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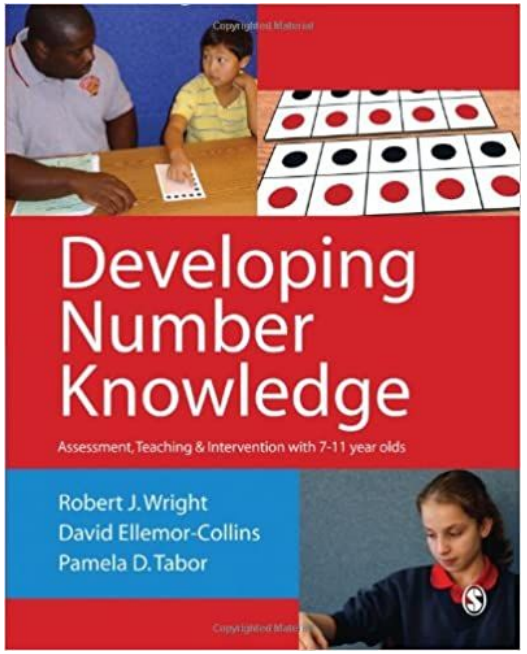

KCM Favorites

Developing Number Knowledge

by Robert Wright, David Ellemor-Collins, and Pamela Tabor

Welcome!

Your host

Cindy Aossey

Regional Consultant
Kentucky Center for Mathematics
cindy.aossey@outlook.com

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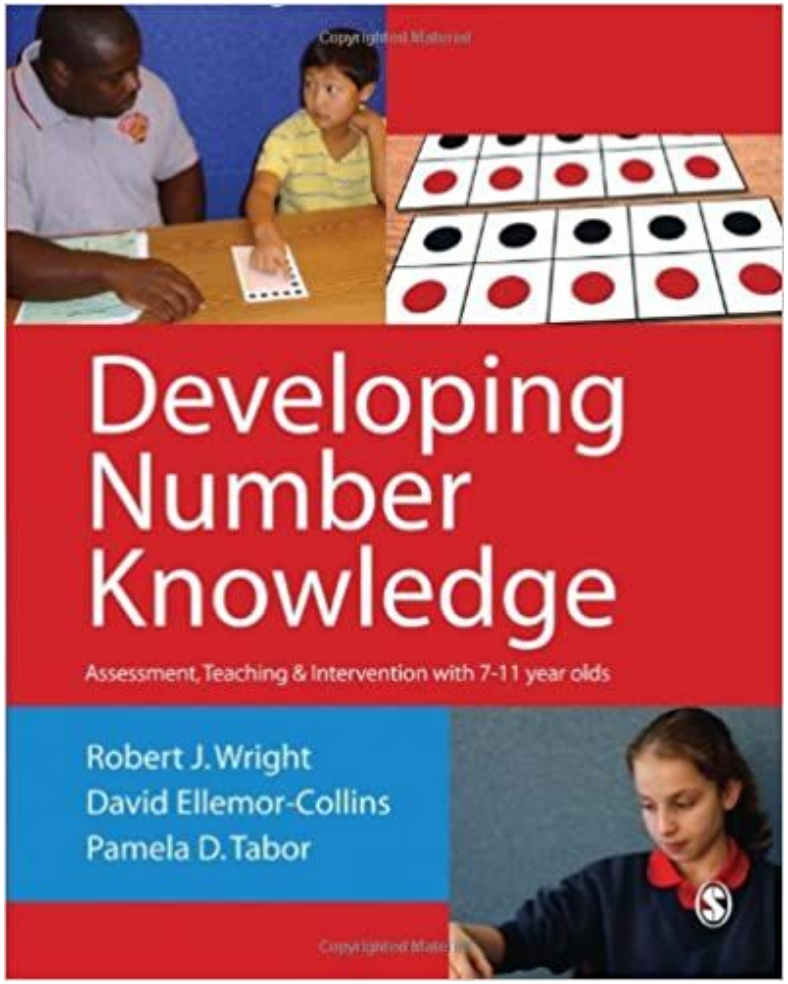
Developing Number Knowledge

Dr. Robert Wright



SAGE Publications, 2012

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Math Recovery Conference 2018

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On stage together!

David Ellemor-Collins, Pam Tabor and Robert Wright

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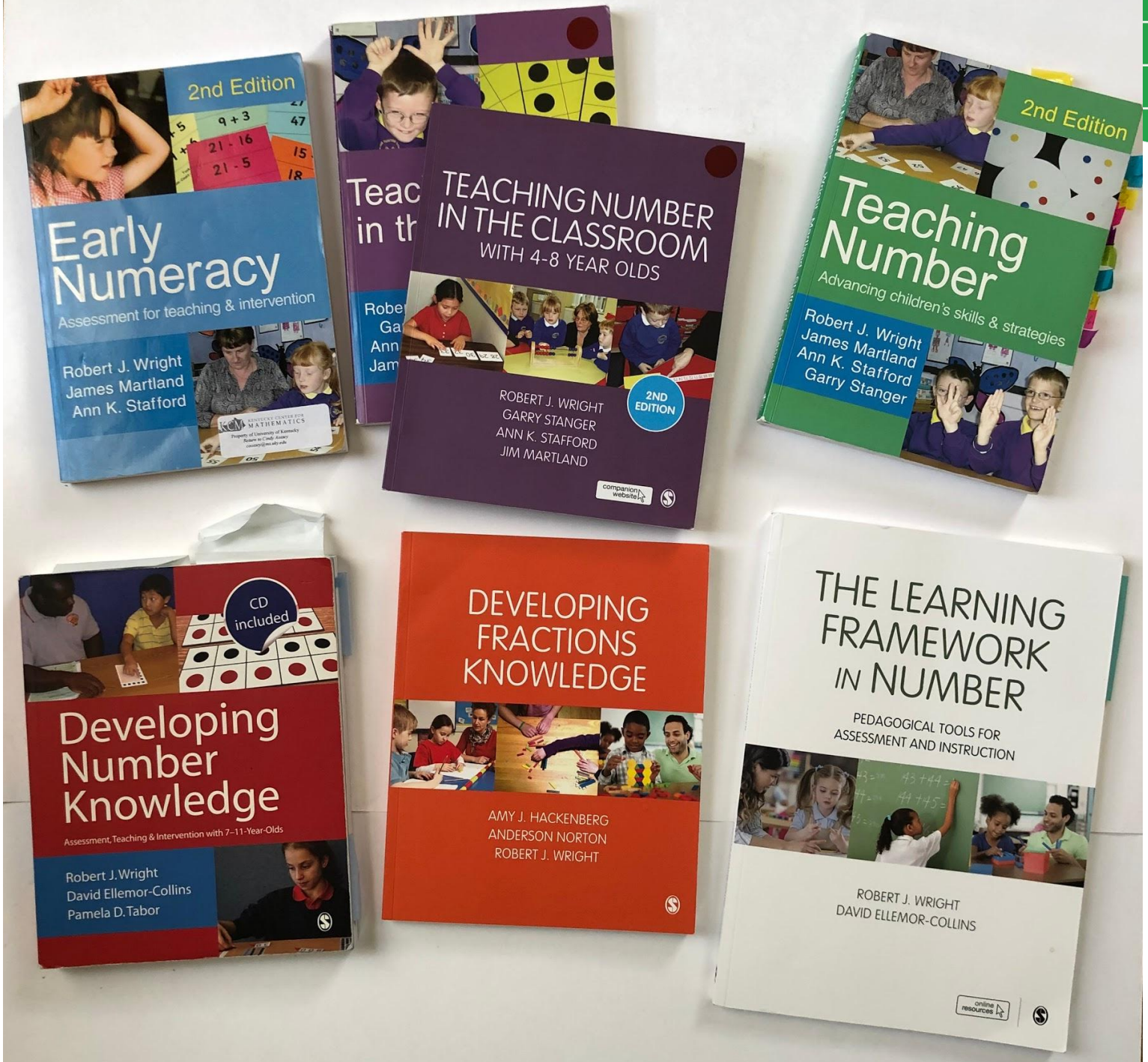

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1993

Dr. Robert J. Wright (Bob) is the originator of the Maths Recovery Programme, at Southern Cross University, Lismore, Australia.

1993-94

2003

The US Math Recovery Council® was formed as a non-profit.

Pictured left to right: Jane Camper, Kurt Kinsey, Petey MacCarty, Bob Wright, Jim Martland, Sybil Sevic, Gary Stanger & Ann Stafford-Dillon.

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Math Recovery comes to Kentucky!

