CHAPTER 4:

LINEAR FUNCTIONS OF 2 VARIABLES

4.1 RATES OF CHANGES IN DIFFERENT DIRECTIONS

From Precalculus, we know that $y = f(x)$ is a linear function if the rate of change of the function is constant. I.e., for every unit that we move in the x direction, the *rise* in the y direction is constant.

Example

Consider the function given by the following table:

x	0	1	2	3
$f(x)$	2	5	8	11

For each movement of 1 in the x direction, the output $f(x)$ from the function increases by 3. Hence, the function is linear with slope (rate of change) equal to three.

Example

Consider the function given by the following table:

x	0	1	2	3
$f(x)$	1	3	9	16

From 0 to 1, the output $f(x)$ increases by 2. From 1 to 2, the output $f(x)$ increases by 6. Hence the rate of change of the function is not constant and the function is not linear.

RATES OF CHANGE IN THE X AND Y DIRECTIONS

In the case of functions of two variables we also have a notion of rate of change. However, there are now two variables, x and y, and so we will consider two rates of change: a rate of change associated with movement in the x direction and a rate of change associated with movement in the y direction.

Example

Consider the situation we saw in the previous chapter where both parents in a household work and the father earns \$5.00 an hour while the mother earns \$10.00 an hour. As before we shall be using:

$x =$ number of hours that the mother works in a week

$y =$ number of hours that the father works in a week

$z = f(x, y) =$ weekly salary for the family

For each increase of one unit in x, the value of the function increases by 10. That is, for each hour that the mother works in a week, the weekly family salary increases by 10 dollars. Hence the rate of change in the x direction (we refer to this as the slope in the x direction or m_x) is 10. Similarly, for each increase of one unit in y, the value of the function increases by 5. That is, for each increase of one in the number of hours the father works the family salary increases by 5 dollars. Hence the rate of change in the y direction (we refer to this as the slope in the y direction or m_y) is 5.

4.2 DEFINITION OF A LINEAR FUNCTION OF TWO VARIABLES

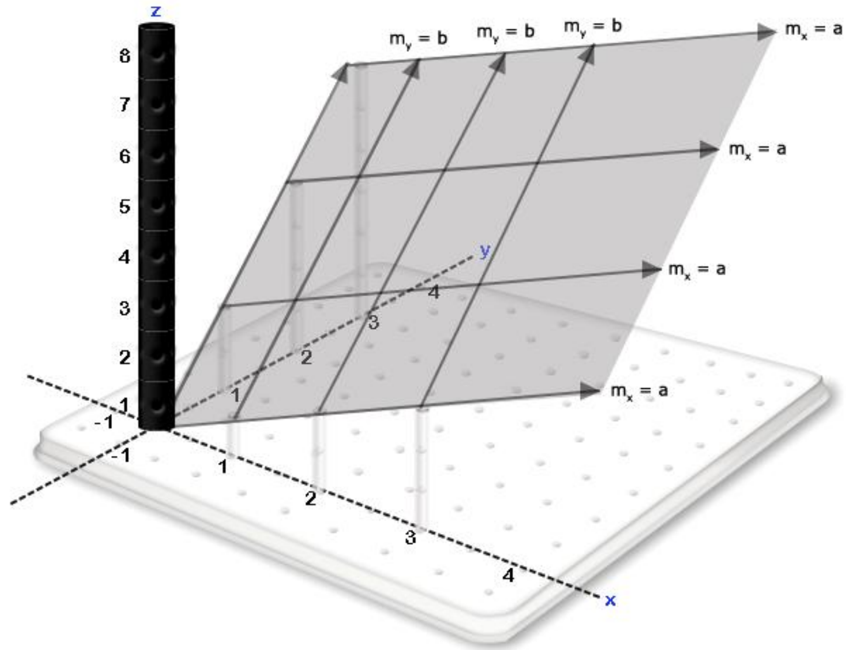
DEFINITION

A function of two variables is said to be *linear* if it has a constant rate of change in the x direction and a constant rate of change in the y direction. We will normally express this idea as m_x and m_y are constant.

4.3 RECOGNIZING A LINEAR FUNCTION OF TWO VARIABLES

SURFACES

If a linear function is represented with a surface, the surface will have a constant slope in the x direction and the y direction. Such a surface is represented below.



