KCM

KENTUCKY CENTER FOR MATHEMATICS

Focus on Geometry

Geometric Measurement

Welcome!

Your host

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KCM Website

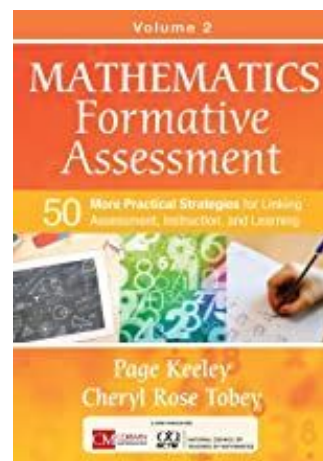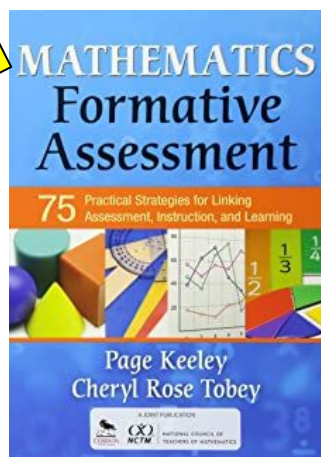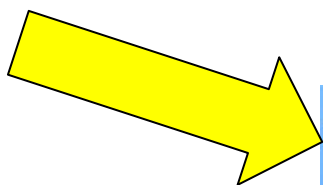## www.kentuckymathematics.org

Agenda

- Standards
- Research
  - Focus on Schema
- Volume
- 2 Dimensional Measurement of Circles
  - Circumference
  - Area
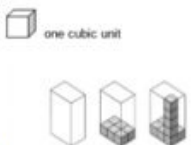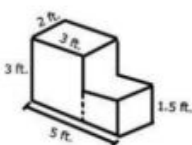
Activator

POMS
Point Of Most Significance

As our summarizer, I will be asking you to tell me what is the **POMS** for you from today's session.

Standards

Measurement and Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	
Standards	**Clarifications**
KY.5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. **MP.6**	a. one cubic unit b. Coherence KY.3.MD.5→ KY.5.MD.3
KY.5.MD.4 Measure volumes by counting unit cubic cm, cubic in, cubic ft. and improvised units. **MP.5, MP.6**	Coherence KY.3.MD.6→ KY.5.MD.4
KY.5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes. b. Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by	For example, students determine the volume of concrete needed to build the steps in the diagram below. 2 ft. 3 ft. 3 ft. 1.5 ft. 5 ft. Coherence KY.4.MD.3→ KY.5.MD.5→KY.6.G.2

Standards	**Clarifications**
adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. **MP.1, MP.4, MP.8**	
Attending to the Standards for Mathematical Practice	
Students use cubes to cover a bottom layer of a rectangular prism, understanding cube as a unit cube (**MP.5**). As students place the cubes in layers to fill the rectangular solid, they notice the number of cubes in each layer can be found by multiplying [number of cubes in one row] x [number of rows] and this product (the base) can be multiplied by how many layers to determine how many unit cubes will fill the container (**MP.8**). Students connect this idea to the formulas for volume and use these formulas to solve problems (**MP.4**). When a three-dimensional shape is not a single rectangular solid, students analyze the shape and its measurements to determine how to decompose the shape and find the volume of each prism (**MP.1**).	

Standards

Geometry
Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.	MP.5. Use appropriate tools strategically.
MP.2. Reason abstractly and quantitatively.	MP.6. Attend to precision.
MP.3. Construct viable arguments and critique the reasoning of others.	MP.7. Look for and make use of structure.
MP.4. Model with mathematics.	MP.8. Look for and express regularity in repeated reasoning.

