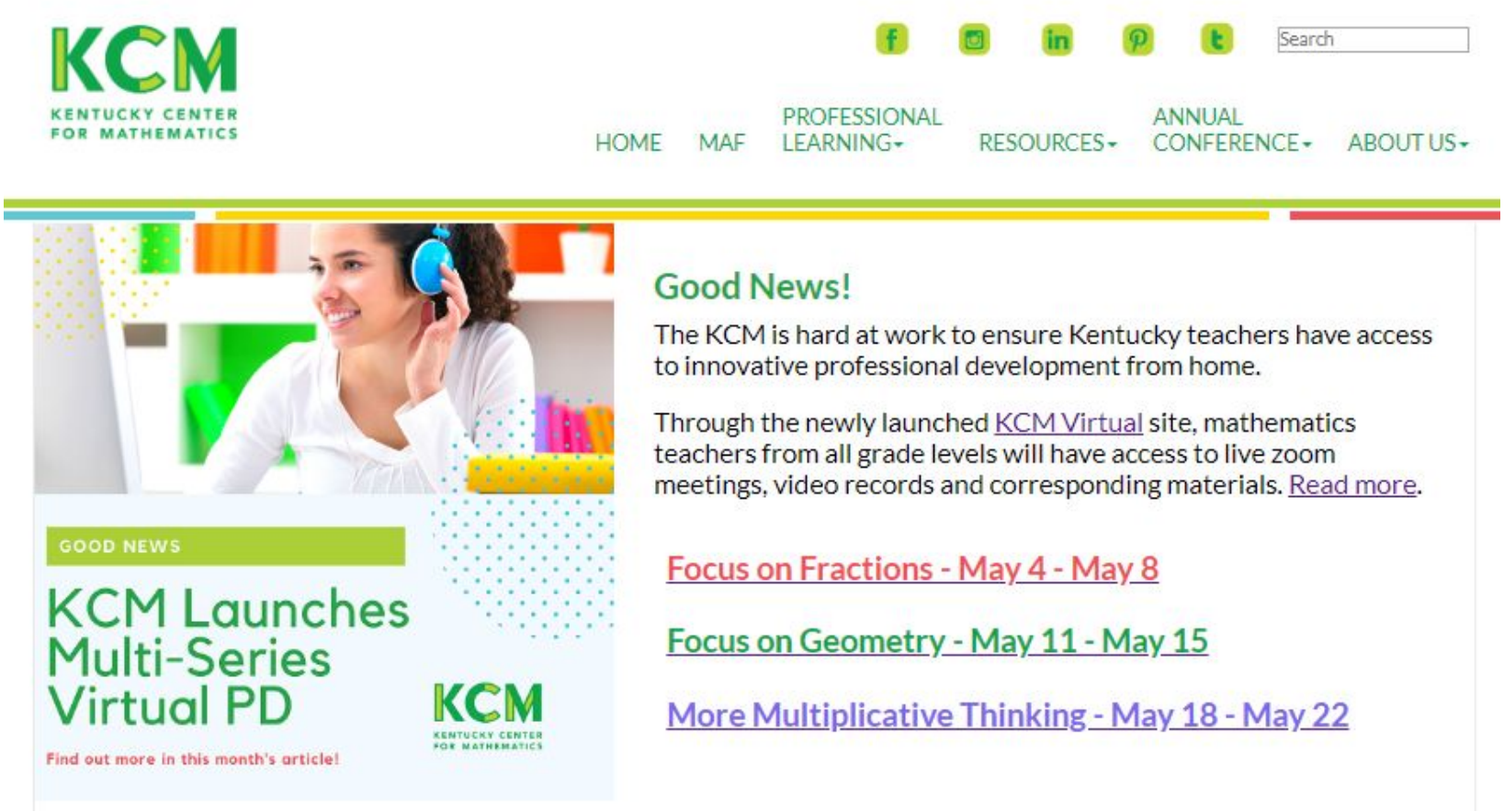
KCM Website

www.kentuckymathematics.org



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Today's Agenda

Research

Standards

Getting there

- Strategies for Multiplication
- Strategies for Division

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Consider this...

Most of us learned to add whole numbers by lining numbers up vertically and adding each column, carrying/borrowing when necessary. But did we understand why it worked? It's most likely that the "how" of doing the computation was "drilled" into our heads, but the "why" was lost.

"Knowledge not supported by understanding is fragile!"

Bill McCallum

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Keep in mind…

The standard algorithm is not a bad thing; it has been in use for years and, when used correctly, works every time.

Although the standard algorithm is often introduced and used in primary grades, *the expectation* (Kentucky Academic Standards) is that students not formally use the an algorithm for addition and subtraction until grade 4, and multiplication and division until grade 5. Why do you think that is the case?

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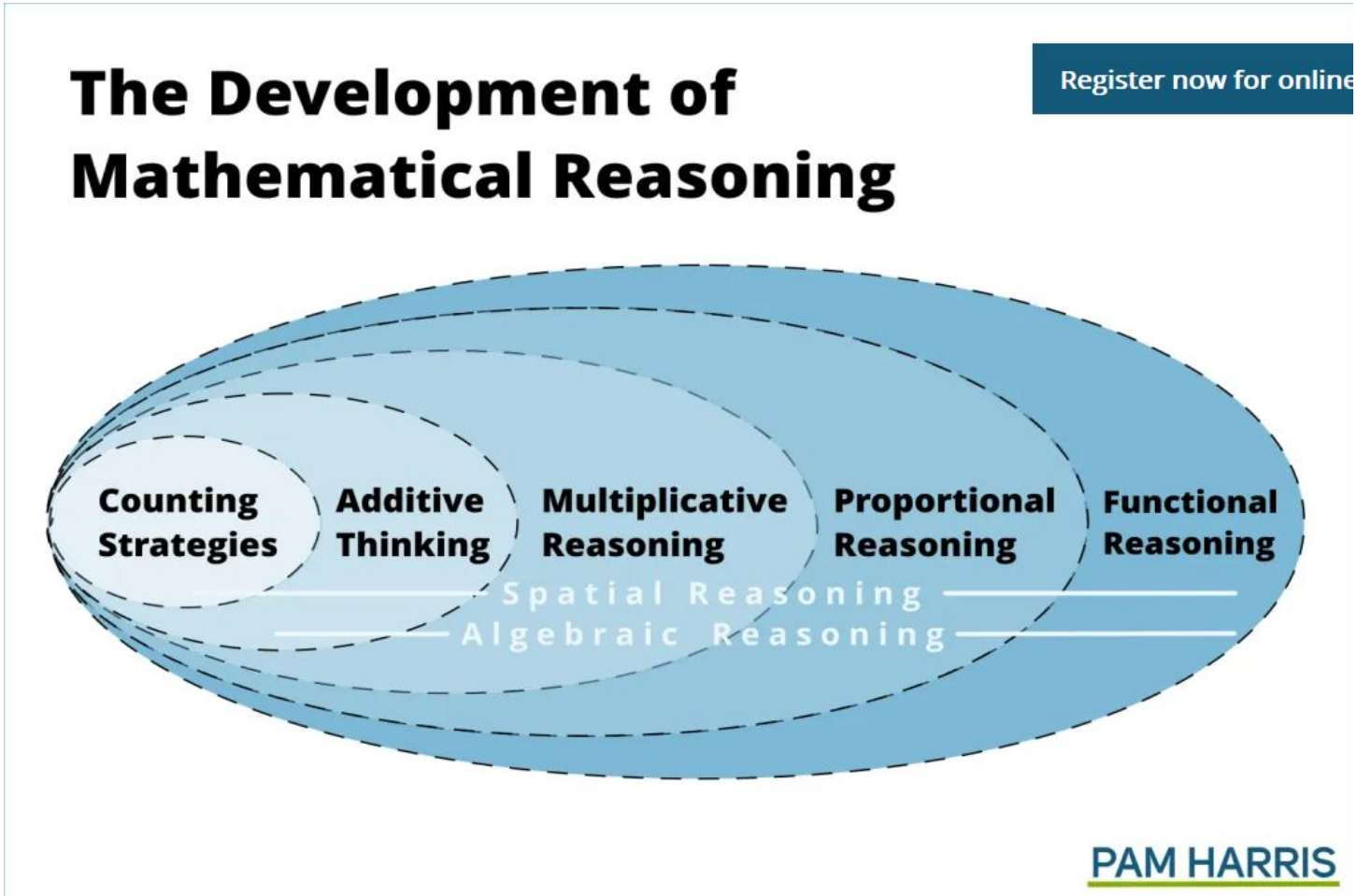
Research from Pamela Weber Harris

https://www.mathisfigureoutable.com/development/



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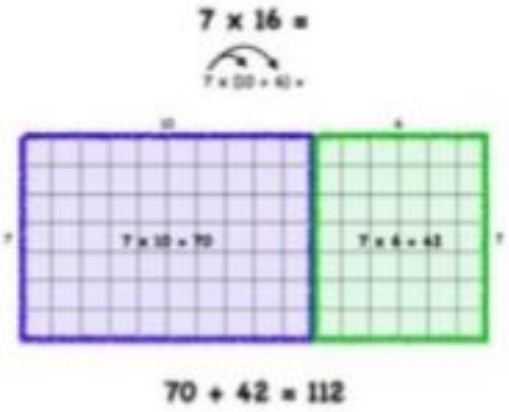
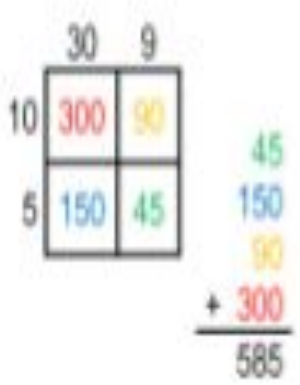
What do the standards say?