2006 - Kurt Kinsey and Petey MacCartey lead first Math Recovery Specialist Course

2007 - I start my personal Math Recovery Journey

2009 - I become a Math Recovery Add&Vantage Champion

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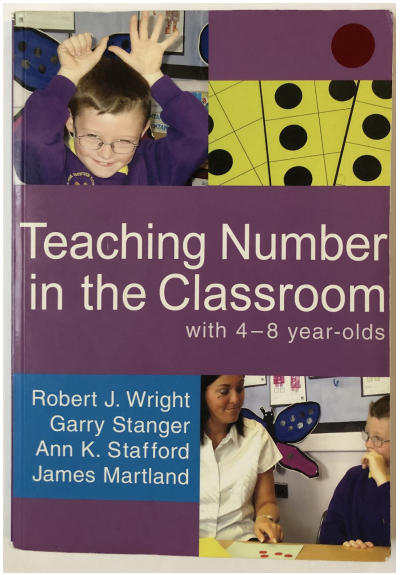
Add+VantageMR®
INFORMED ASSESSMENT FOR MATH TEACHING



Course 1- Addition & Subtraction, Number Words & Numerals, and Structuring

Course 2 - **Place Value** and **Multiplication**

"Dr. Wright is working on a new book…"

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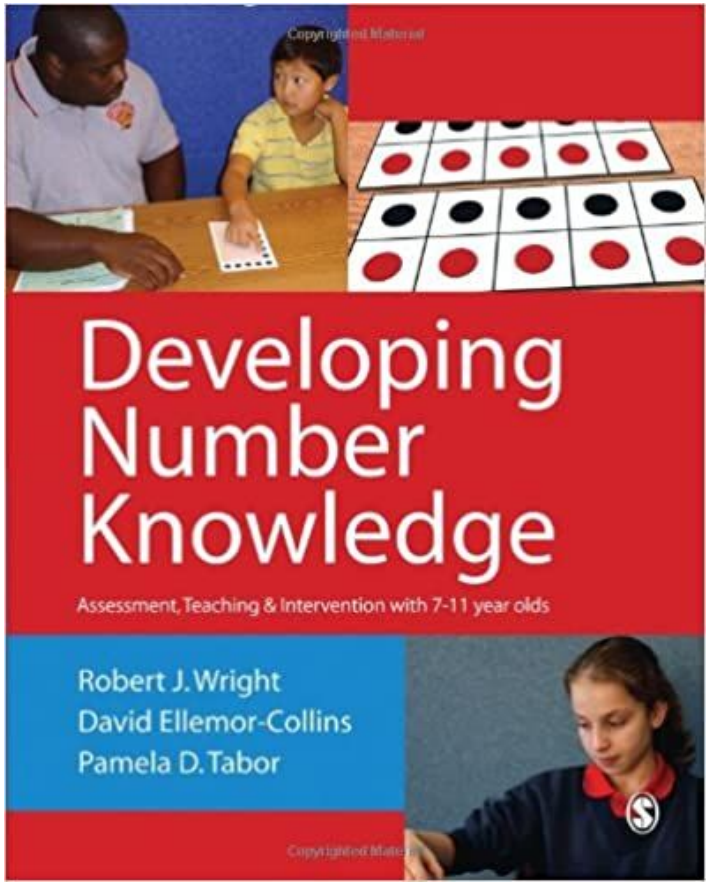
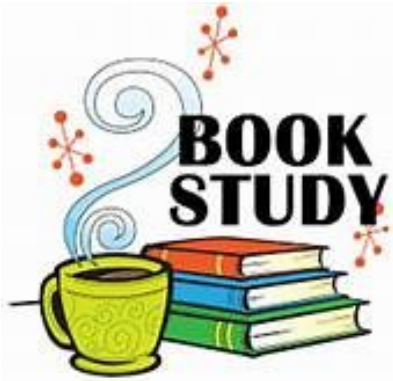
2012 - The wait is finally over!





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A bigger world is revealed!

See Further and more Clearly

- Extend to higher grade level content
- Refine and deepen earlier work

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Book highlights

- Themes of Progressive Mathematization

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Book highlights

- Themes of Progressive Mathematization
- Inquiry vs. Rehearsal Mode
- Six Domains of Instruction:
  - Number Words and Numerals
  - Structuring 1 to 20
  - Conceptual Place Value
  - Addition and Subtraction to 100
  - Multiplication and Division
  - Written Computation

Overview

Assessment Task

Instructional Tasks

Printables for tasks on included CD!

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Themes of Progressive Mathematization

"**Progressive Mathematization** means the development of mathematical sophistication over time: for example, developing from adding counters through to bare numbers…. The themes elaborate on how the domains develop and interweave, ..." (Pg. 15)

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Themes of Progressive Mathematization

Pages 14-19 Link: https://us.sagepub.com/en-us/nam/developing-number-knowledge/book235091#preview

Theme A: Structuring Number

Theme B: Extending the Range of Numbers

Theme C: Decimalizing towards Base-ten Thinking

Theme D: Unitizing and Not Counting by Ones

Theme E: Distancing the Setting of Materials

Theme F: Notating

Theme G: Formalizing

Theme H: Generalizing

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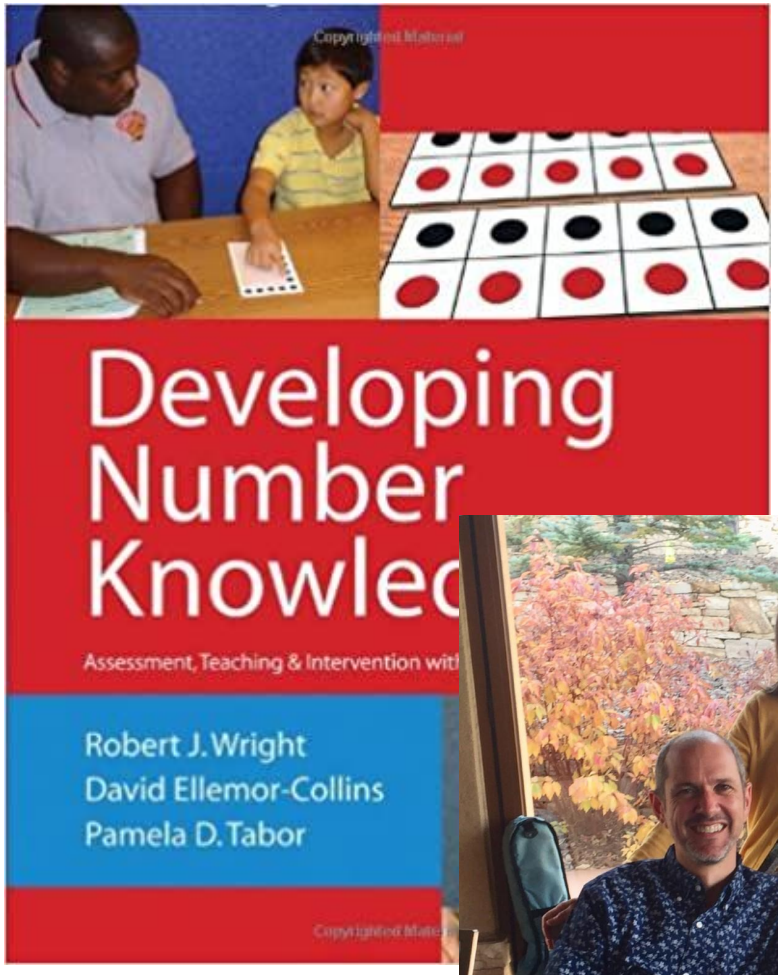
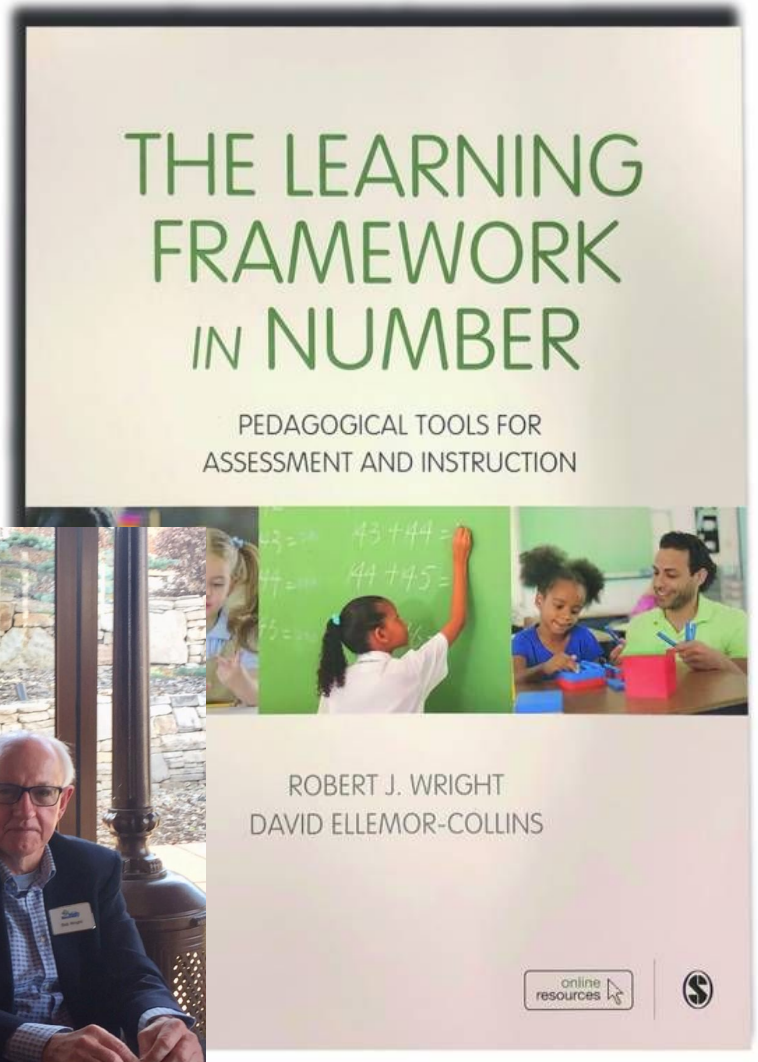
Robert Wright & David Ellmor-Collins





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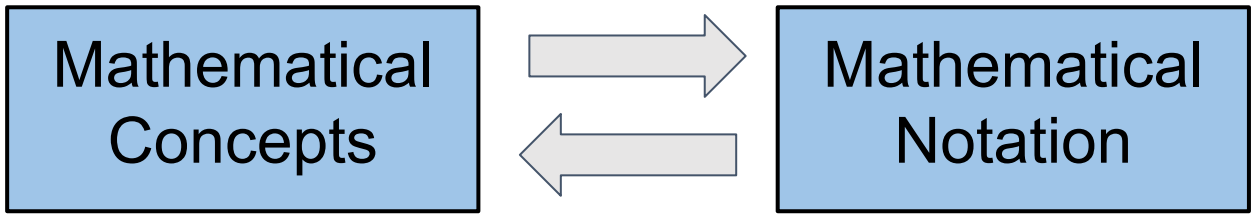
Theme F: Notating

Page 18



"Mathematical Notation and Mathematical Concepts are learned in tandem."