Cluster: Solve real-world and mathematical problems involving area, surface area and volume.	
Standards	**Clarifications**
KY.6.G.1 Find the area of right triangles, other triangles, special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and quadrilaterals; apply these techniques in the context of solving real-world and mathematical problems. **MP.1, MP.6, MP.8**	Area of the listed shapes may be thought of as a rectangle with larger area, subtracting the areas exterior to the actual shape to obtain the true area, or as a composite area of smaller triangles and rectangles which sum to the true area of the given shape. Students recognize given shapes can be combined to find area or decomposed to find area, such as surface area, and one method may be more efficient than the other. Coherence KY.5.NF.4→KY.6.G.1→KY.7.G.6
KY.6.G.2 Find the volume of a right rectangular prism with rational number edge lengths. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with rational number edge lengths in the context of solving real-world and mathematical problems. **MP.2, MP.5, MP.6**	Coherence KY.5.MD.5→KY.6.G.2→KY.7.G.6
KY.6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **MP.4, MP.5, MP.6**	For example, a gardener draws a map of his garden on a coordinate plane with vertices (-2, 7), (-2, -1), (4, 7). What is the base and height of this triangle? What is the area of his garden, assuming each unit on the coordinate plane is 1 meter? Coherence KY.5.G.2→KY.6.G.3
KY.6.G.4 Classify three-dimensional figures including cubes, prisms, pyramids, cones and spheres. **MP.2, MP.3**	Emphasis is on classifying three-dimensional shapes and specifically the attributes of each shape that make it unique to its classification.

Attending to the Standards for Mathematical Practice
Students make sense of real-world problems involving area, volume and surface area. Students begin to understand any shape can be thought of as a series of simpler shapes, merely stitched together to form a composite shape (**MP.1**). They begin to visualize the volume of any given shape as a bounded region, filled with smaller cubes of equal size (**MP.2**) and understand, by doing so, they approximate the volume of a three-dimensional shape with integer edge lengths (**MP.5**) and then, continue this reasoning by precisely finding the volume of figures with rational edge lengths (**MP.1, MP.6, MP.8**). Generalizing the study of geometric shapes to the coordinate plane gives students a tool to precisely calculate side lengths and area of shapes. When two different units are given within a problem, students know to use previous knowledge of conversions to make the units match before solving the problem (**MP.4, MP.5, MP.6**).

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Standards

Geometry
Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Cluster: Solve real-life and mathematical problems involving angle measure, area, surface area and volume.

Standards	Clarifications

KY.7.G.4 Use formulas for area and circumference of circles and their relationships.
  a. Apply the formulas for the area and circumference of a circle to solve real-world and mathematical problems.
  b. Explore and understand the relationship between the radius, diameter, circumference and area of a circle.
MP.1, MP.2, MP.8
	Circle Formulas: $C = d\pi \quad C = 2r\pi \quad A = \pi r^2$
	Note: Calculating the radius or diameter of a circle given its area is not expected, as finding the square root of a number is reserved for 8th grade.
	  a. Both area and circumference are represented; students recognize when circumference is needed and when area is needed.
	  b. Emphasis is on calculating area given diameter; finding circumference given radius or diameter; and finding radius or diameter given circumference. Special attention given to the relationship between diameter and circumference as a ratio that leads to pi.
	Coherence KY.7.G.4 → KY.8.G.9

Attending to the Standards for Mathematical Practice

A student who merely memorizes the area and circumference formulas for a circle or the area, volume and surface area formulas of other shapes does not have a deep, conceptual understanding of the basis for these equations. Exploring the relationships between radius, diameter, area and circumference limits the confusion inherent in rote memorization, because students are given a context to the concepts (**MP.2, MP.8**). Solving real-world situations involving these quantities gives the student context for their understanding of the mathematics (**MP.1**). In addition, precise drawing or manipulation of technology lends itself to generate definitions (**MP.6**). Students continue their work from grade 6 from solving area problems involving triangles and rectangles to those involving more complex shapes, such as rhombi or trapezoids (**MP.4**). Students may mischaracterize volume and surface area of three dimensional shapes, leading them to develop ways to decide upon whether a situation calls for the volume of a figure, or the surface area of a figure (**MP.3**). The use of nets and other appropriate tools gives students a structure to foster greater understanding of the concept of surface area (**MP.5**).