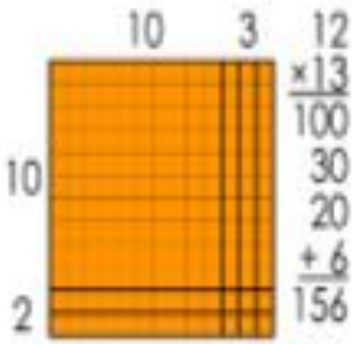
KY.4.NBT.5 Multiply whole numbers

- Up to four digit number by a one-digit number
- Two-digit number by two-digit number

Multiply using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.







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What the standards say?

KY.4.NBT.6 Divide up to four-digit dividends by one-digit divisors. Find whole number quotients and remainders using

- strategies based on place value
- the properties of operations
- the relationship between multiplication and division

Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.

$5{,}500 \div 4 = ?$

	1,000	300	70	5	
4	1,000 x 4	300 x 4	70 x 4	5 x 4	1,000 300 70 + 5
	4,000	1,200	280	20	1,375

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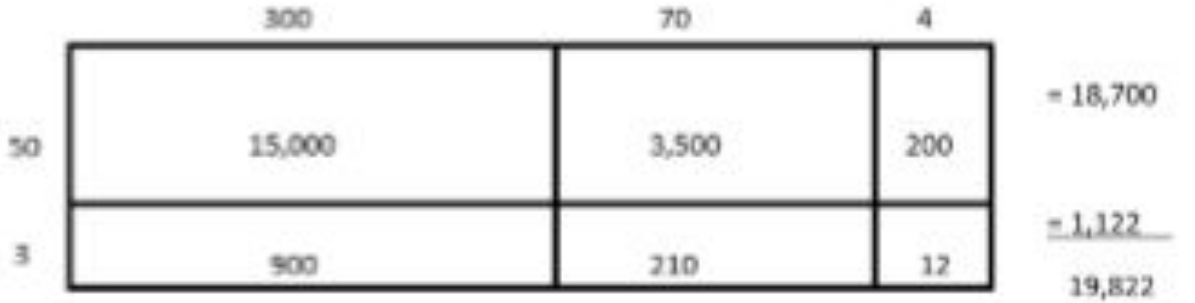
What do the standards say?

KY.5.NBT.5 Fluently multiply multi-digit whole numbers (not to exceed four-digit by two-digit multiplication) using an algorithm.



374
× 53
12 (3 × 4)
210 (3 × 70)
900 (3 × 300)
200 (50 × 4)
3,500 (50 × 70)
15,000 (50 × 300)
19, 822

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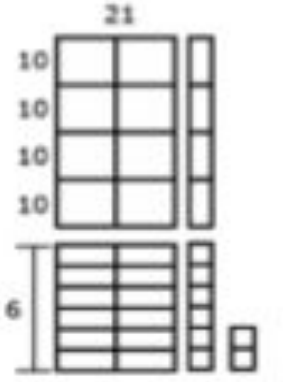
What do the standards say

KY.5.NBT.6 Divide up to four-digit dividends by two-digit divisors.

a. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors using…

- strategies based on place value
- the properties of operations
- the relationship between multiplication and division

b. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.



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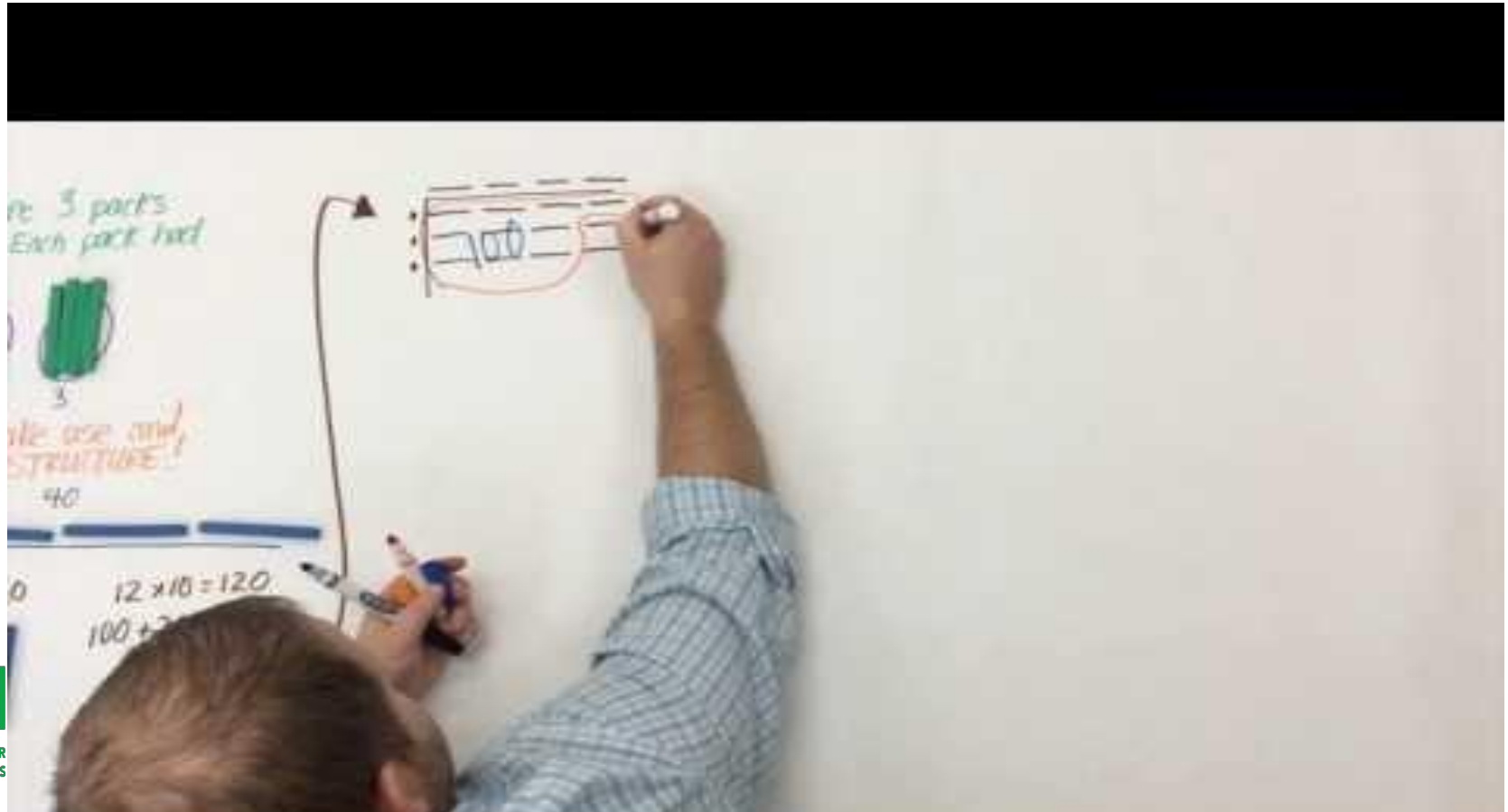
The Progression of Multiplication

[The Progression of Multiplication HD]



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Three Act Tasks

Sugar Packets

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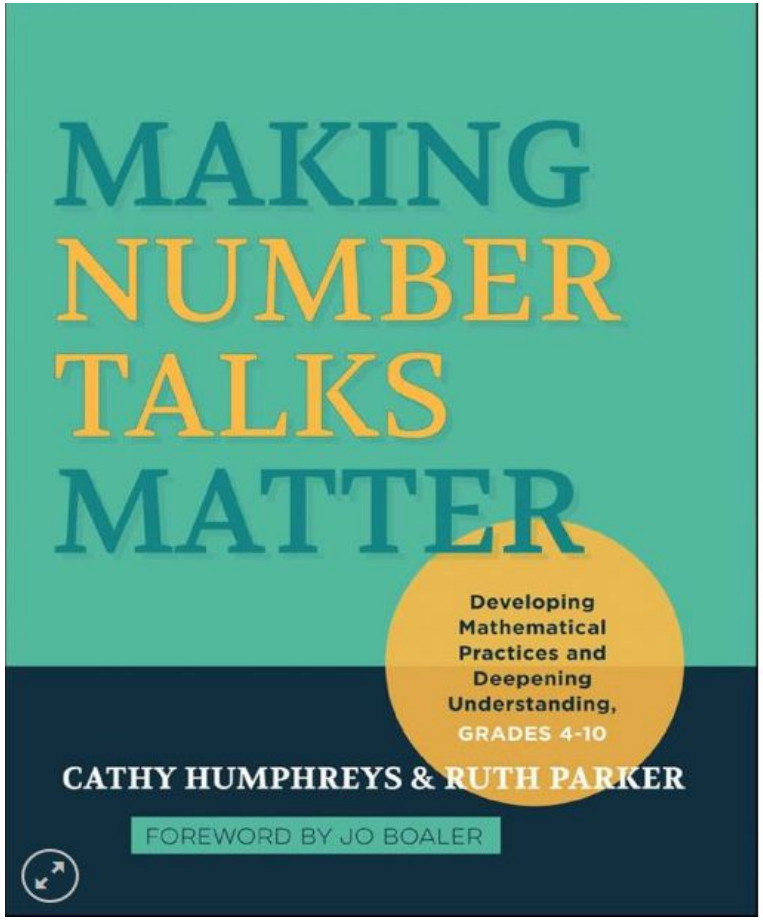
Four Strategies for Multiplication