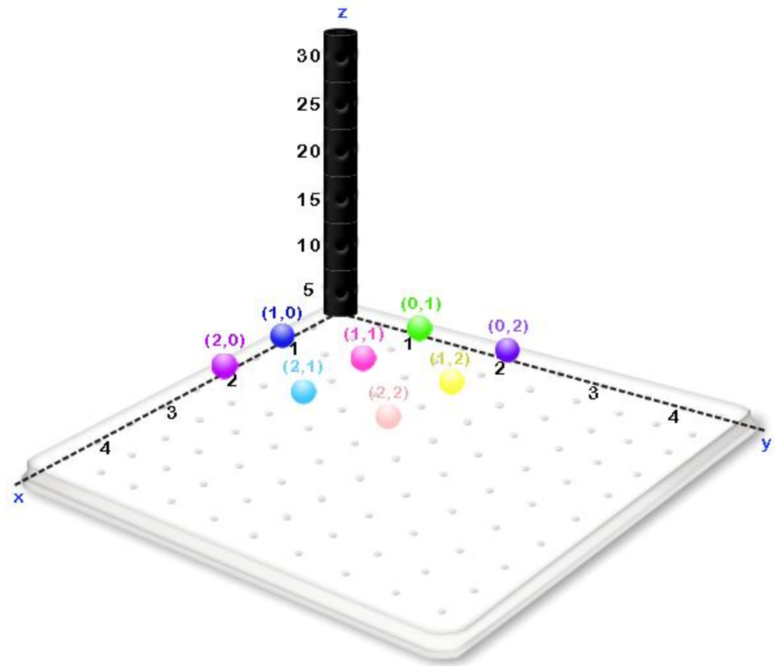
On the above surface, in the *x* directions the cross sections are a series of lines with slope *a* and in the *y* direction, the cross sections are a series of lines with slope *b*. This is consistent with constant slopes in the *x* and in the *y* directions. If we visualize a surface with constant slopes in the *x* and *y* directions, the surface that represents a linear function in 3 dimensions will always be a plane.

TABLES

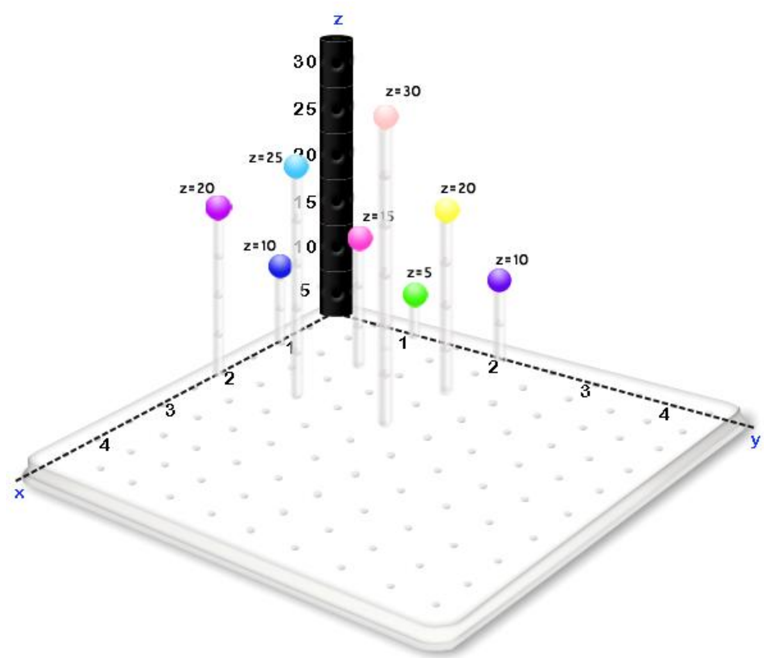
Assume that the function *f* is represented by the following table:

	$y = 0$	$y = 1$	$y = 2$	$y = 3$
$x = 0$	0	5	10	15
$x = 1$	10	15	20	25
$x = 2$	20	25	30	35
$x = 3$	30	35	40	45

Geometrically, we can see the information contained in the table by first placing a point for each (*x*, *y*) in the table on the *xy* plane of our 3-space



Then we can raise each point to its appropriate *z* value (height) in 3 dimensions.



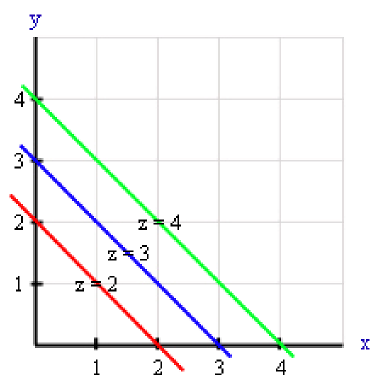
If this is a plane, any two points in the *x* direction should give us the same slope. For example, if we go from (0, 0, 0) to (1, 0, 10), we can obtain one slope in the *x* direction. This slope is equal to $m = \frac{rise}{run} = \frac{10}{1} = 10$. If we go from (1, 1, 15) to (3, 1, 35), this will provide another slope in the x direction. This slope is equal to $m = \frac{rise}{run} = \frac{20}{2} = 10$. If we obtain the slope in this manner for any two points that are oriented in the x direction, we will find that they all result in a slope of 10. Hence, the slope in the x direction for the data points in this table are all equal to 10.