Research

Conceptual understanding in math is the creation of a robust framework representing the numerous and interwoven relationships between mathematical ideas, patterns, and procedures. This framework can be used to coherently integrate new knowledge and solve unfamiliar problems.
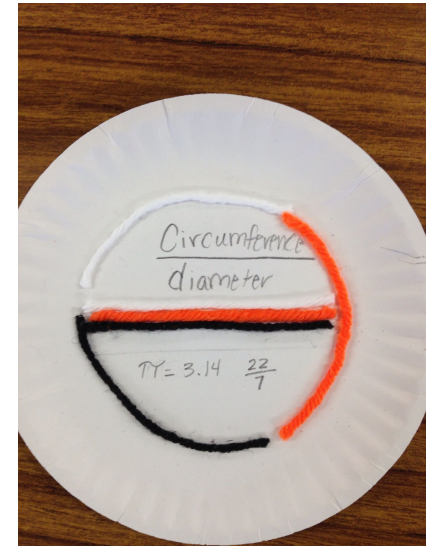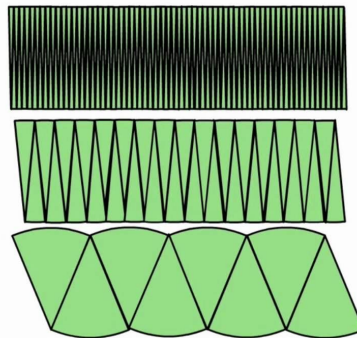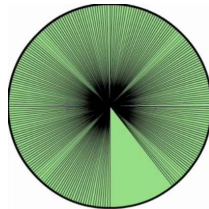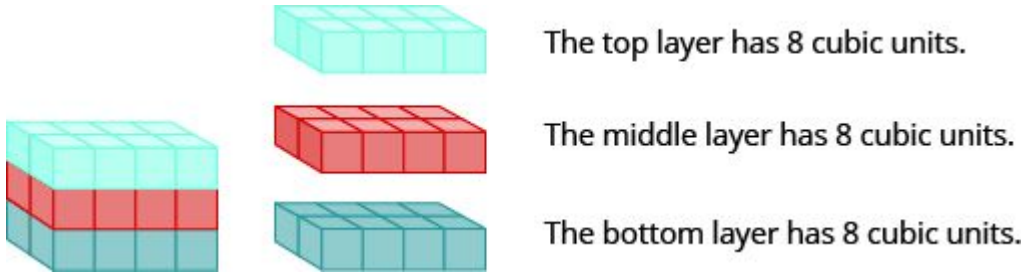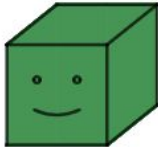
https://blog.mindresearch.org/blog/schemas-deep-conceptual-understanding

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Research

https://blog.mindresearch.org/blog/schemas-deep-conceptual-understanding

Research

Let's build schema!

The top layer has 8 cubic units.

The middle layer has 8 cubic units.

The bottom layer has 8 cubic units.

Geometric Measurement Volume

Meet Cubey. Cubey is a cube. He is one inch long, one inch wide, and one inch high. Say "Hi!" to Cubey. Did he say "hi" back? No? That's because you're talking to a computer screen. Don't worry. Cubey won't judge you.

(Go to the next slide.)

https://teacher.desmos.com/activitybuilder/custom/5adcb99681d4cb0a4e40b34c

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Geometric Measurement Volume