- Break a factor into addends
- Factor a factor
- Round a factor and adjust
- Halving and doubling



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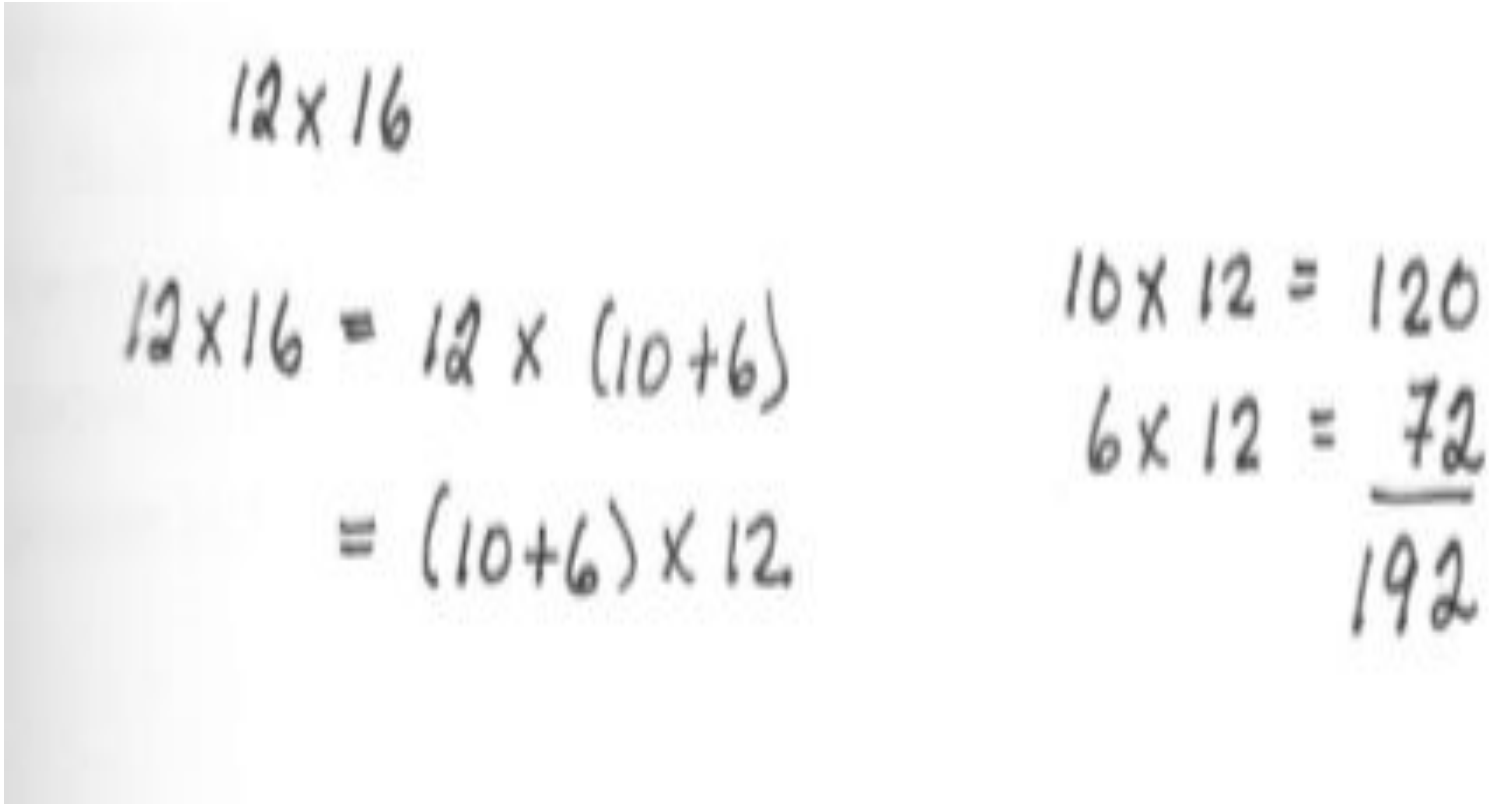
Break a Factor into two or more addends



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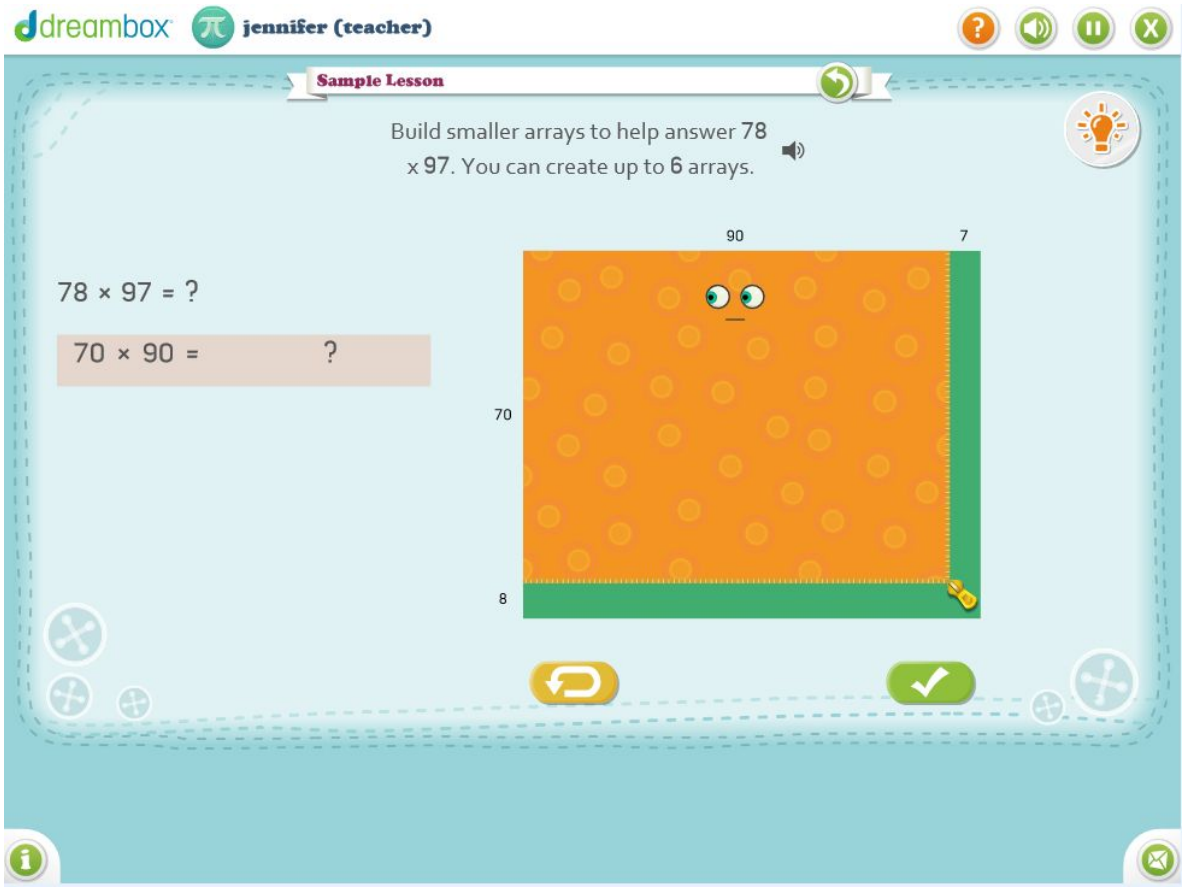
Dreambox Teacher Tools

https://www.dreambox.com/teachertools



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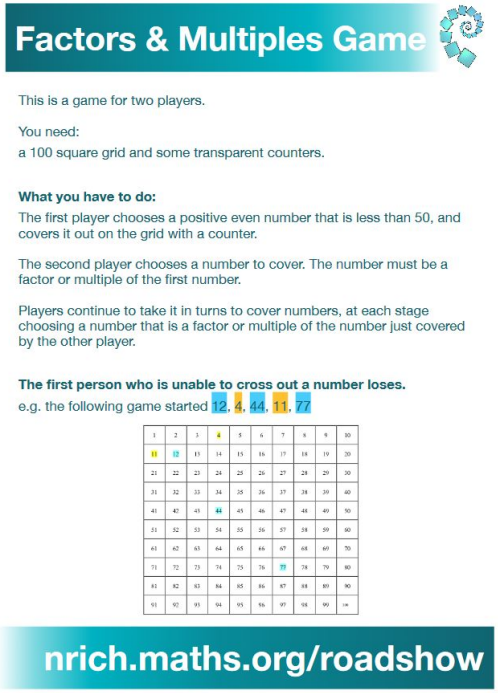
Factors and Multiples Game

[Factors and Multiples Game]



Factors & Multiples Game

This is a game for two players.

You need:
a 100 square grid and some transparent counters.

What you have to do:

The first player chooses a positive even number that is less than 50, and covers it out on the grid with a counter.