If this is a plane, any two points in the y direction should also give us the same slope. For example, if we go from (0, 0, 0) to (0, 1, 5), we can obtain one slope in the y direction. This slope is equal to $m = \frac{rise}{run} = \frac{5}{1} = 5$. If we go from (1, 1, 15) to (1, 3, 25), this will provide another slope in the y direction. This slope is equal to $m = \frac{rise}{run} = \frac{30}{3} = 5$. If we obtain the slope in this manner for any two points that are oriented in the y direction, we will find that they all result in a slope of 5. Hence, the slope in the y direction for the data points in this table are all equal to 5.

As the slopes in the x direction that are associated with these points and the slopes in the y direction associated with these points are both constant, this plane is consistent with the definition of a linear function.

CONTOUR DIAGRAMS

Assume that the function f is represented by the following contour diagram:

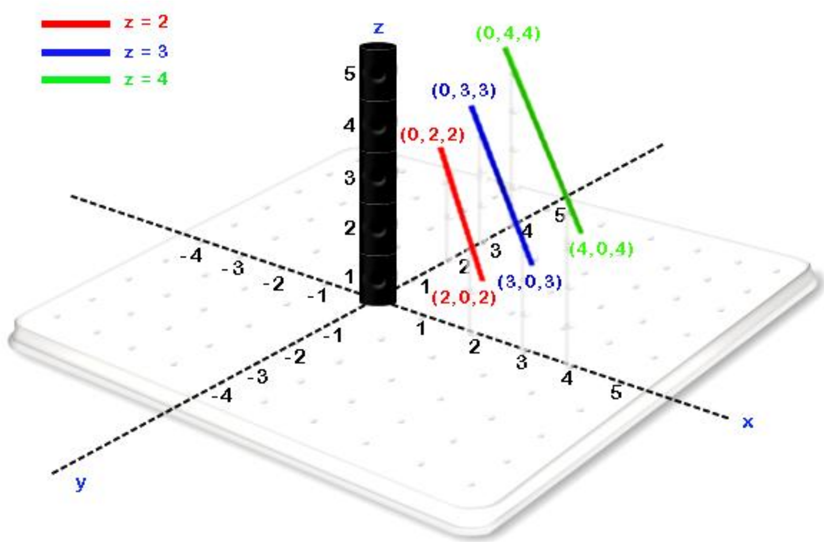


If this is a plane, any two points in the x direction should give us the same slope. For example, if we go from (2, 0, 2) to (2, 0, 3), we can obtain one slope in the x direction. This slope is equal to $m = \frac{rise}{run} = \frac{1}{1} = 1$. If we go from (1, 2, 3) to (2, 2, 4), this will provide another slope in the x direction. This slope is equal to $m = \frac{rise}{run} = \frac{1}{1} = 1$. If we obtain the slope in this manner for any two points that are oriented in the x direction, we will find that they all result in a slope of 1. Hence, the slopes in the x direction for the data points in this table are all equal to 1.