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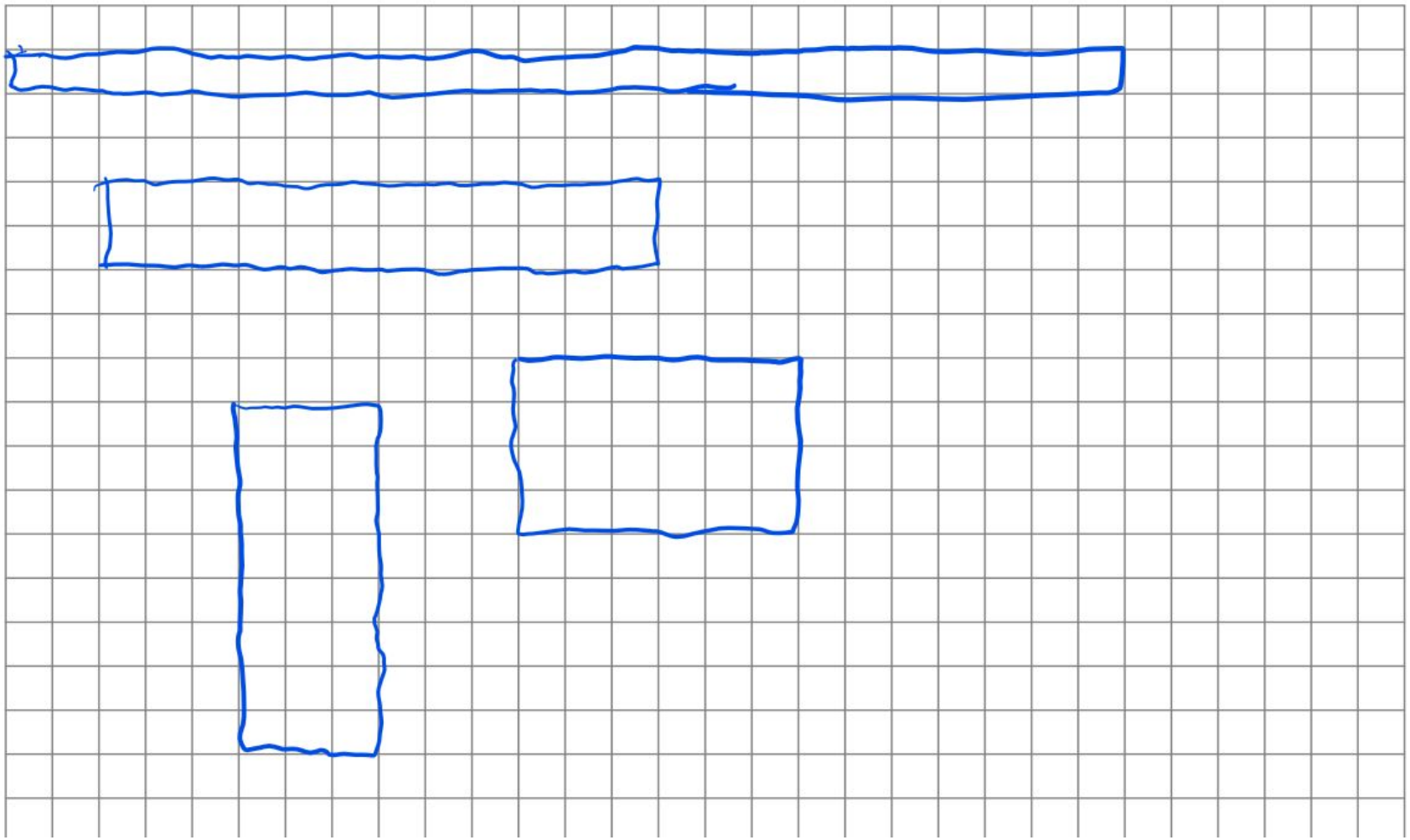
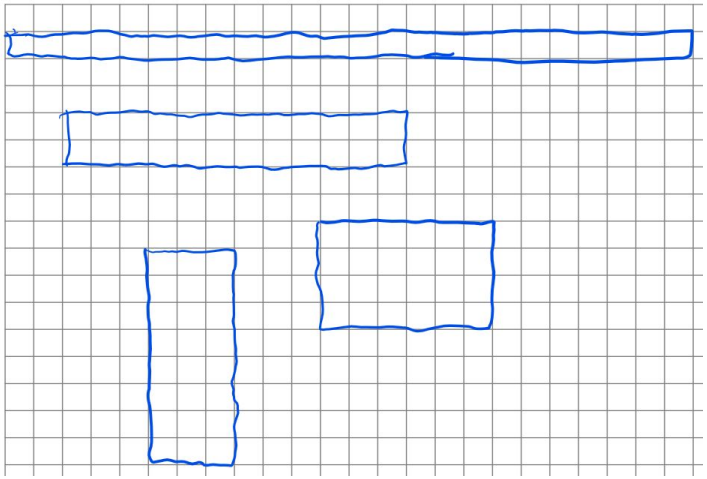
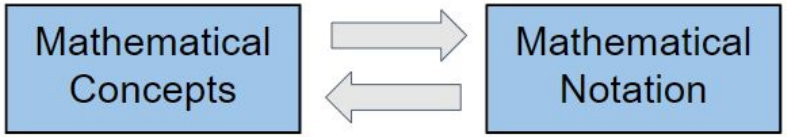
Example: Finding factor pairs of 24



Example: Finding factor pairs of 24



1×24
2×12
3×8
4×6



How might organizing the factor pairs of 24 in this way deepen students' conceptual understanding?

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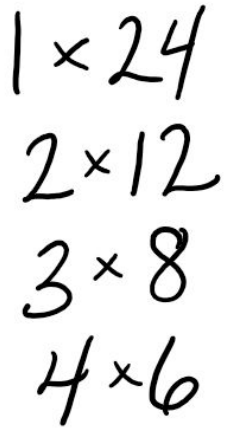
Example: List factor pairs of 24



Help students:

- see that expressions can communicate a RELATIONSHIP rather than being a problem to solve
- find relationships between factor pairs
- know if all factor pairs have been identified

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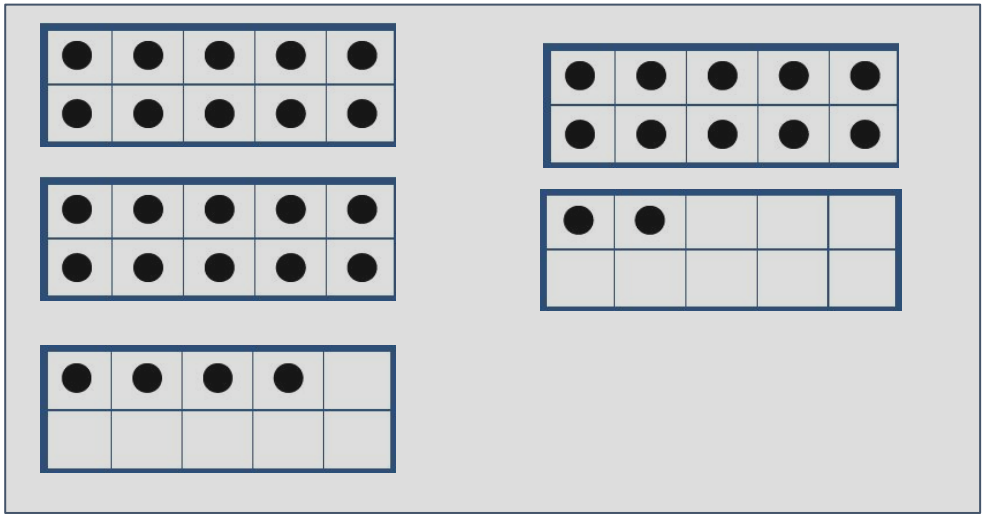
Example: Adding 2 digit numbers