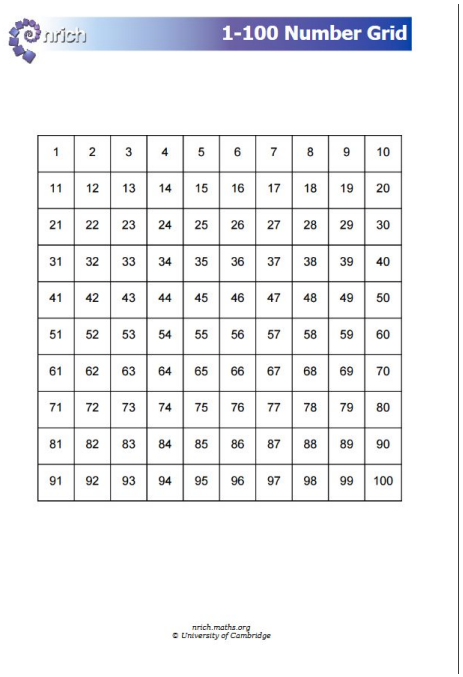
The second player chooses a number to cover. The number must be a factor or multiple of the first number.

Players continue to take it in turns to cover numbers, at each stage choosing a number that is a factor or multiple of the number just covered by the other player.

The first person who is unable to cross out a number loses.

e.g. the following game started 12, 4, 44, 11, 77

nrich.maths.org/roadshow



nrich **1-100 Number Grid**

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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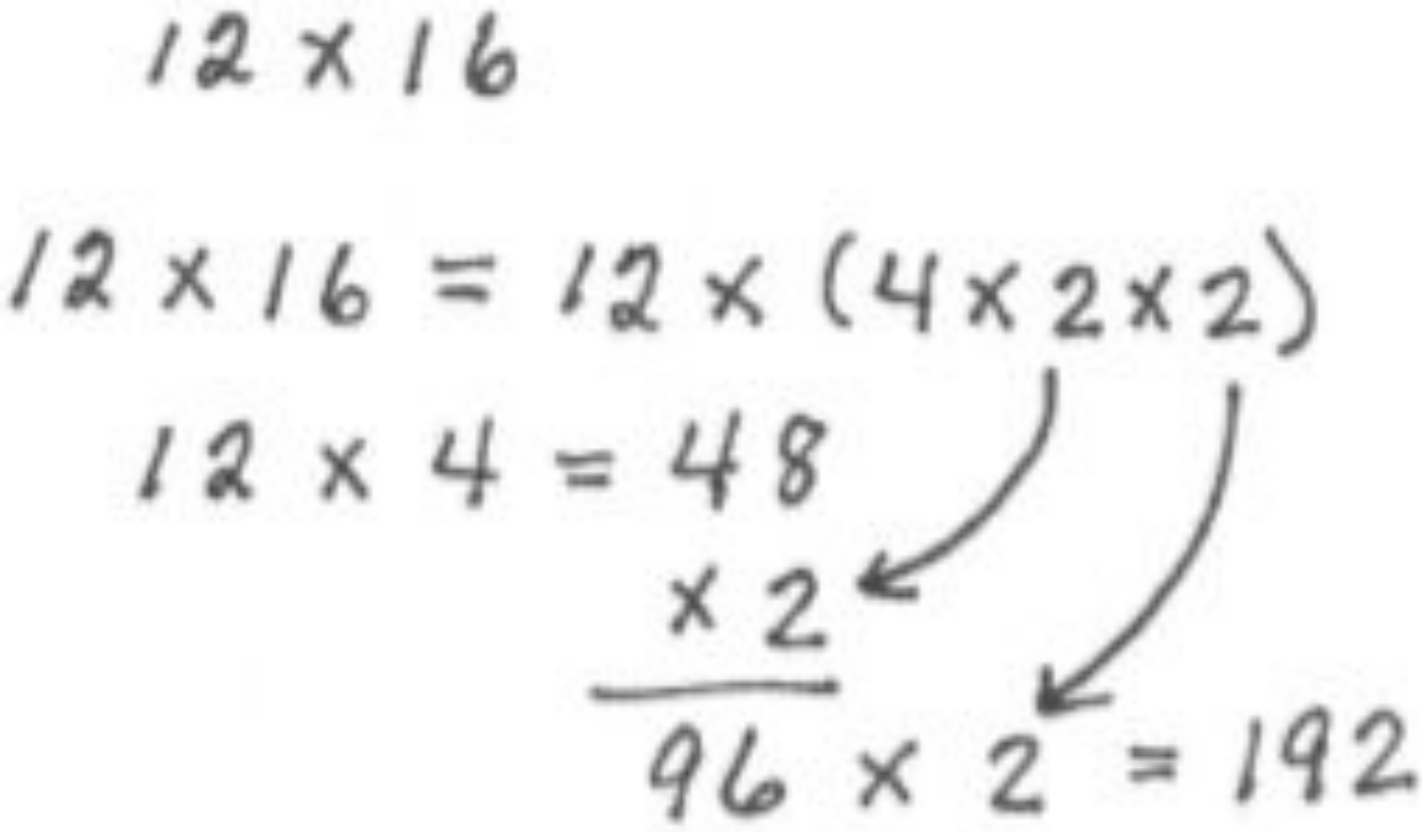
Factor a factor



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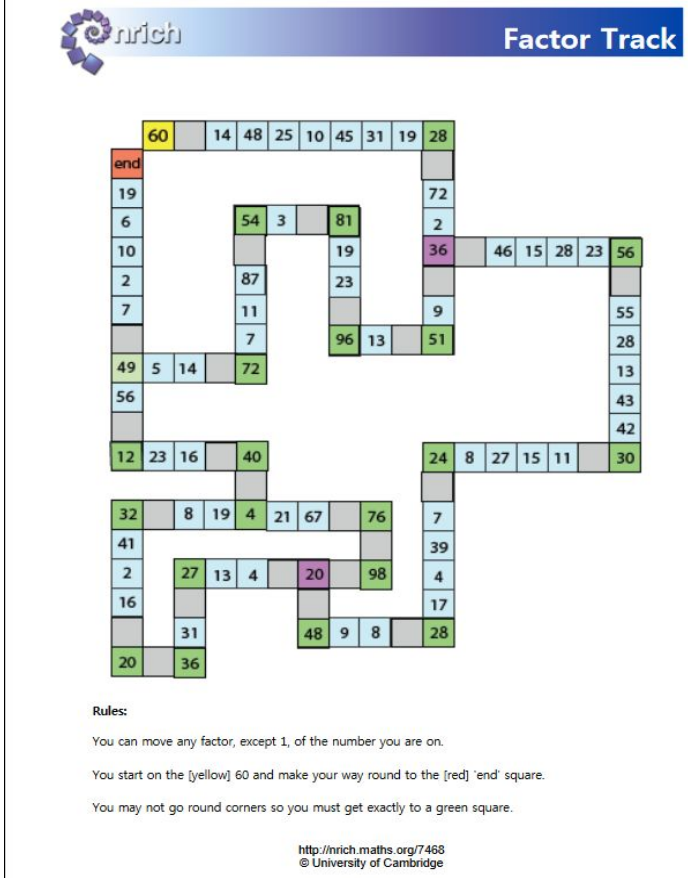
Factor Track

[Factor Track]



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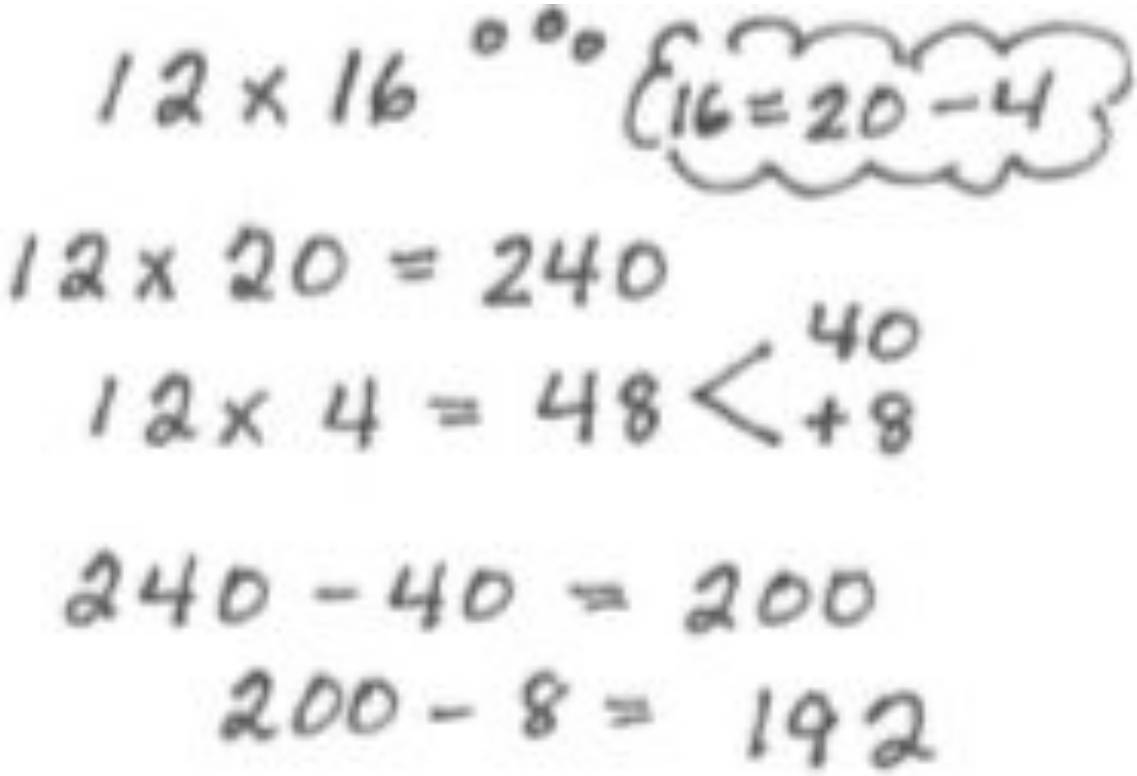
Round a factor and adjust



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Multiplication Race

Multiplication Race

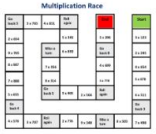
2 x 3 digit

Multiplication Race

Materials: game board, number cube, one counter for each player, calculator

1. Each player places a counter on the box marked 'Start'.
2. Take turns to roll a number cube and move forward that number of spaces along the path. Solve the multiplication problem you land on or follow the instruction you land on.
3. Partners use a calculator to check each other's work. A player who gives an incorrect product must miss a turn.
4. Continue until one player reaches the box marked 'End'.