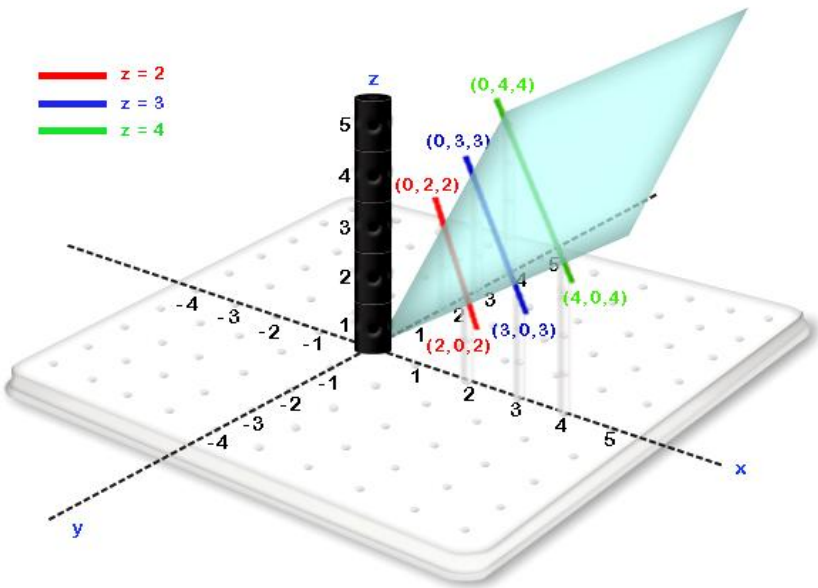
If this is a plane, any two points in the *y* direction should also give us the same slope. For example, if we go from (2, 0, 2) to (2, 1, 3), we can obtain one slope in the *y* direction. This slope is equal to $m = \frac{rise}{run} = \frac{1}{1} = 1$. If we go from (1, 1, 2) to (1, 3, 4), this will provide another slope in the *y* direction. This slope is equal to $m = \frac{rise}{run} = \frac{2}{2} = 1$. If we obtain the slope in this manner for any two points that are oriented in the *y* direction, we will find that they all result in a slope of 1. Hence, the slope in the *y* direction for the data points in this table are all equal to 1.

As the slopes in the *x* direction that are associated with these points and the slopes in the *y* direction associated with these points are both constant, this contour is consistent with the definition of a linear function.

Geometrically, we can see the information contained in the contour by moving each contour on the *xy* plane to its appropriate height in 3-space

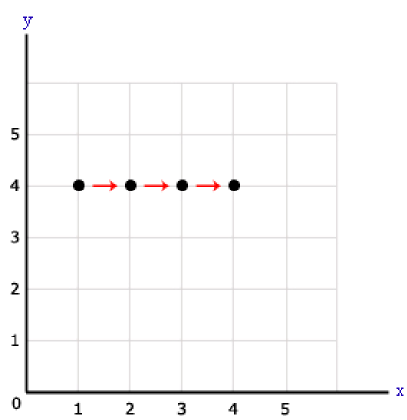


With these contours in 3-space, we can see that the contour diagram is consistent with the following plane.

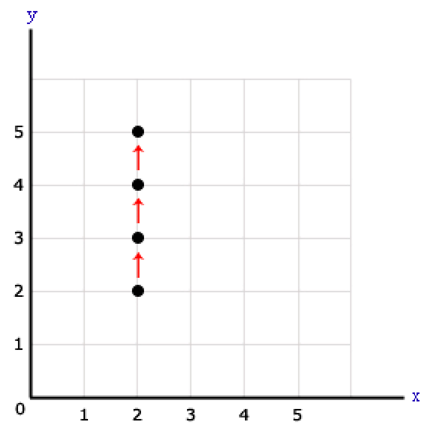


FORMULAS

Given a formula, the key to determining whether slopes are constant in the *x* and *y* directions lies in the fact that when we move in the *x* direction, *y* is constant and when we move in the *y* direction *x* is constant.



For example, the above trajectory starts at (1, 4) and moves in the *x* direction through the points (2, 4), (3, 4) and (4, 4). While *x* increases with a trajectory in the *x* direction, the value of *y* remains constant at *y = 4*.

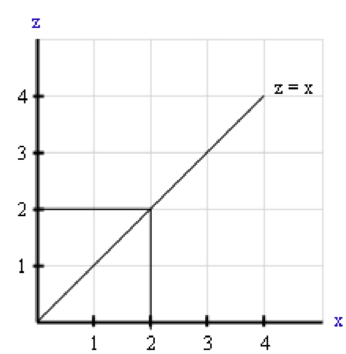


In the above case, the trajectory starts at (2, 2) and moves in the *y* direction through the points (2, 3), (2, 4) and (2, 5). While *y* increases with a trajectory in the *y* direction, the value of *x* remains constant at $x = 2$.

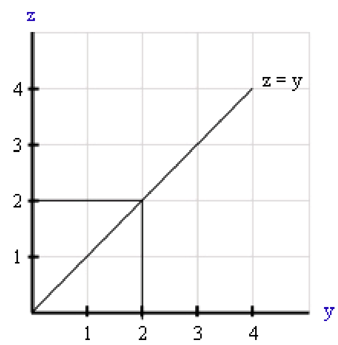
Example Exercise 4.3.1: Given the function represented with the formula $z = f(x,y) = x + y$, determine whether the formula represents a linear function.