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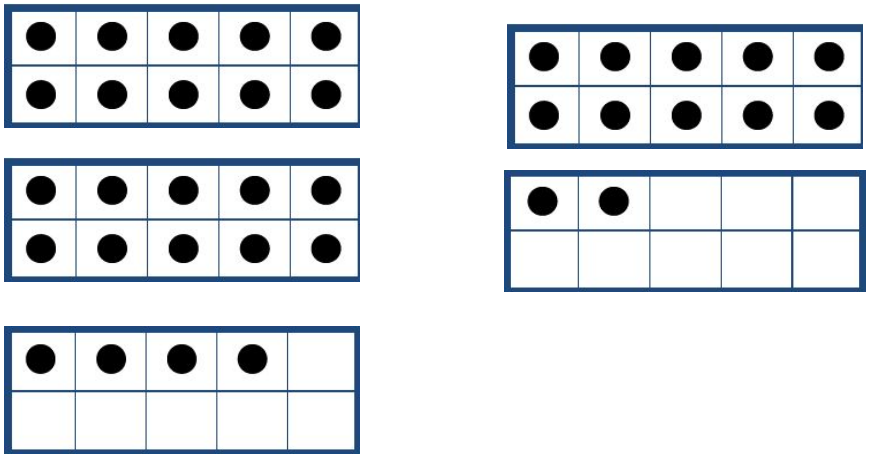
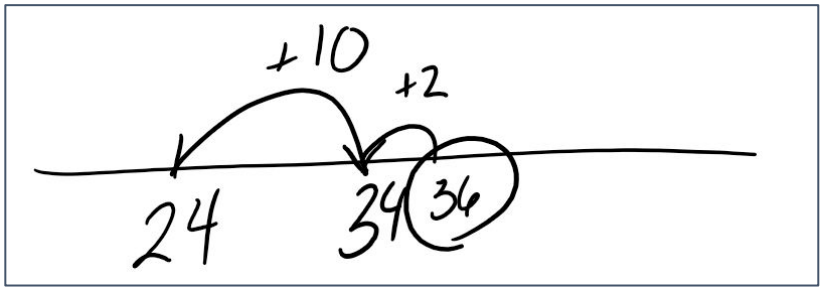
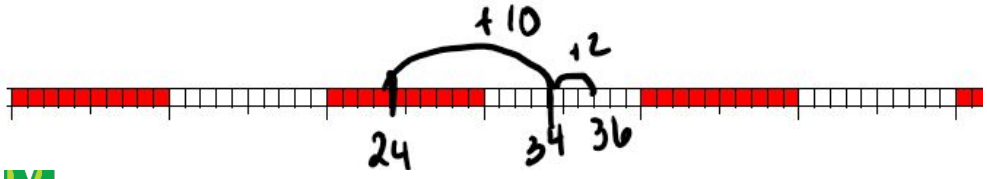
Example: Adding 2 digit numbers



"24 and 10 is 34.
2 more is 36."



24 + 10 → 34 + 2 → 36



24 + 10 = 34
34 + 2 = 36

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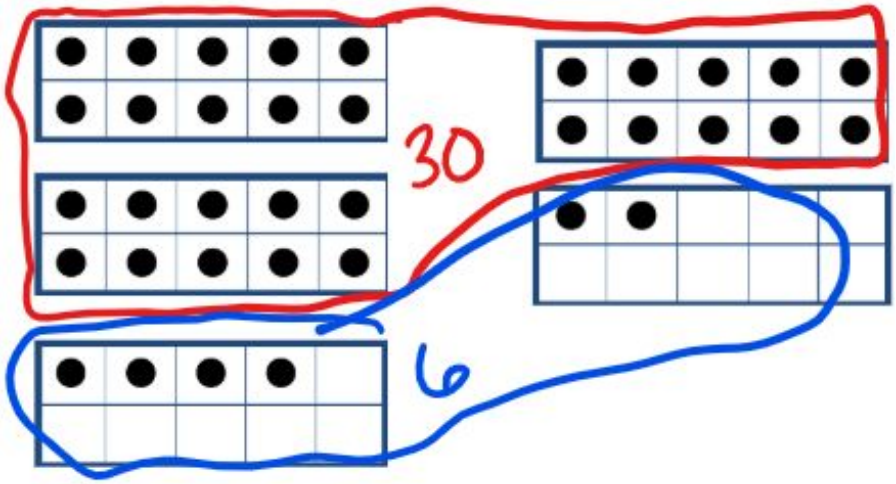
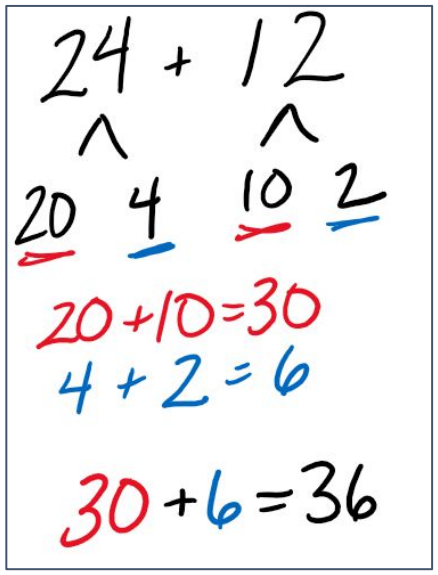
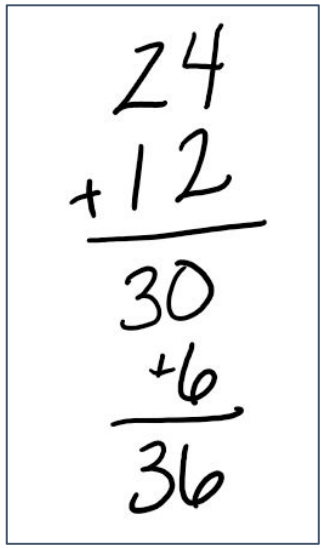
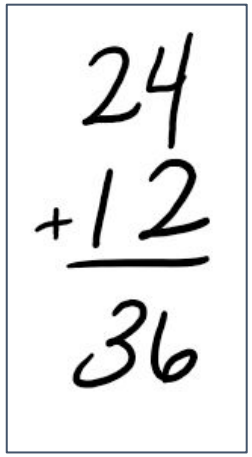
Example: Adding 2 digit numbers



"I added 20 and 10 to get 30.
I added the 4 and 2 to get 6.
That gives 36"







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Theme F: Notating

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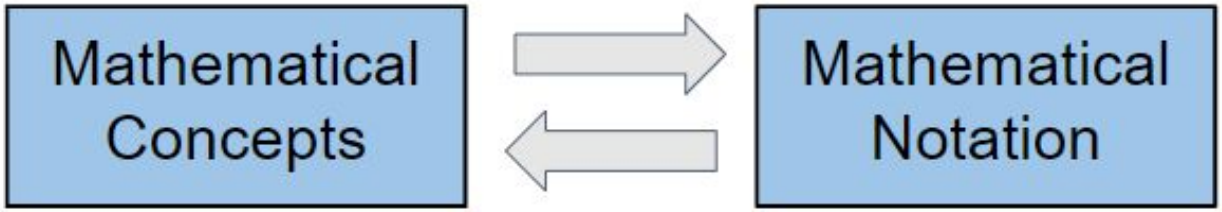

"If too much new notation is introduced without opportunities to grapple with the companion concepts, most students will not make the leap. Instead, the notation remains disconnected from the realm of what makes sense for students, and becomes a syntactical game following someone else rules. However, if new notation is not introduced, students' conceptual development will be limited."

Themes of Progressive Mathematization

Pages 14-19 Link: https://us.sagepub.com/en-us/nam/developing-number-knowledge/book235091#preview

Theme A: Structuring Number

Theme B: Extending the Range of Numbers

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Book highlights

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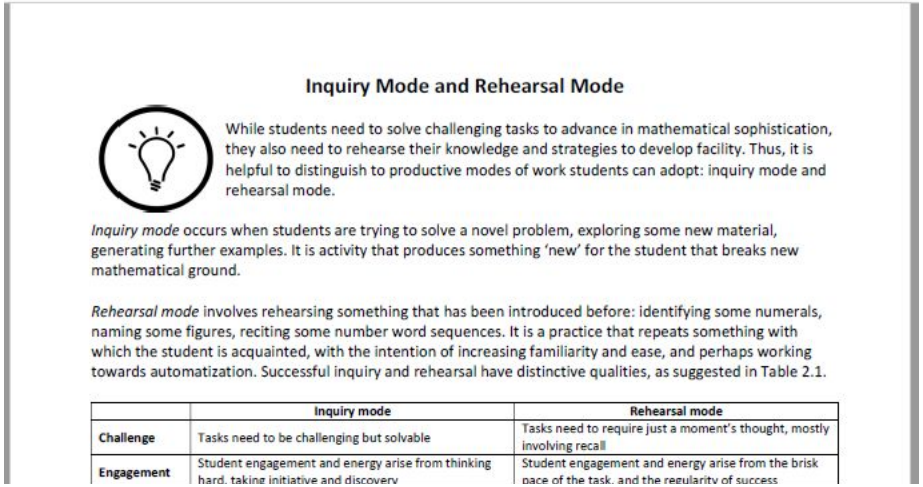
Inquiry vs. Rehearsal Mode

Page 23



Inquiry Mode and Rehearsal Mode

While students need to solve challenging tasks to advance in mathematical sophistication, they also need to rehearse their knowledge and strategies to develop facility. Thus, it is helpful to distinguish to productive modes of work students can adopt: inquiry mode and rehearsal mode.

Inquiry mode occurs when students are trying to solve a novel problem, exploring some new material, generating further examples. It is activity that produces something 'new' for the student that breaks new mathematical ground.

Rehearsal mode involves rehearsing something that has been introduced before: identifying some numerals, naming some figures, reciting some number word sequences. It is a practice that repeats something with which the student is acquainted, with the intention of increasing familiarity and ease, and perhaps working towards automatization. Successful inquiry and rehearsal have distinctive qualities, as suggested in Table 2.1.