Extension: Create your own Multiplication Race board and try it out with a partner.



2 x 3 digit

Multiplication Race

Go back 5	83 x 764	94 x 653	Roll again	End		Start
72 x 654			25 x 348	25 x 292		13 x 121
69 x 763		Miss a turn	36 x 896	Go back 8		21 x 242
58 x 982		47 x 358		94 x 695		34 x 615
47 x 884		58 x 312		83 x 772		45 x 672
35 x 653		Go back 5	69 x 467	72 x 563	Roll again	56 x 511
Go back 4						Go back 3
24 x 574	13 x 709	Roll again	92 x 772	89 x 346	Miss a turn	78 x 524	67 x 494

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Halving and doubling

$$12 \times 16$$

$$12 \times 16 = 24 \times 8$$

$$= 48 \times 4$$

$$= 96 \times 2$$

$$= 192$$

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Double and Halve

Double and Halve

ver. 1

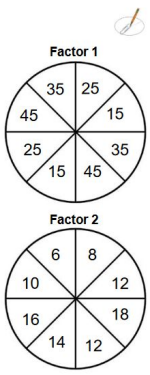
Double and Halve

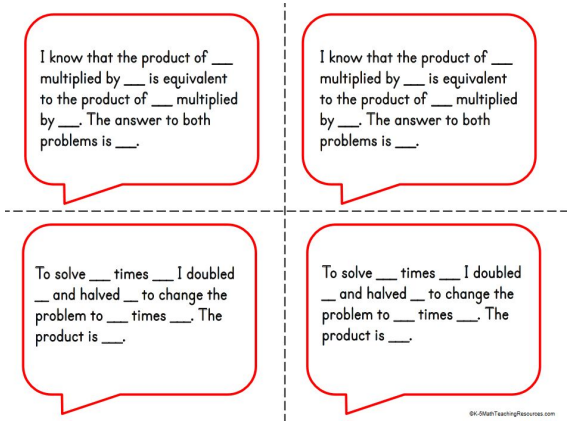
Materials: 10 counters per player, 2 paper clips, 2 pencils, Double and Halve board

1. Work with a partner. Collect 10 counters each.
2. Take turns to spin a paper clip on each spinner. Use the two numbers the paper clips land on to create a multiplication problem.
3. Double one factor and halve the other to change the problem to one with an equivalent product that is easy to solve mentally. Explain your strategy.
4. Place a counter on the multiplication fact on the board. If the multiplication fact is already covered play passes to the next player.
5. Continue playing until one player has placed all ten counters on the board.

Double and Halve

30 x 3	50 x 3	70 x 3	90 x 3
30 x 4	50 x 4	70 x 4	90 x 4
30 x 5	50 x 5	70 x 5	90 x 5
30 x 6	50 x 6	70 x 6	90 x 6
30 x 7	50 x 7	70 x 7	90 x 7
30 x 8	50 x 8	70 x 8	90 x 8
30 x 9	50 x 9	70 x 9	90 x 9





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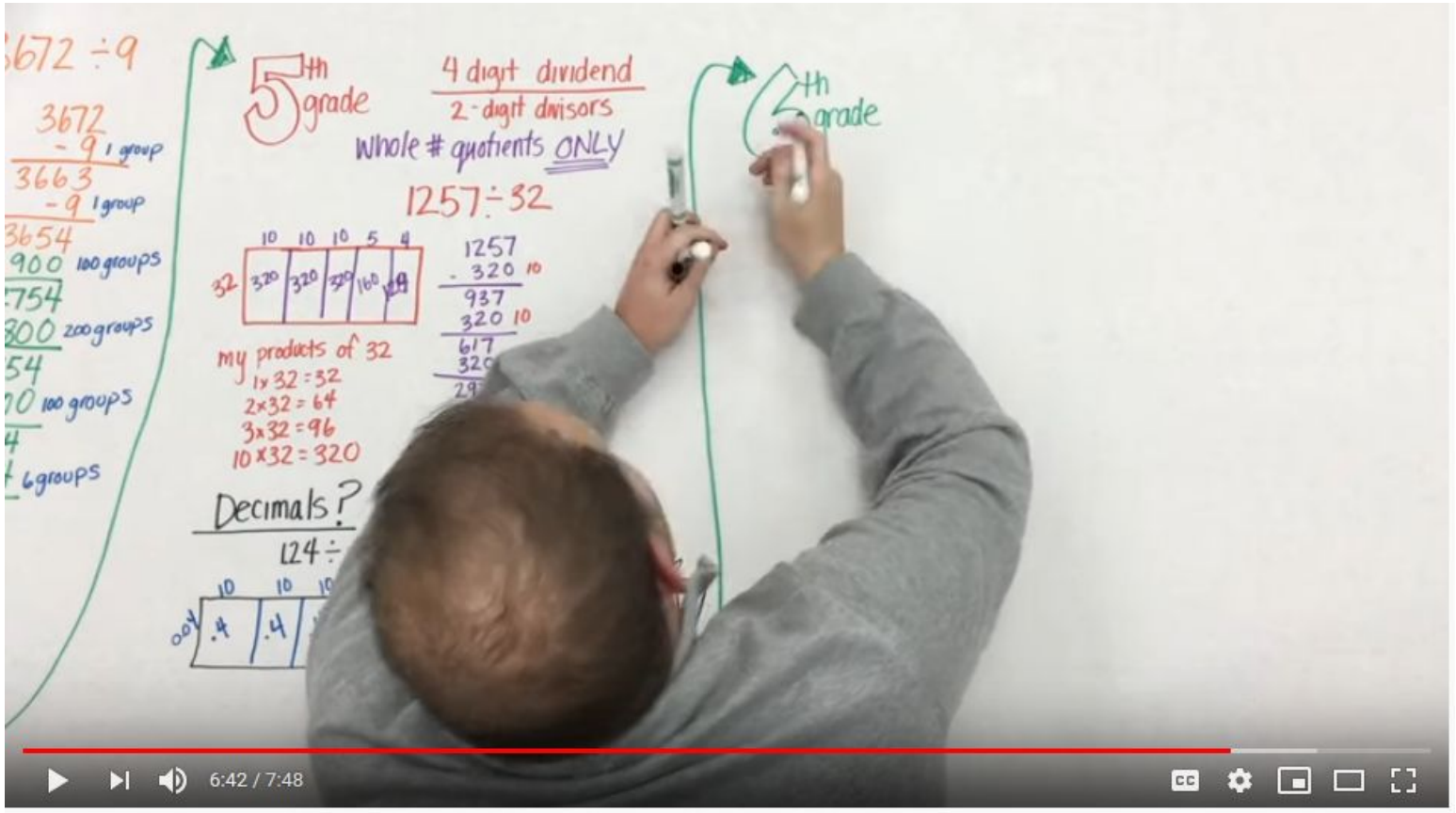
Progression of Division

[The Progression of Division HD]



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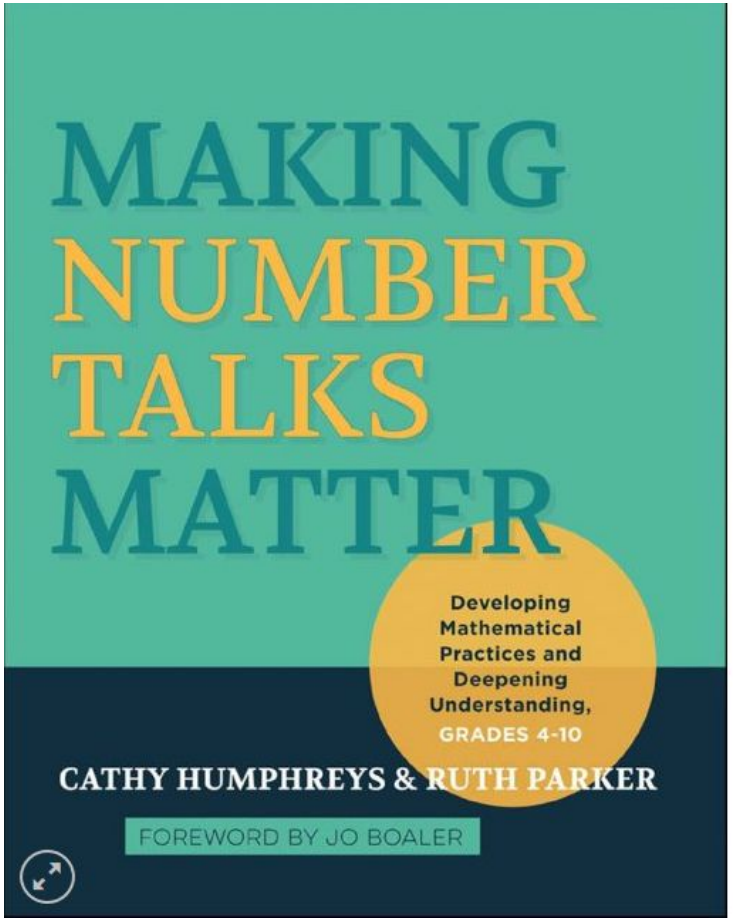
Strategies for Division