https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Cubes/

Geometric Measurement Volume

https://gfletchy.com/packing-sugar/
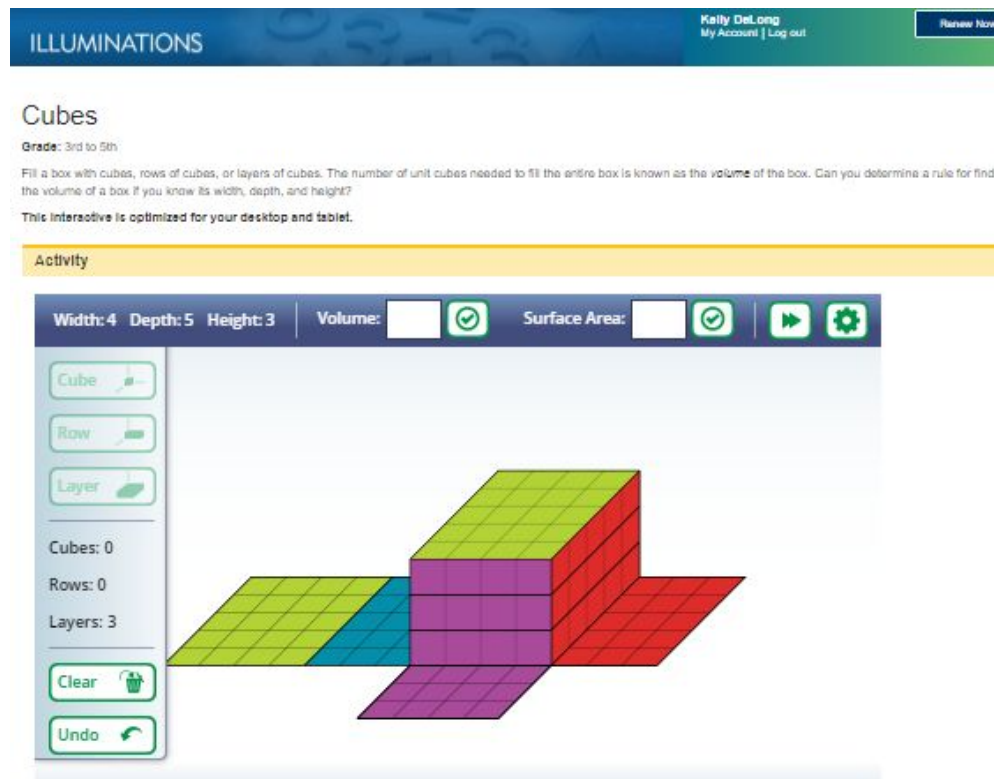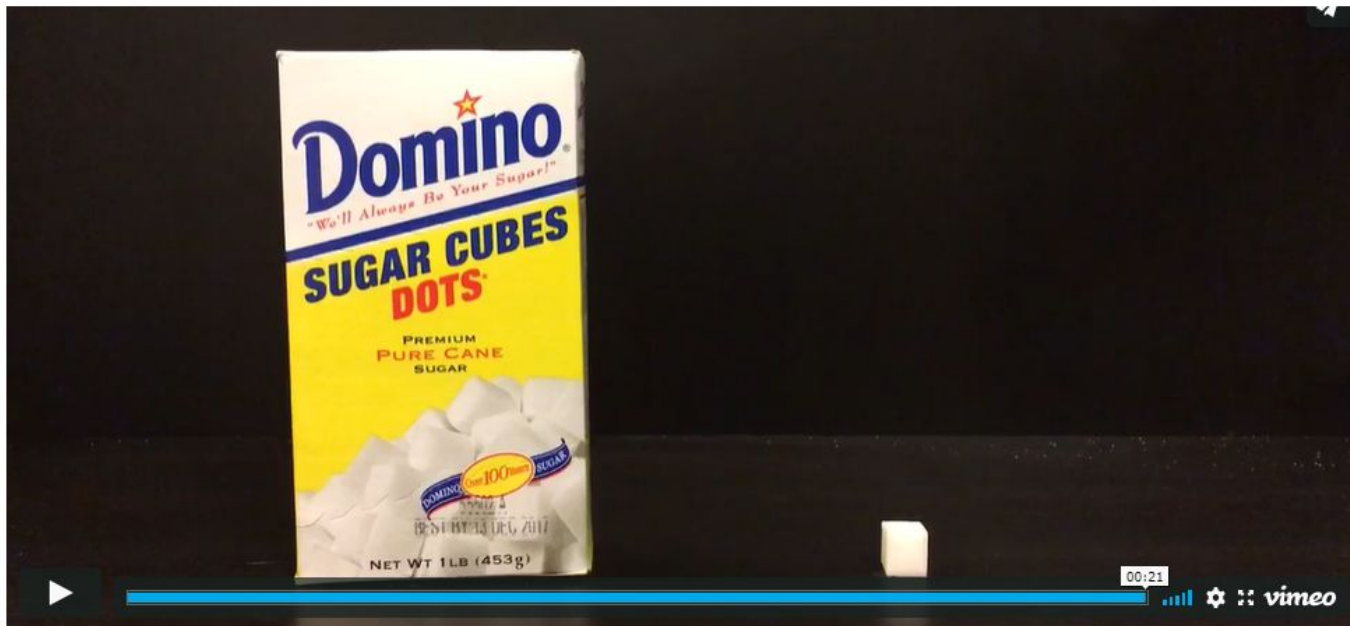
Geometric Measurement Volume
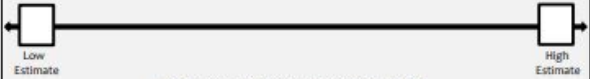
Name: _____   Date: _____