Solution:

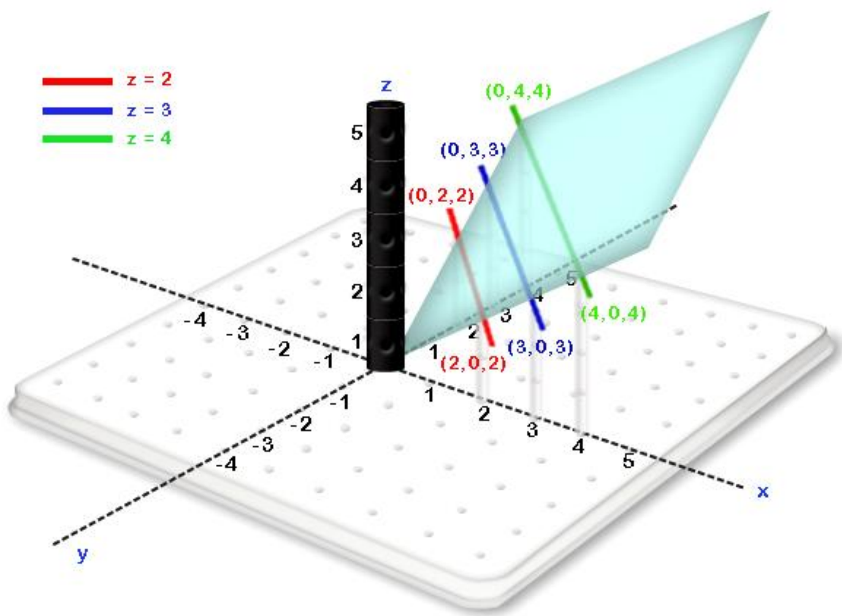
Assume that we start at any point (a, b) on the *xy* plane. If we move in the *x* direction then *y* remains constant in $y = b$ and the cross section of the curve will be $z = x + b$ where *b* is constant. This cross section is a line with slope equal to 1. Hence the slope in the *x* direction is equal to 1 for every point (a, b). For example, when we start at the point (0, 0) and move in the *x* direction, our trajectory will be associated with the cross section $y = 0$ where $z = x + 0$.



If we move in the *y* direction then *x* remains constant in $x = a$ and the cross section of the curve will be $z = a + y$ where *a* is constant. This cross section is a line with slope equal to 1. Hence the slope in the *y* direction is equal to 1 for every point (a, b). For example, when we start at the point (0, 0) and move in the *y* direction, our trajectory will be associated with the cross section $x = 0$ where $z = 0 + y$.



Hence, the function represented by the formula $z = f(x,y) = x + y$ has constant slopes in both the x direction and the y direction and does represent a linear function. If we note that the point (0, 0, 0) satisfies the function and place the appropriate cross sections in the x and y directions, we can see that the associated plane will have the following form:



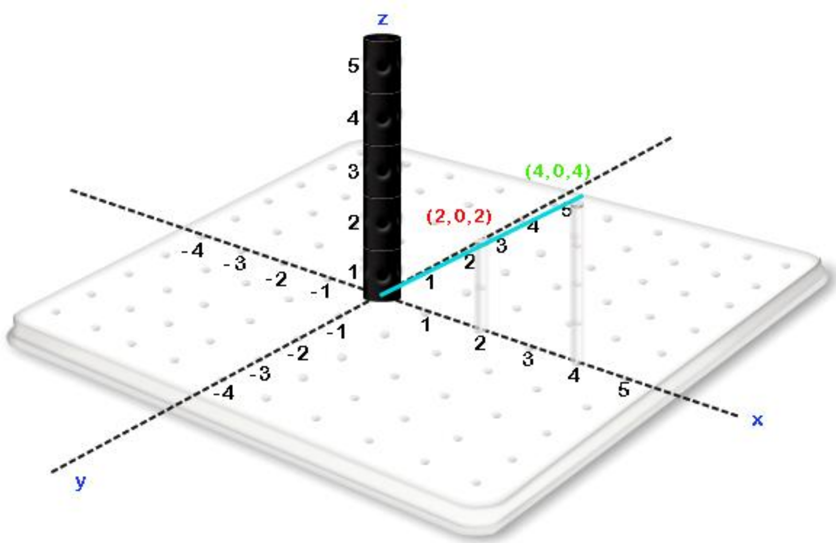
Example Exercise 4.3.2: Given the function represented with the formula $z = f(x,y) = x + y^2$, determine whether the formula represents a linear function.

Solution:

Assume that we start at any point (a, b) on the xy plane. If we move in the x direction then y remains constant in $y = b$ and the cross section of the curve will be $z = x + 3b^2 + 1$ where b is constant. This cross section is a line with slope equal to 2. Hence the slope in the x direction is equal to 1 for every point (a, b).