- Multiply instead
- Chunk out
- Make a tower
- Halving and halving



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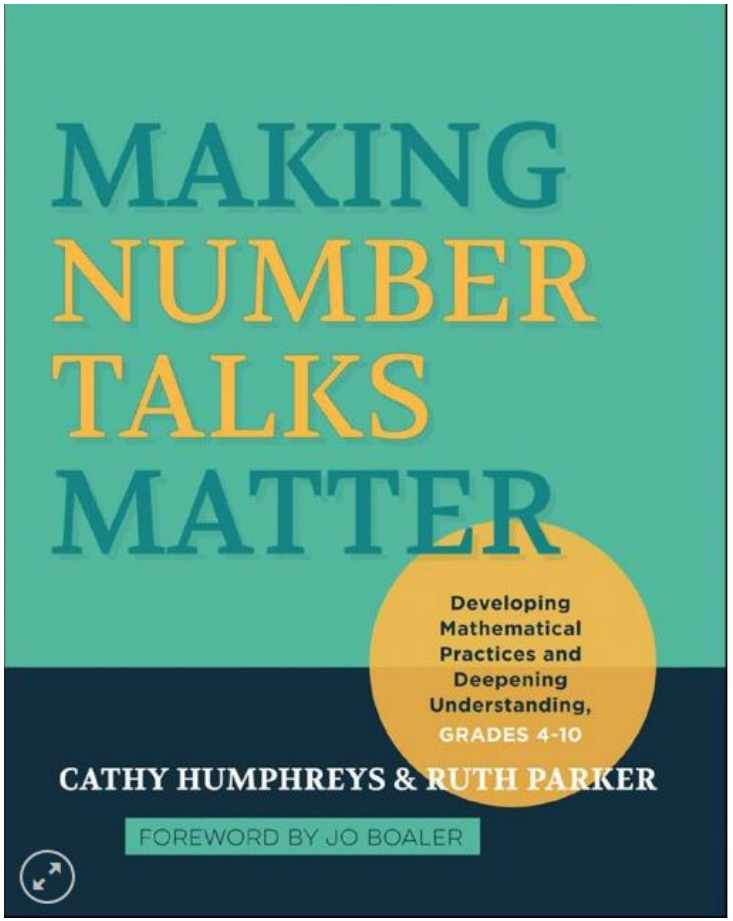
Strategies for Division

"...when computers and calculators truly come of age in the schools, paper and pencil long division will probably be 'as dead as a dodo bird'".

Richard Anderson

"...there is no longer a job in the world--not one single job--where someone does long division with paper and pencil; not one job, that is, other than teaching,"

Maier 1982



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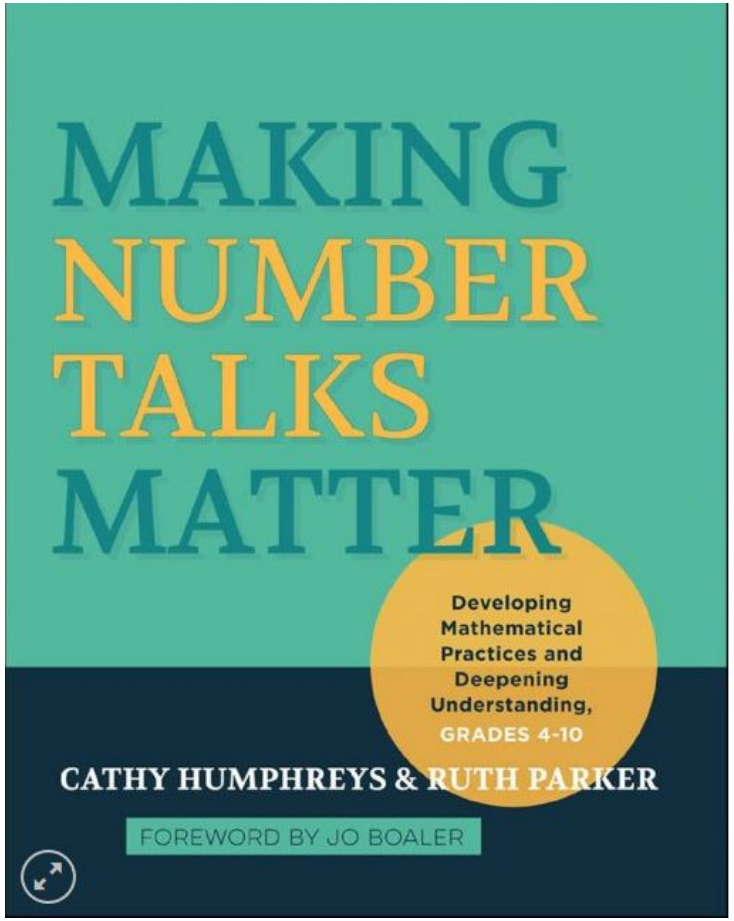
Strategies for Division

Through Number Talks, students can make sense of division (the operation--not the standard algorithm)--and, by maintaining a focus on the relationships between quantities, learn how to size up a problem to determine a reasonable "ballpark" answer.

Cathy Humphreys and Ruth Parker



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Multiply instead

17/3

“I know 3 times 5 is 15, so I have 5 groups of 3; then I have 2 left, so remainder of 2.

Origo at Home

Each of the weeks below provides five daily activities organized around a specific math topic. To help establish a rhythm for learning at home, each day of the week has a focus:

- Monday — Read and Discuss (Grades K–2) or Watch and Talk (Grades 3–5)
- Tuesday — Hands-on Math
- Wednesday — Problem-solving
- Thursday — Game Day (digital practice)
- Friday — Practice

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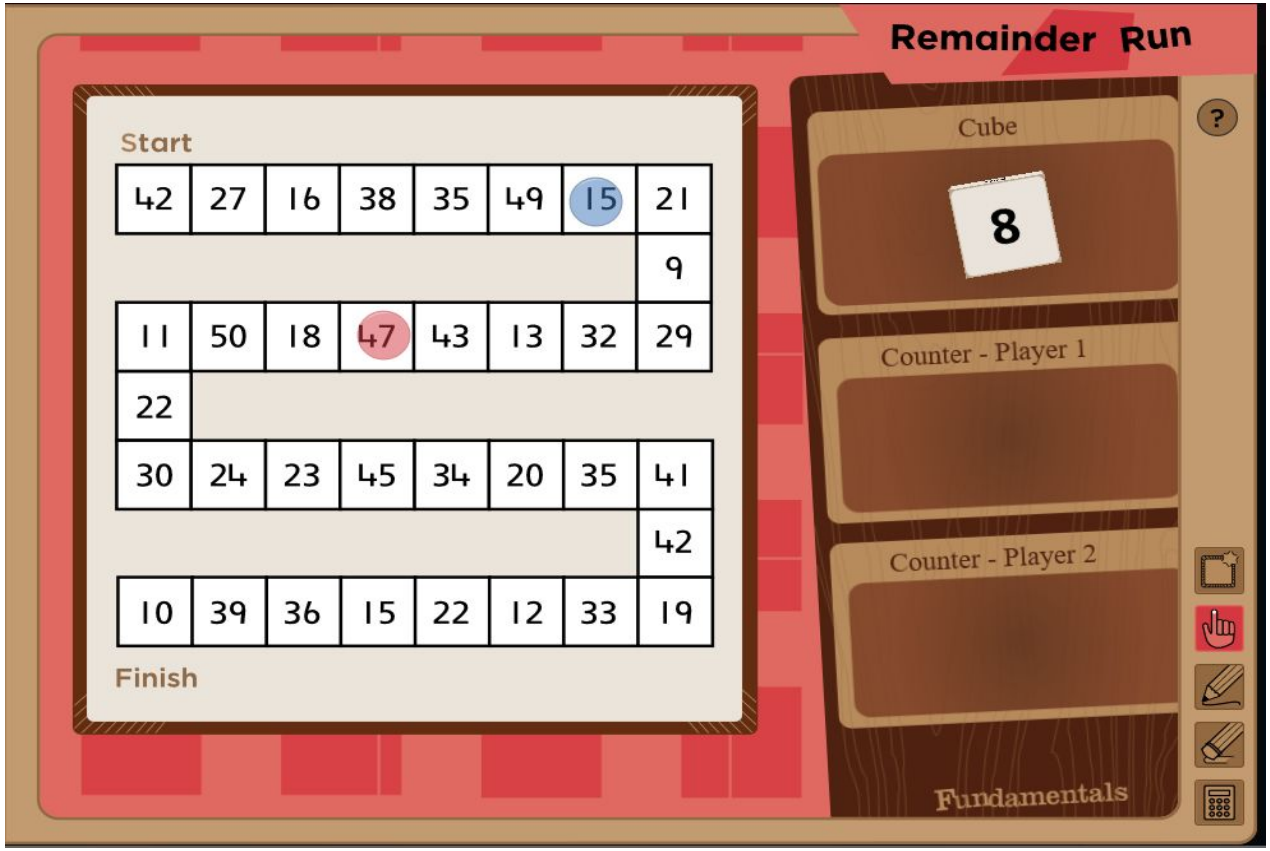
Origo at Home