	Inquiry mode	Rehearsal mode
Challenge	Tasks need to be challenging but solvable	Tasks need to require just a moment's thought, mostly involving recall
Engagement	Student engagement and energy arise from thinking hard, taking initiative and discovery	Student engagement and energy arise from the brisk pace of the task, and the regularity of success

	Inquiry mode	Rehearsal mode
Challenge	Tasks need to be challenging but solvable	Tasks need to require just a moment's thought, mostly involving recall
Engagement	Student engagement and energy arise from thinking hard, taking initiative and discovery	Student engagement and energy arise from the brisk pace of the task, and the regularity of success
Autonomy	Students need autonomy in approaching the task, and preferably autonomy in checking their solutions	The task is mostly externally directed – for example, by a teacher, a game, a computer, or a worksheet. Answers are checked immediately.
Time	Time needs to be relatively long, long enough to exercise student's persistence and initiative	Time needs to be relatively short, sufficient to practice without getting tired or distracted
Pace	Pace varies with the ebb and flow of students' inventions and puzzling	Pace is kept fairly brisk and even
Follow-up	Inquiry work is well served with follow-up sharing, discussion and debate, to bring communal mathematical reasoning to bear on students' work	Rehearsal work can be follow-up with revisiting two or three of the items that caused difficulty

Book highlights

- Themes of Progressive Mathematization
- Inquiry vs. Rehearsal Mode
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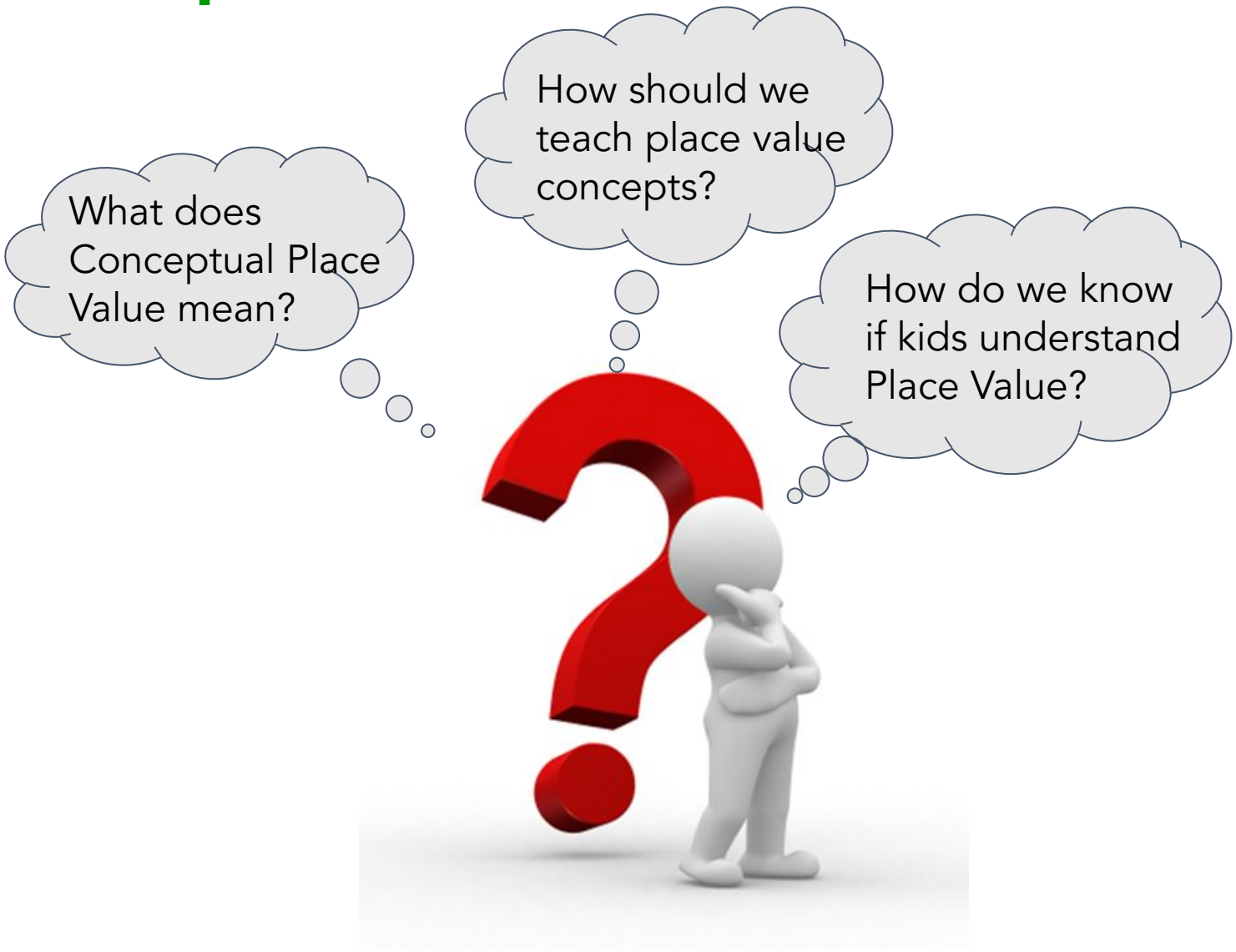
Conceptual Place Value



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Conceptual Place Value

Chapter 5, page 77

"Thus, being able to flexibly increment and decrement by ones and tens, and later also by hundreds, is critical knowledge for developing facile mental computation. We refer to this critical knowledge as *conceptual place value*. (CPV)."

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Conceptual Place Value

Chapter 5

“twenty-eight, twenty-nine, twenty-ten… wait, that’s thirty!”

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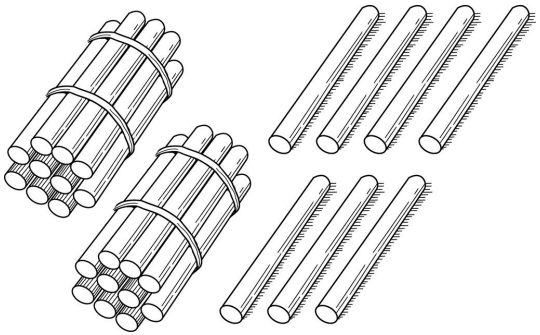

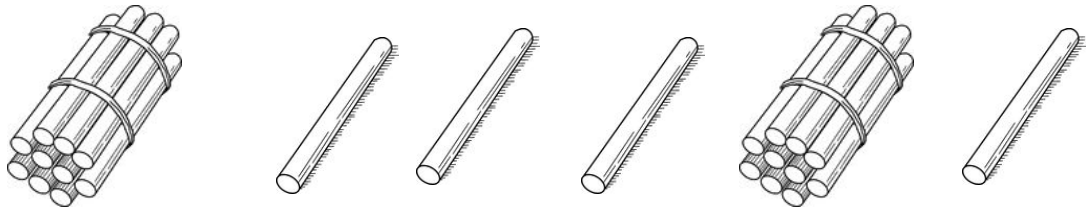

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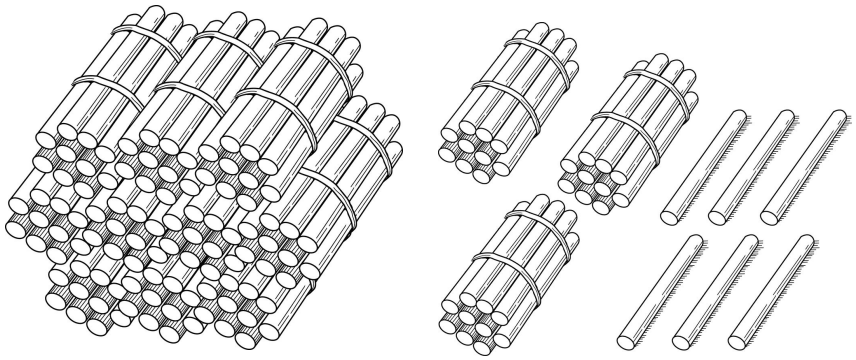
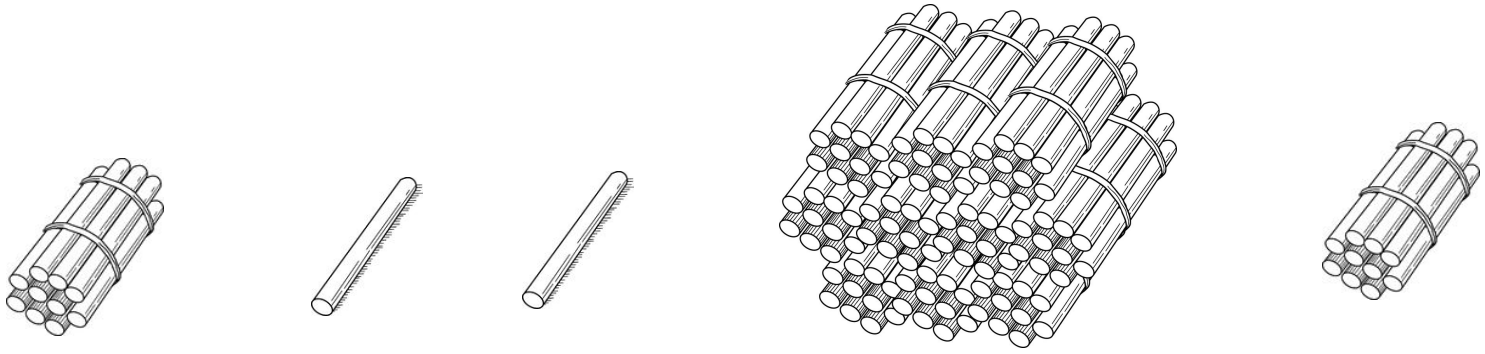
https://etc.usf.edu/clipart/galleries/723-counting - Free Education Clip Art





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ACTIVITY IA5.6: Crazy Grid

Intended learning: To add ten or multiples of ten to any number.

Instructional mode: Shorter, rehearsal mode for individuals, small gr

Materials: Crazy grid drawn on the board or from Resource CD needed.