1. What did you notice?

2. What do you wonder?

3. Main Question:

4. Estimate

Low Estimate ◄───────────────────────────► High Estimate

place your best estimate on the number line and label

5. What information would you like to know?

6. Answer

https://gfletchy.com/3-act-lessons/

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Geometric Measurement Volume

https://gfletchy.com/got-cubes/

Geometric Measurement Literature Integration
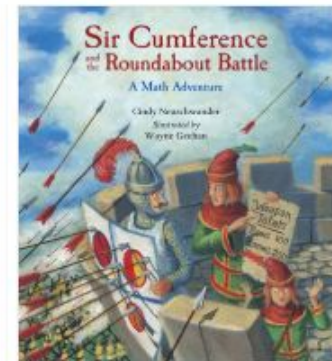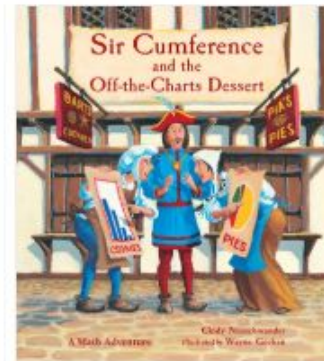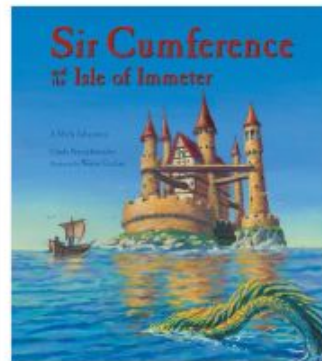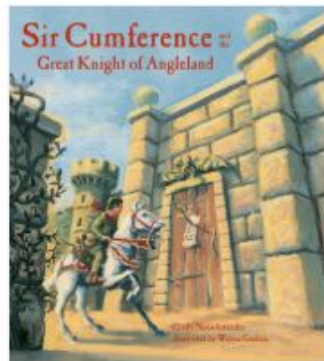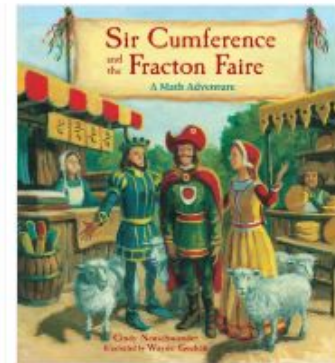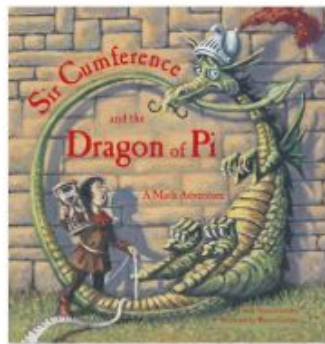
Kids LOVE this math literature series!