[ORIGO at Home - Interactive Digital K-6 Math plans & resouces for home]



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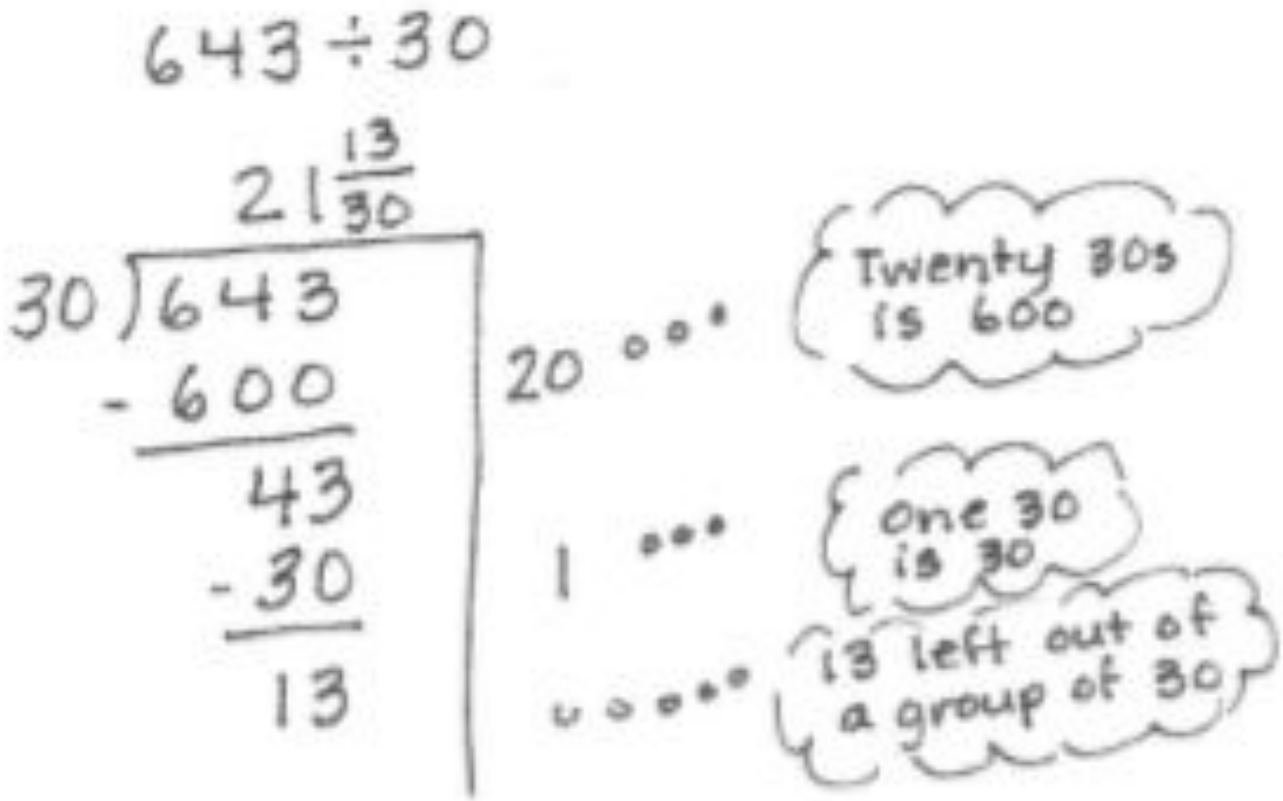
Chunk out



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Chunk it out/ Partition the Dividend

Division Strategy: Partition the Dividend

Division Strategy: Partition the Dividend

Materials: Division Equations Board

1. Work with a partner. Choose a line of four problems from the board (vertically, horizontally or diagonally) that you will both solve.
2. Solve each problem by breaking the dividend into parts that are easy to divide. Solve the easier problems and then add the partial quotients.

Example: 72÷5

72
50+22 (÷5)
10 + 4R2 = 14 R2

or

$$5\overline{)50+22}\quad 10 + 4\ R2 = 14\ R2$$

256÷7

256
210+46 (÷7)
30 + 6R4 = 36 R4

or

$$7\overline{)210 + 46}\quad 30 + 6\ R4 = 36\ R4$$

3. Check your work with your partner. Then repeat with another line of four problem

ver. 1

101 ÷ 9	91 ÷ 7	104 ÷ 8	125 ÷ 5
64 ÷ 4	58 ÷ 4	48 ÷ 3	79 ÷ 6
73 ÷ 6	97 ÷ 8	67 ÷ 5	41 ÷ 3
49 ÷ 3	120 ÷ 9	72 ÷ 5	84 ÷ 6

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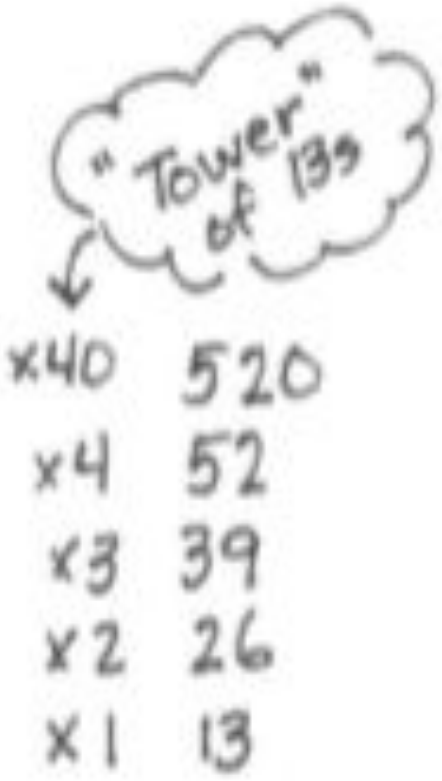
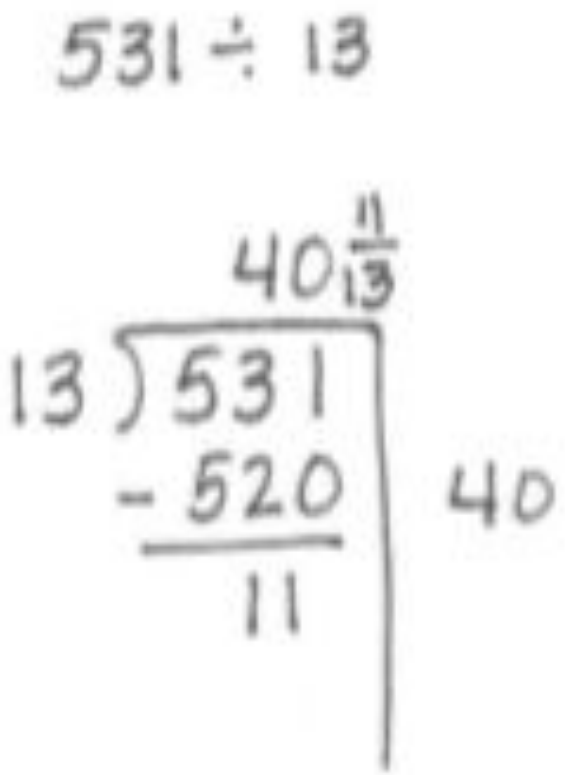
Make a tower





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Dreambox Teacher Tools

https://www.dreambox.com/teachertools

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Partial Quotients

https://www.k-5mathteachingresources.com/support-files/division-strategy-partial-quotients-ver.3.pdf

Division Strategy: Partial Quotients

Materials: Division equations boards

1. Work with a partner. Choose a line of four problems from the board (horizontally, vertically or diagonally) that you will both solve using the partial quotients algorithm.

Step 1: Write a list of easy facts for the divisor.

Step 2: Subtract from the dividend an easy multiple of the divisor (e.g. x 10, x 100, x 200 etc.) Record the partial quotient in a column to the right of the problem.

Step 3: Repeat until the dividend has been reduced to zero, or the remainder is less than the divisor.

Step 4: Add the partial quotients to find the final quotient.

Example: 3,863 ÷ 16

Easy Facts for 16

10 x 16 = 160
20 x 16 = 320
30 x 16 = 480
100 x 16 = 1,600
200 x 16 = 3,200

```
230 r3
16) 3683
  - 3200   200 x
    483
  - 320     20 x
    163
  - 160     10 x
      3
```

3. Check your work with your partner.

4. Repeat with another line of four problems.

3,292 ÷ 16 =	3,624 ÷ 17 =	3,155 ÷ 15 =	2,929 ÷ 14 =
6,835 ÷ 17 =	3,973 ÷ 13 =	4,836 ÷ 16 =	3,919 ÷ 19 =
4,591 ÷ 15 =	4,834 ÷ 16 =	3,828 ÷ 19 =	4,580 ÷ 15 =
4,984 ÷ 16 =	3,463 ÷ 34 =	1,831 ÷ 16 =	3,768 ÷ 18 =