Example: $y = 0 \Rightarrow z = f(x, 0) = x + 0^2 = x$. This is $z = x$.

x	z
2	2
4	4

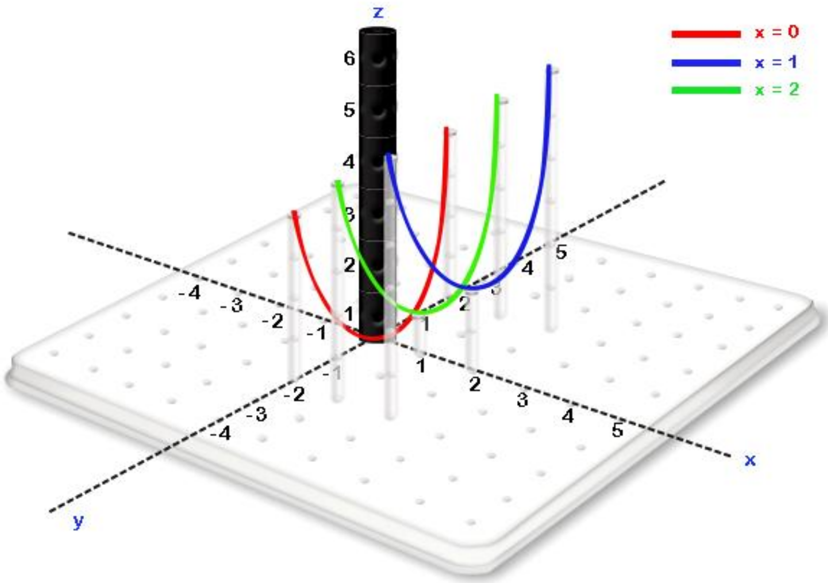


If we move in the y direction then x remains constant in $x = a$ and the cross section of the curve will be $z = x + a^2$ where a is constant. This cross section is a parabola. As a parabola is not a curve with a constant slope, the slope in the y direction is not constant.

If $x = 0 \Rightarrow z = 0 + y^2 = y^2$. This is the equation of a parabola that open toward them $z > 0$ with vertex (0, 0) on the yz plane.

If $x = 1 \Rightarrow z = 1 + y^2$.This is the equation of a parabola that open toward them $z > 1$ with vertex (0, 1) on the yz plane.

If $x = 2 \Rightarrow z = 2 + y^2$.This is the equation of a parabola that open toward them $z > 2$ with vertex (0, 2) on the yz plane.



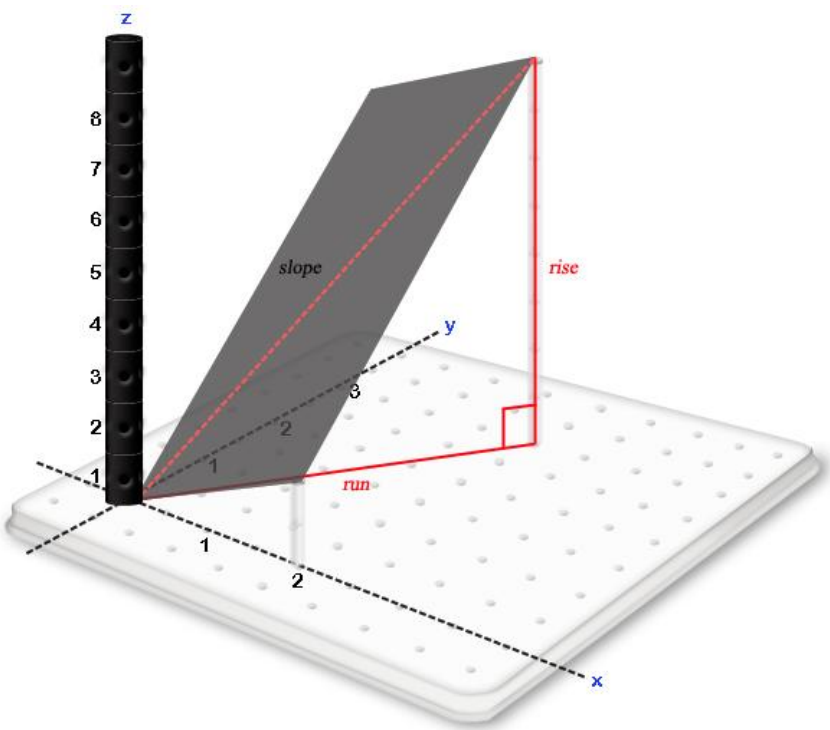
Hence, the function represented by the formula $z = f(x,y) = x + y^2$ has a constant slope in the x direction but does not have a constant slope in the y direction and does not represent a linear function.