Geometric Measurement Literature Integration
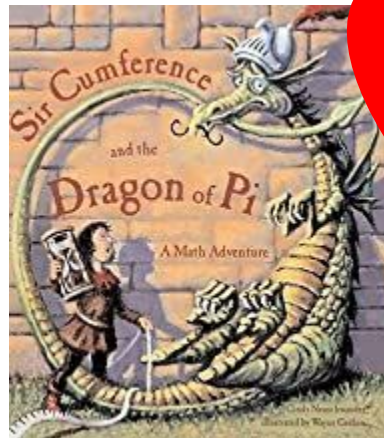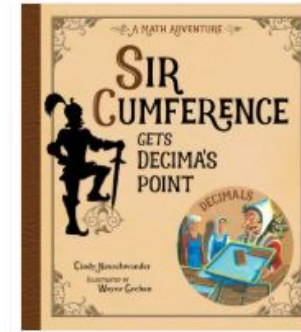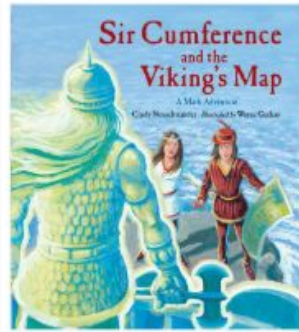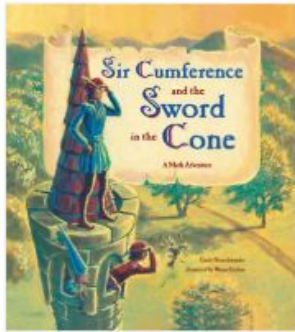
Geometric Measurement Circumference
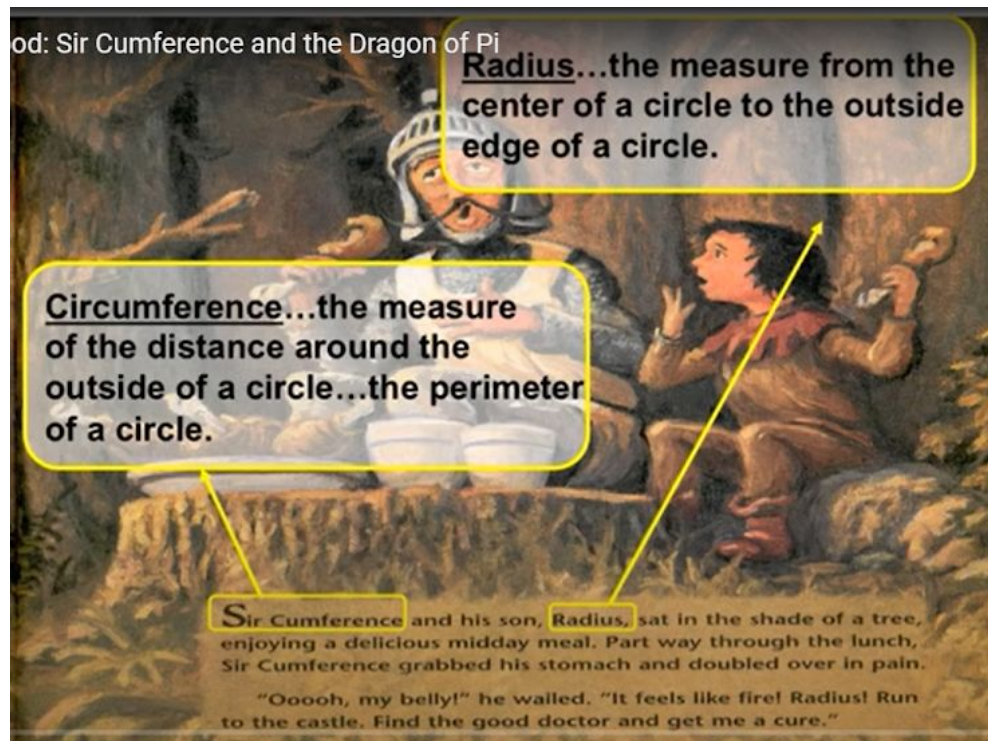
https://www.youtube.com/watch?v=h7TR7zFOEZg

Geometric Measurement Circumference

https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Circle-Tool/

Geometric Measurement
Area of a Circle

https://www.youtube.com/watch?v=9u5hmhRTq1k
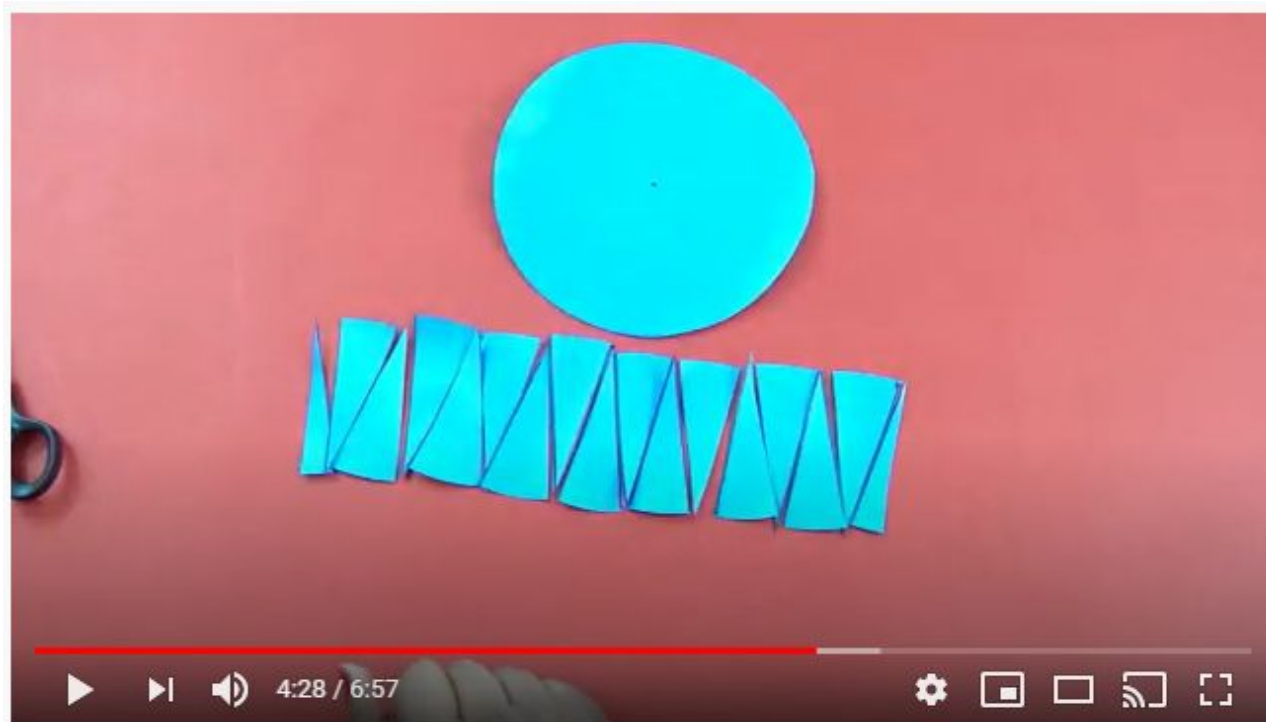
Research