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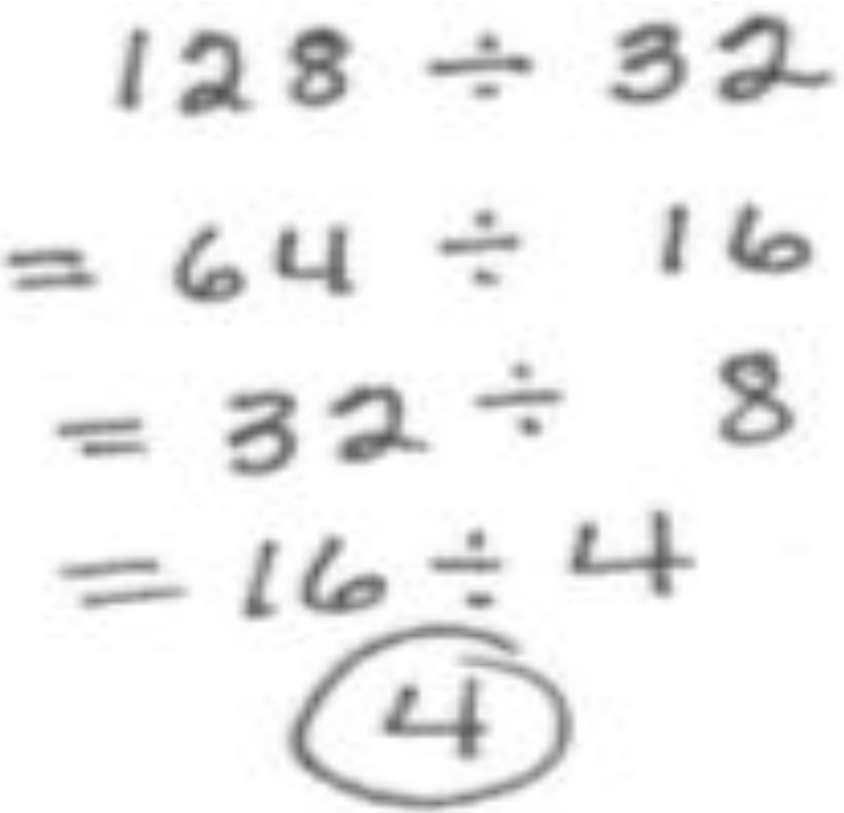
Halving and halving



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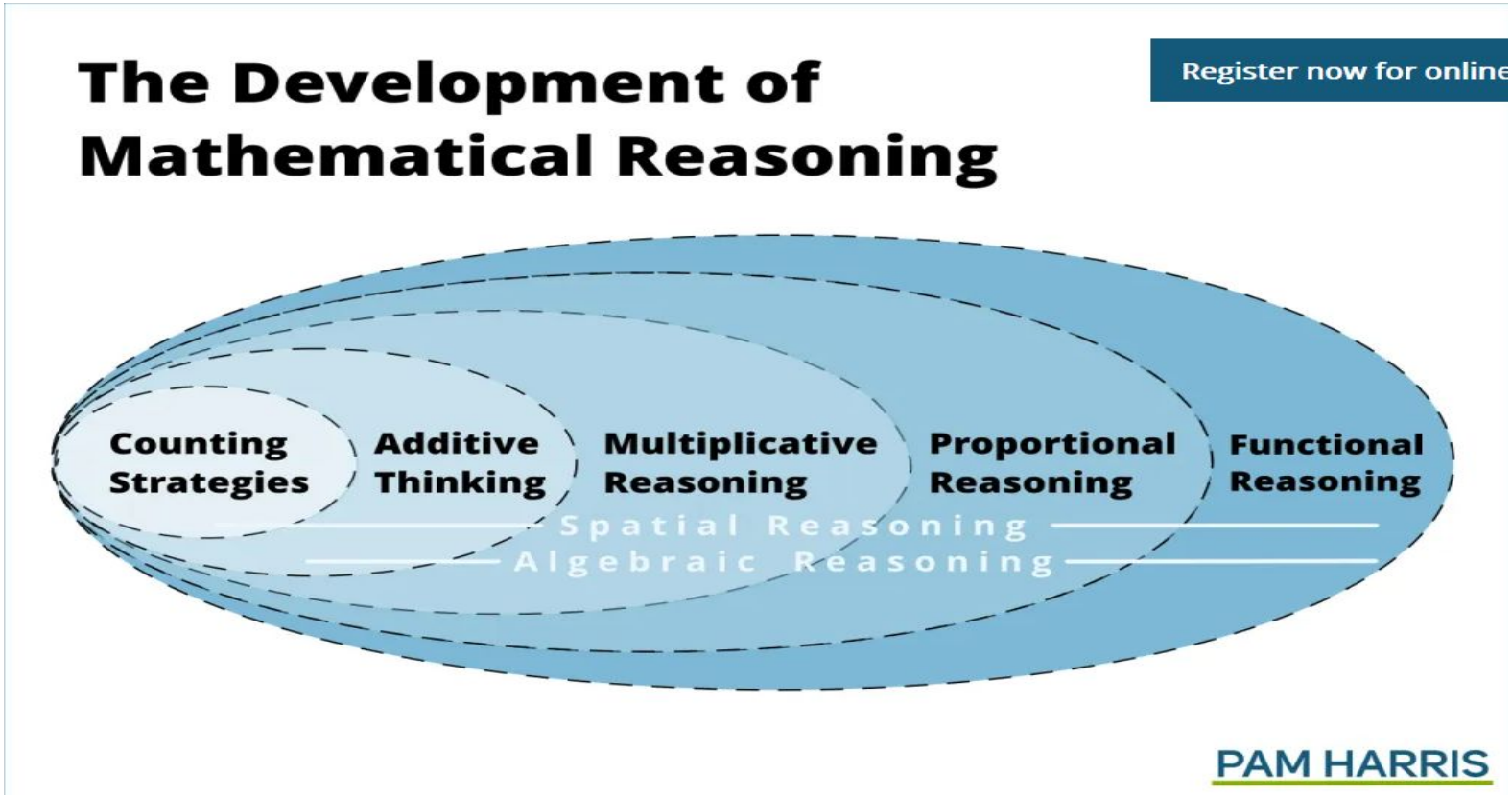
All of these strategies are useful with any numbers, including fractions, decimals, and negative numbers!

These strategies help students make use of properties of operations, which leads to algebraic understanding.



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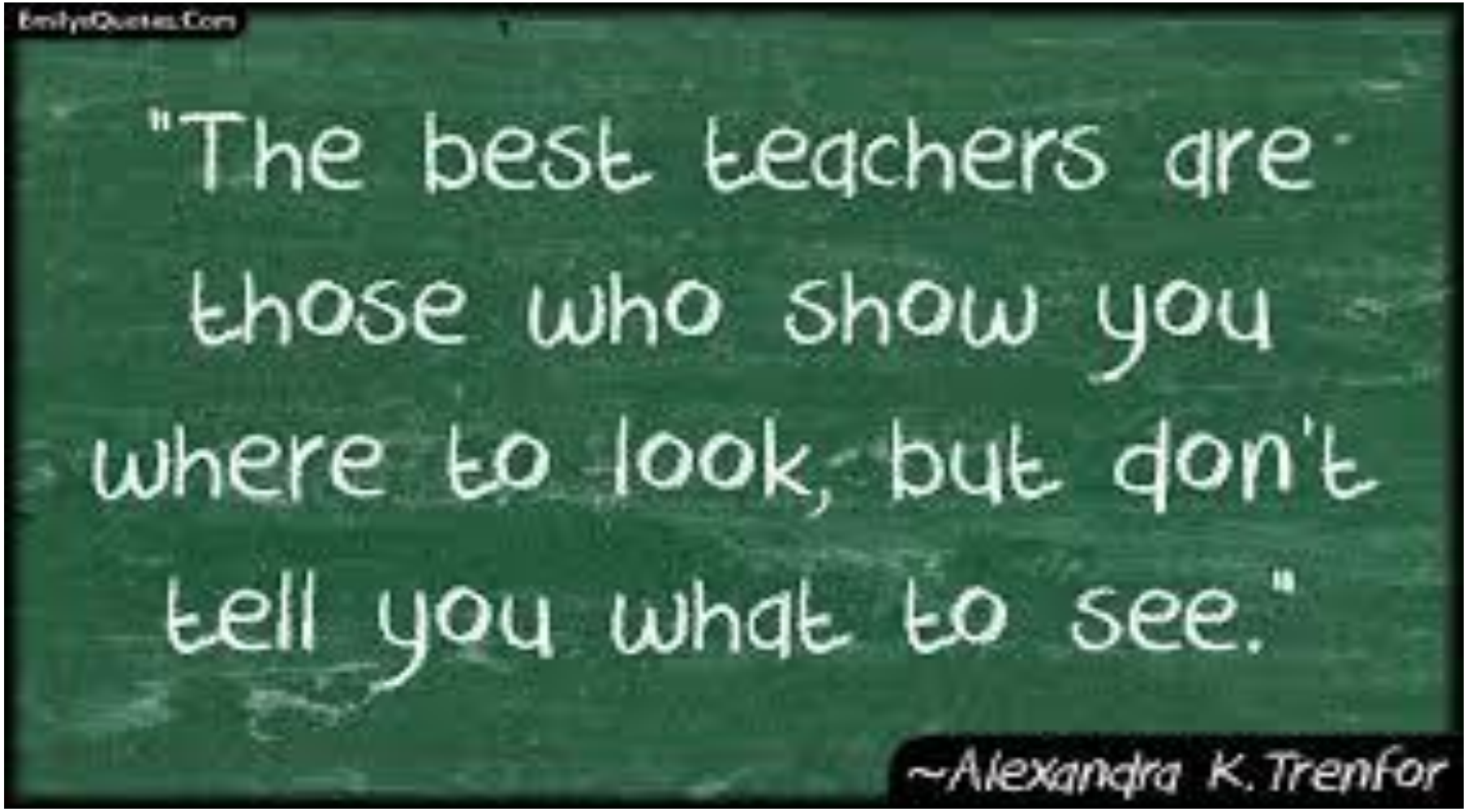

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