Description: Present a grid with one number displayed (see Figure 5.7 *print a hundred grid, and the printer went crazy. For some reason, almost no you help me fill in the missing numbers?*

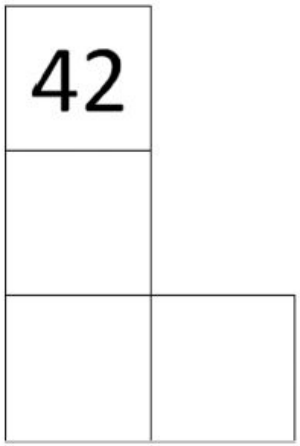
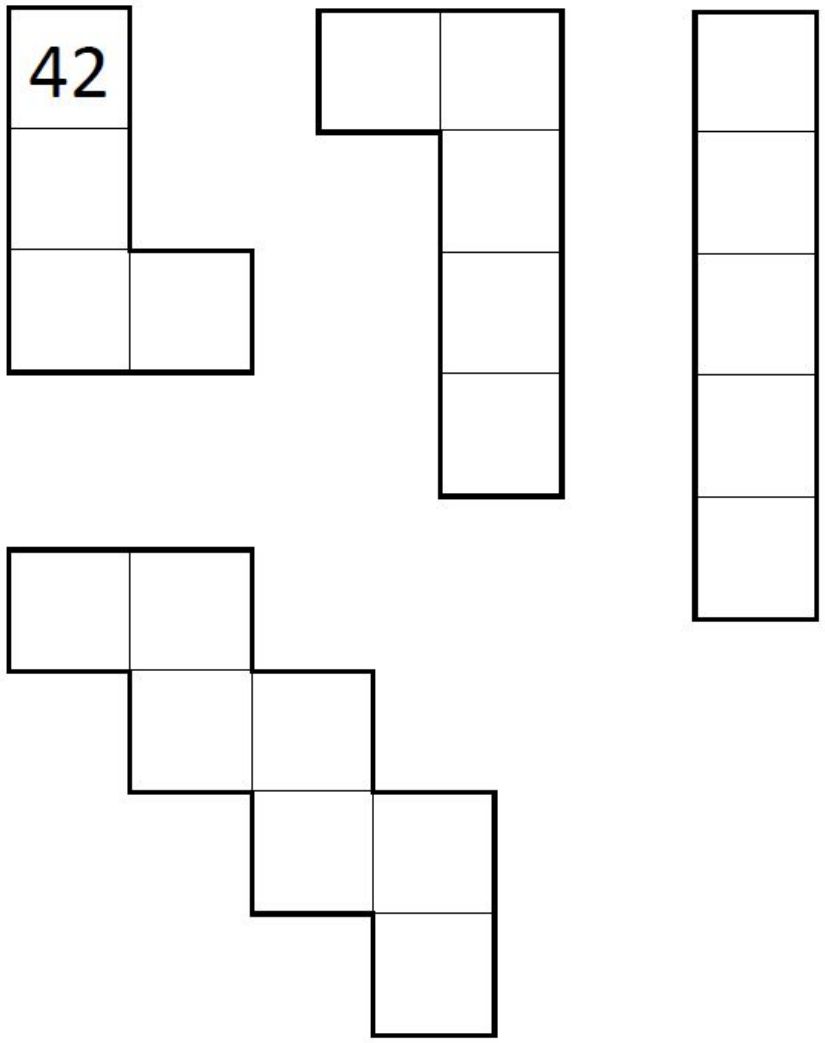


Figure 5.7 A sample crazy grid

Responses, variations and extensions:

- This task is designed for students who have worked patterns on t familiar with the format of the hundred grid.
- Examples of extensions are to begin with a number: (a) in the l requires hurdling the centuple (87); and (c) that is in the hundre the centuple (493).
- As students develop facility, have them insert only one specified mi all the missing numbers.



https://jamboard.google.com/d/1zSZl6XNzxun5-mve1D8Nwooff3reVsjlnAfcYYGVKXs/viewer?f=0

Conceptual Place Value

Chapter 5, page 77

"Thus, being able to flexibly increment and decrement by ones and tens, and later also by hundreds, is critical knowledge for developing facile mental computation. We refer to this critical knowledge as *conceptual place value*. (CPV)."

"As students become proficient…, they"

- develop a sense of the relative sizes of numbers
- learn ways of relating multi-digit numbers to each other
- decimalize their approach to multi-digit numbers, habitually organizing numbers in terms of their base-ten units of ones, tens, hundreds and so on."

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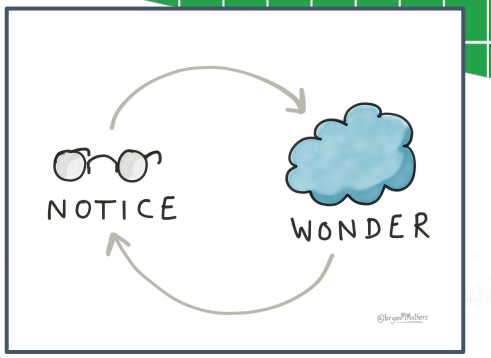


Table 5.1 Distinguishing conceptual place value from conventional place value

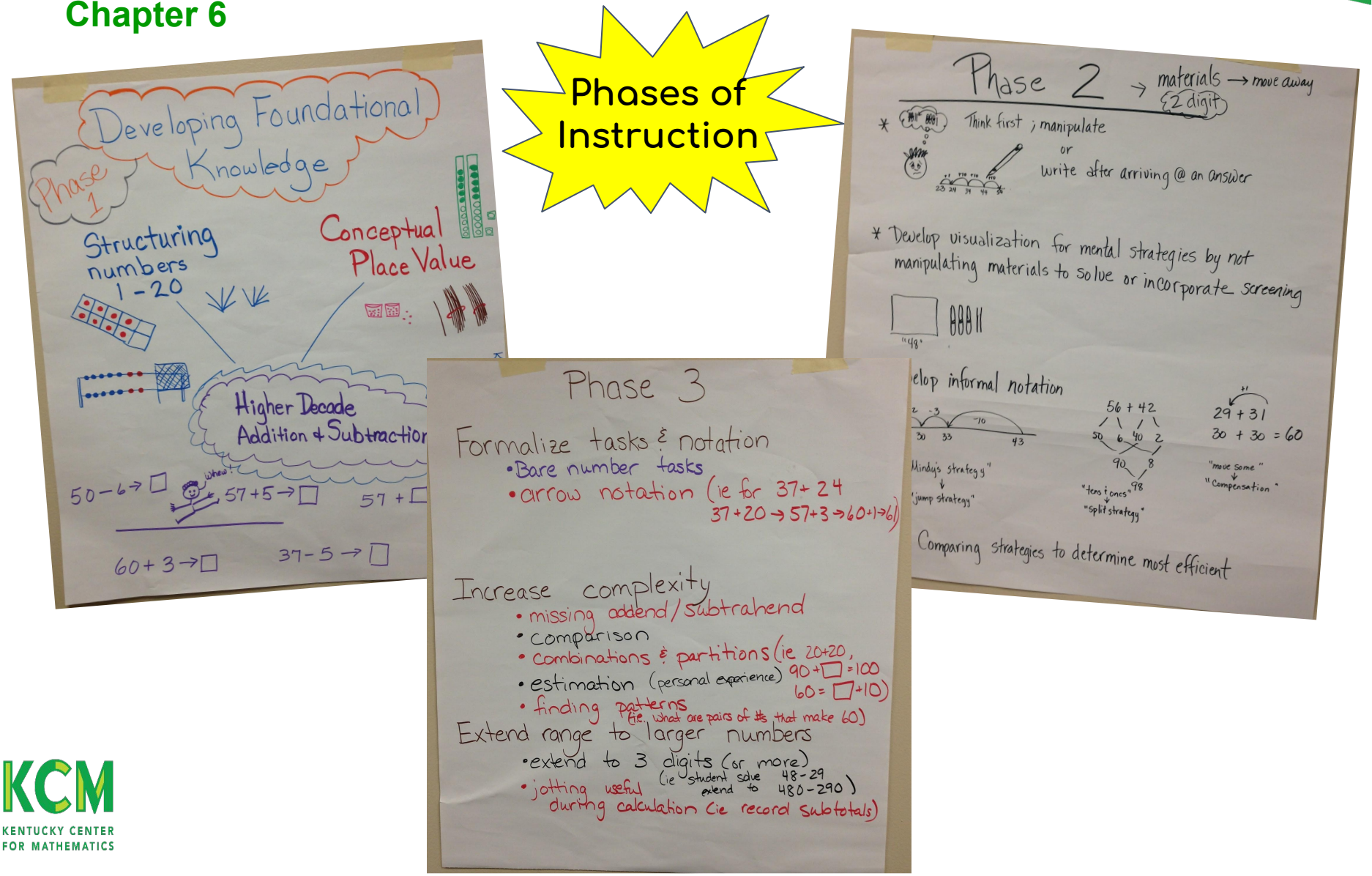
Conceptual place value	Conventional place value
Numbers are presented and discussed in their full value: 20 as twenty or two tens; 21 as twenty-one, or twenty and one.	Numbers may be explicitly presented or discussed in terms of digits: 20 has 2 in the tens column; 21 has 1 in the ones column.
Tasks involve increments/decrements in sequence. For example, from 611, ten less is 601, ten less is 591, one less is 590.	Typically, tasks are not presented as a sequence of increments/decrements.
Solving tasks essentially involves inquiry or problem-solving.	Solving tasks might require following a convention or rehearsing a given procedure.
Answering tasks might involve using knowledge of the number sequence.	Answers are unlikely to relate tasks to the number sequence.
Answers do not involve exchanging units. For example, students solve 195 and ten more is 205, but do not need to explain this by trading 10 tens for 1 hundred.	Answers involve explicitly exchanging or trading: 10 ones for 1 ten, 10 tens for 1 hundred.
Attention is on structuring numbers around dynamic relationships of ones, tens and hundreds.	Attention is on manipulating numbers in terms of the formal place value system.
The aim is to cultivate strong mental strategies.	The aim is to prepare students to use the standard algorithms.