How did we build schema for geometric concepts?
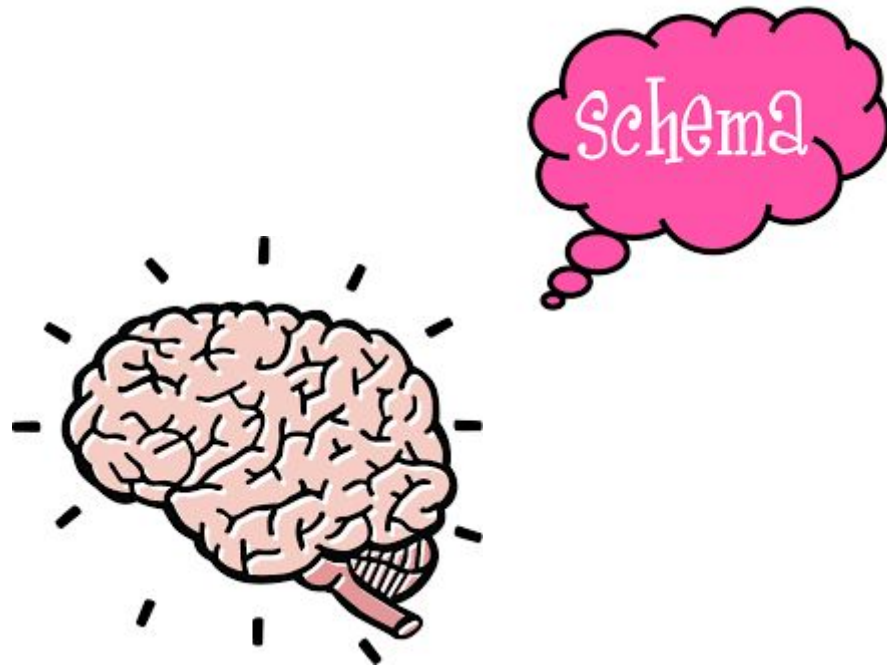
Summarizer

POMS
Point **O**f **M**ost **S**ignificance

As our summarizer, I will be asking you to tell me what is the **POMS** for you from today's session.
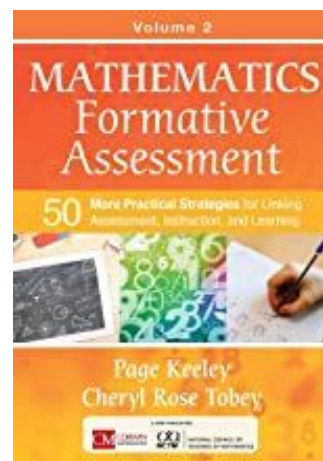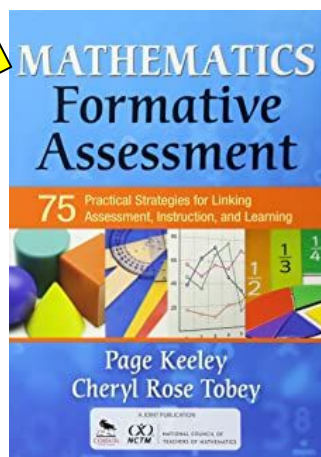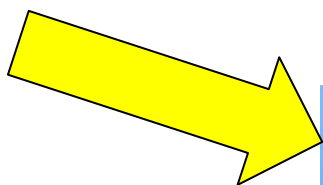
Developing Geometric Thinking

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KCM is here to support you!

Contact me

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