Starts first grade

Common Core 4th grade

Addition and Subtraction to 100

Chapter 6



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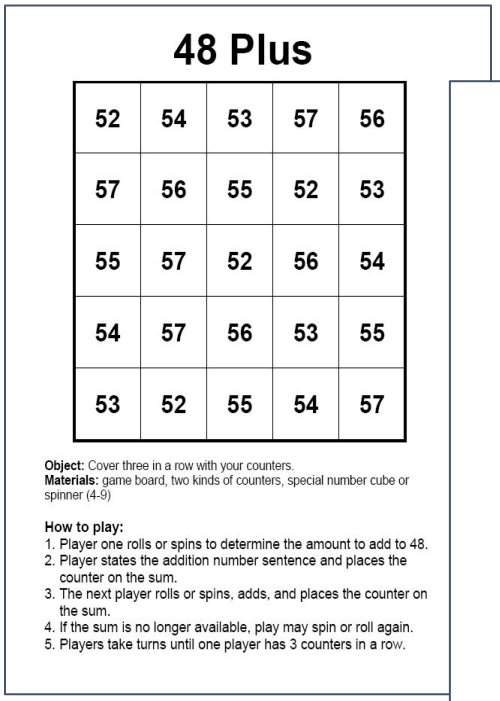
CD Resources

Example: Chapter 6

- 20 plus game.pdf
- 32 minus, 51 minus.pdf
- 48 Plus, 67 Plus.pdf
- 70 plus game.pdf
- Add or Subtract 12.pdf
- How Many More to 100.pdf
- How Many More to Make 60.pdf
- Jumping to 50 Game.pdf
- Mini Ten Frames.pdf
- Spinners 0-9, 1-9 Choose, 1-6, 4-9.pdf

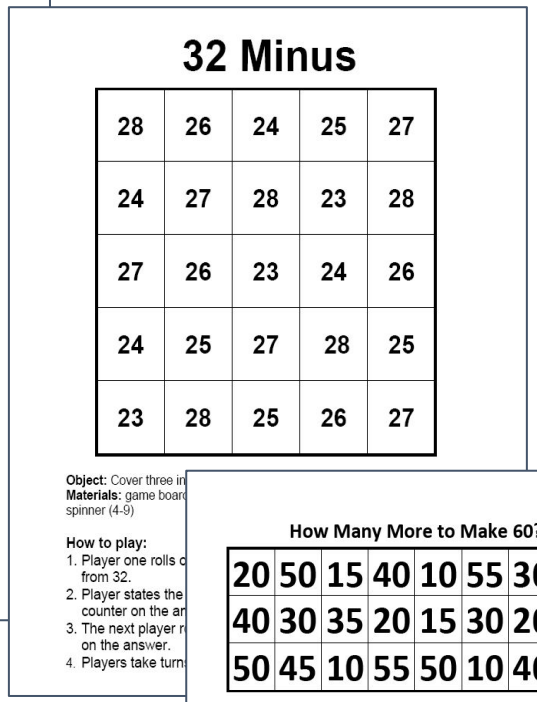


48 Plus

52	54	53	57	56
57	56	55	52	53
55	57	52	56	54
54	57	56	53	55
53	52	55	54	57

Object: Cover three in a row with your counters.
Materials: game board, two kinds of counters, special number cube or spinner (4-9)

How to play:
1. Player one rolls or spins to determine the amount to add to 48.
2. Player states the addition number sentence and places the counter on the sum.
3. The next player rolls or spins, adds, and places the counter on the sum.
4. If the sum is no longer available, play may spin or roll again.
5. Players take turns until one player has 3 counters in a row.

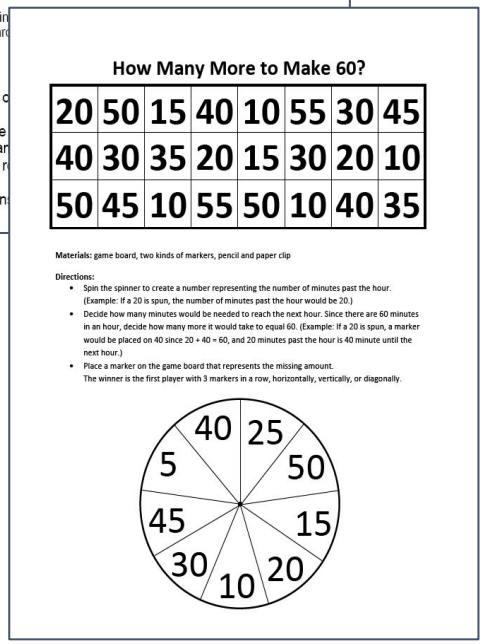


32 Minus

28	26	24	25	27
24	27	28	23	28
27	26	23	24	26
24	25	27	28	25
23	28	25	26	27

Object: Cover three in
Materials: game board
spinner (4-9)

How to play:
1. Player one rolls
from 32.
2. Player states the
counter on the ar
3. The next player r
on the answer.
4. Players take turn



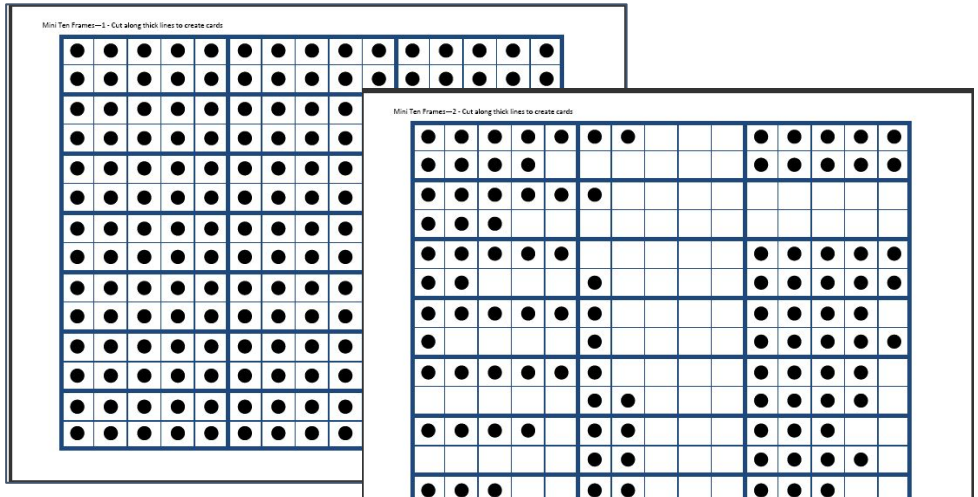
How Many More to Make 60?

20	50	15	40	10	55	30	45
40	30	35	20	15	30	20	10
50	45	10	55	50	10	40	35

Materials: game board, two kinds of markers, pencil and paper clip

Directions:
- Spin the spinner to create a number representing the number of minutes past the hour. (Example: If a 20 is spun, the number of minutes past the hour would be 20.)
- Decide how many minutes would be needed to reach the next hour. Since there are 60 minutes in an hour, decide how many more it would take to equal 60. (Example: If a 20 is spun, a marker would be placed on 40 since 20 + 40 = 60, and 20 minutes past the hour is 40 minute until the next hour.)
- Place a marker on the game board that represents the missing amount.

The winner is the first player with 3 markers in a row, horizontally, vertically, or diagonally.



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Book highlights

- Themes of Progressive Mathematization
- Inquiry vs. Rehearsal Mode
- Six Domains of Instruction:
  - Number Words and Numerals
  - Structuring 1 to 20
  - Conceptual Place Value
  - Addition and Subtraction to 100
  - Multiplication and Division
  - Written Computation

Overview

Assessment Task

Instructional Tasks

Printables for tasks on included CD!

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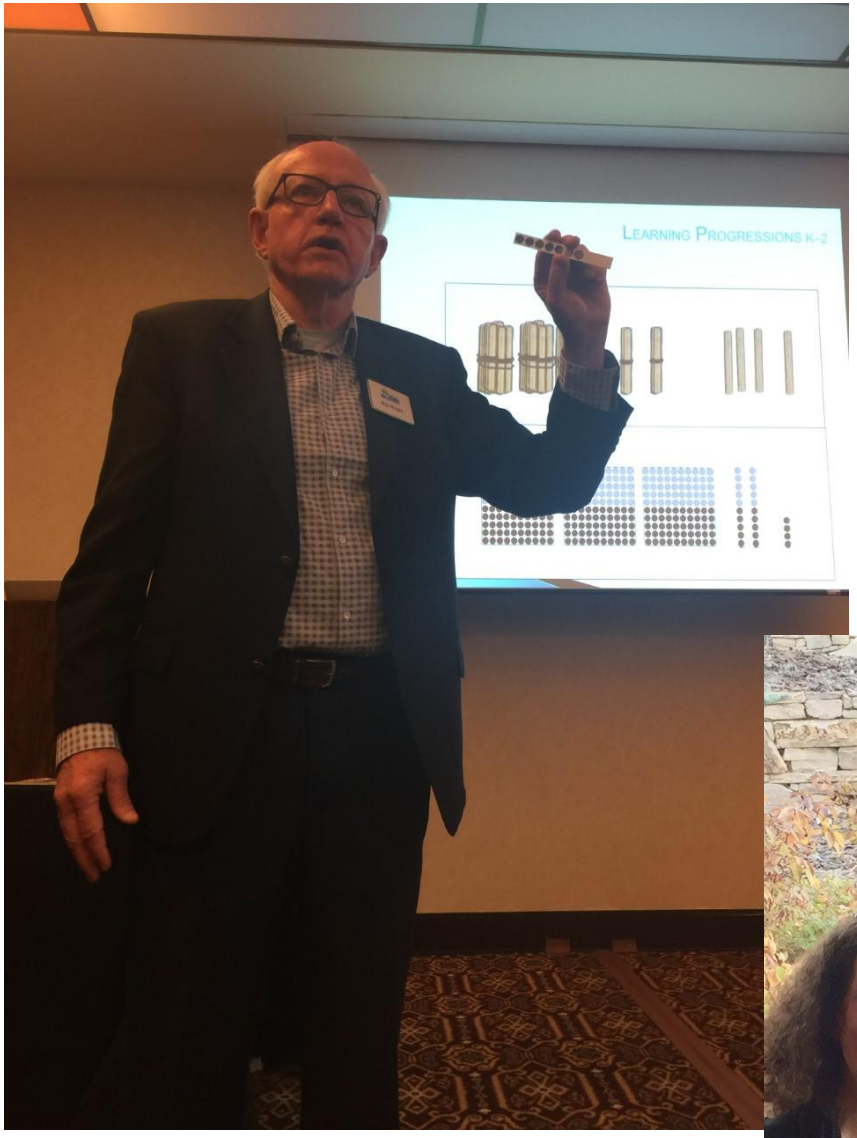

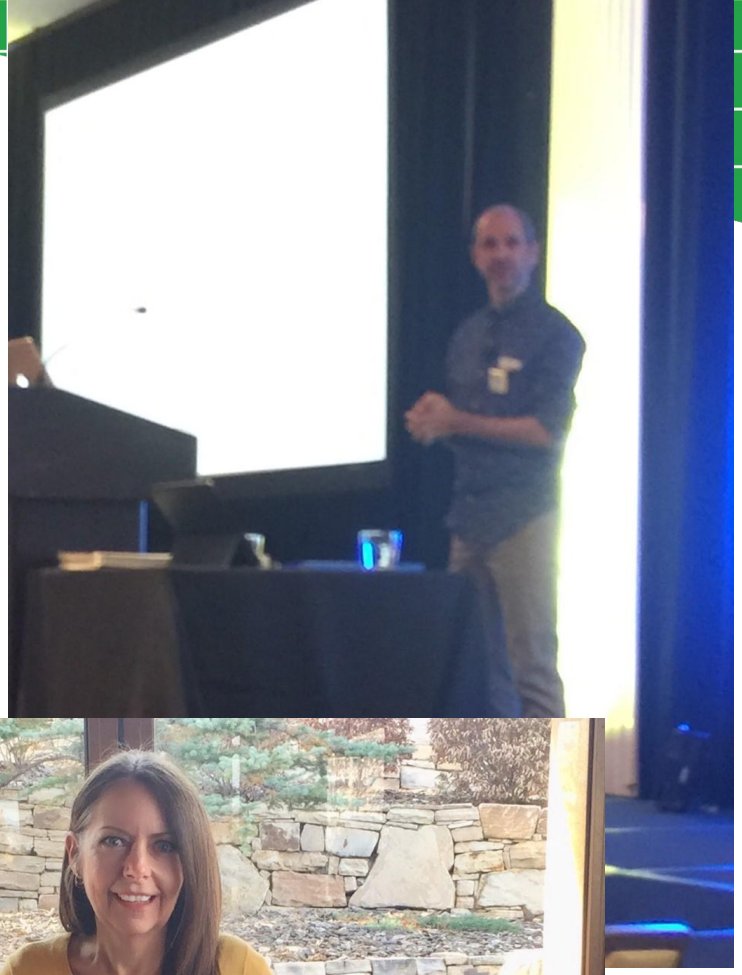

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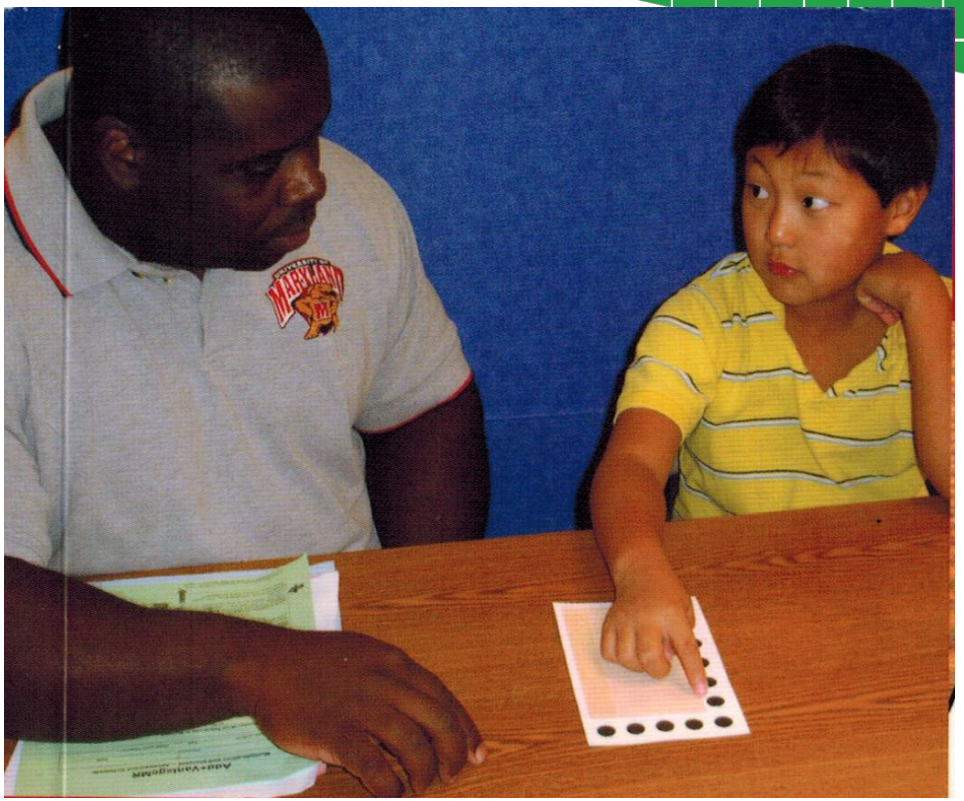

"Invite student to problem-solve, visualize, organize, justify and generalize - this is the kind of activity that is likely to help them advance mathematically." (Pg. 15)

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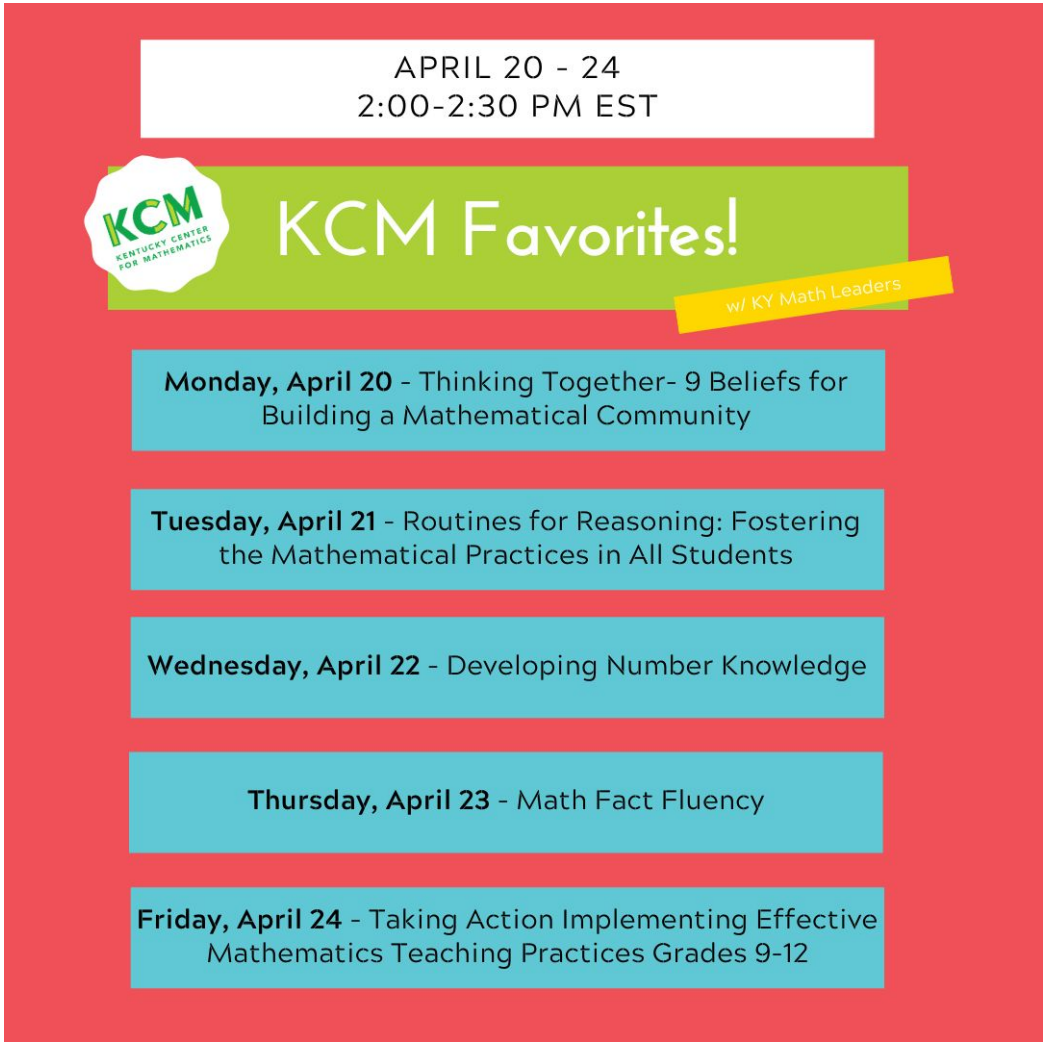

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KCM is here to support you!

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