4.4 MOVEMENT ON PLANES AND THE ASSOCIATED CHANGE IN HEIGHT

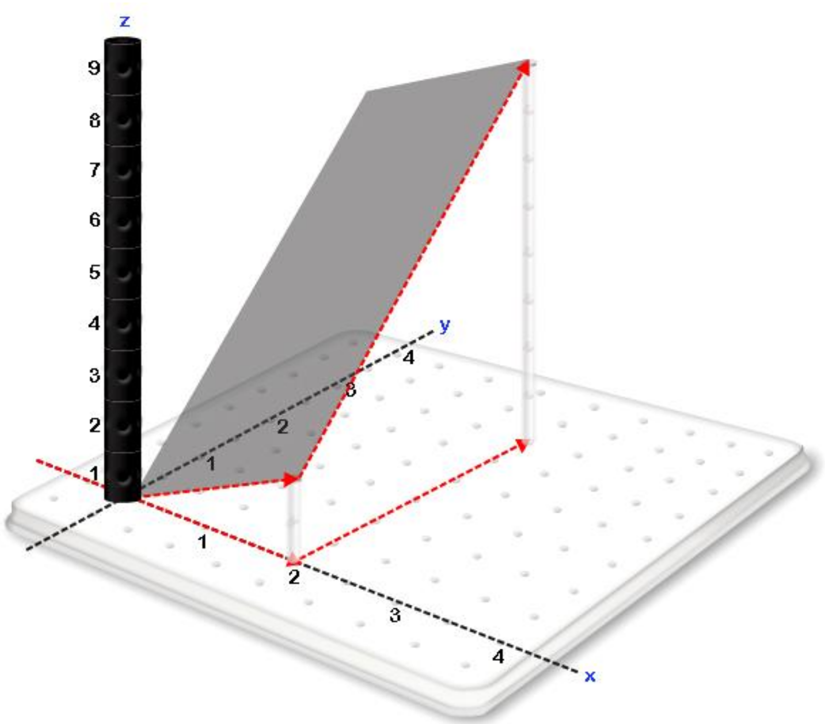
Example Exercise 4.4.1: Given a plane with $m_x = 1$ and $m_y = 2$, find the difference in height between $(0, 0, f(0, 0))$ and $(2, 3, f(2, 3))$.

Solution:

1. Identify the right triangle whose *rise* and *run* are associated with this slope.

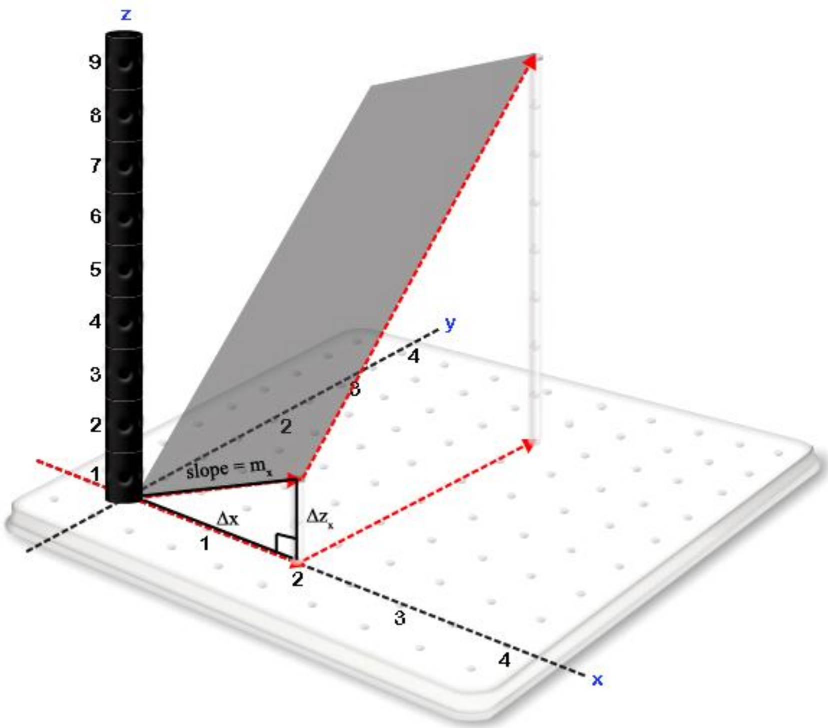


2. Divide the trajectory into movement in the *x* direction and movement in the *y* direction.



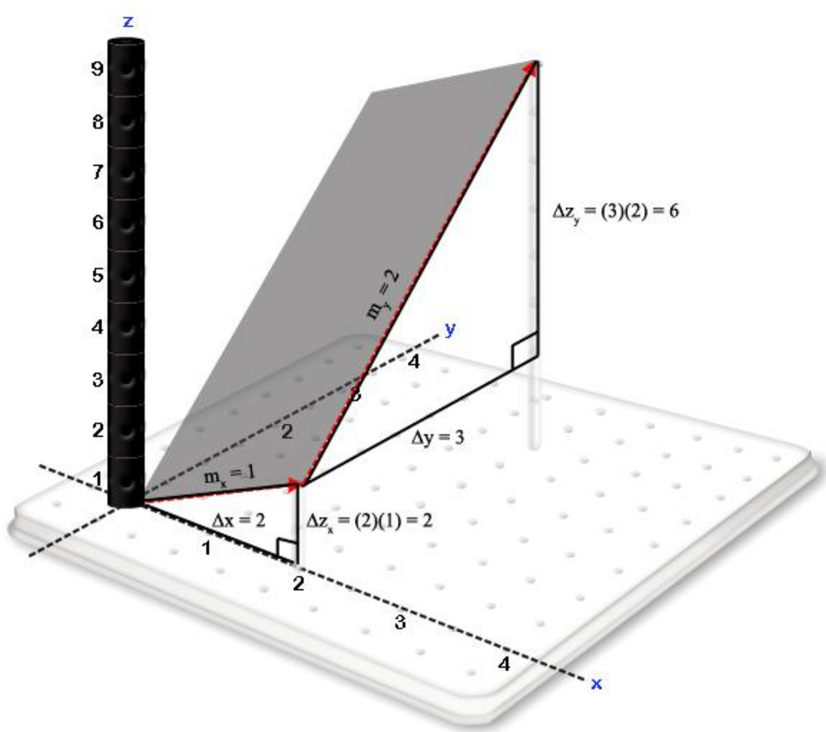
3. Find the change in height associated with each component of the trajectory:

Rise in the *x* direction:



$$\Delta x = 2, m_x = 1 \rightarrow \Delta z_x = 2$$

Rise in the *y* direction:

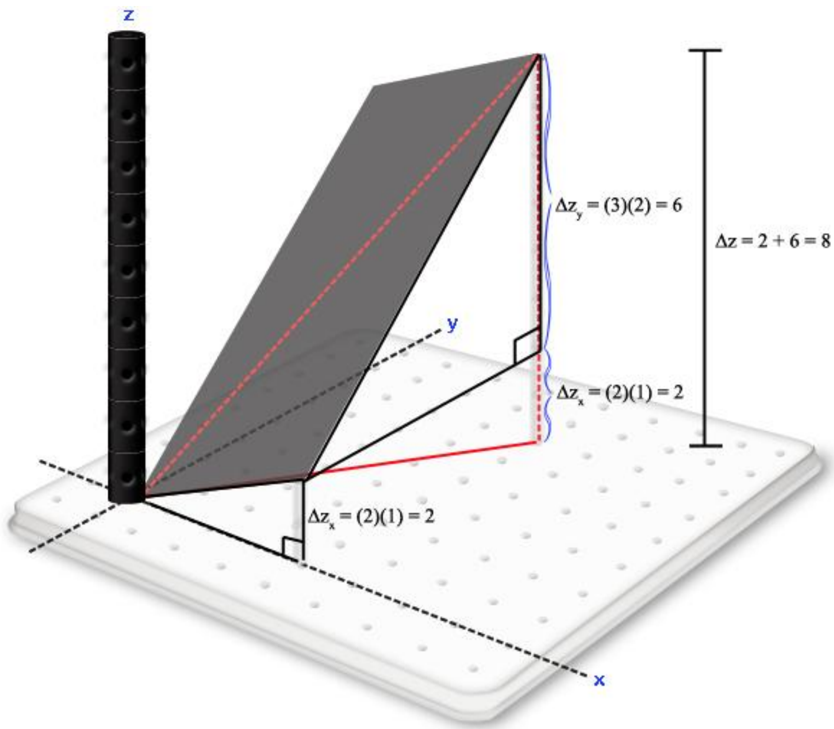


$$\Delta y = 3, m_y = 2 \rightarrow \Delta z_y = 6$$

4. Find the overall change in height:

   Taking both components of the change in height, the total difference in height is $\Delta z_x + \Delta z_y = 8$

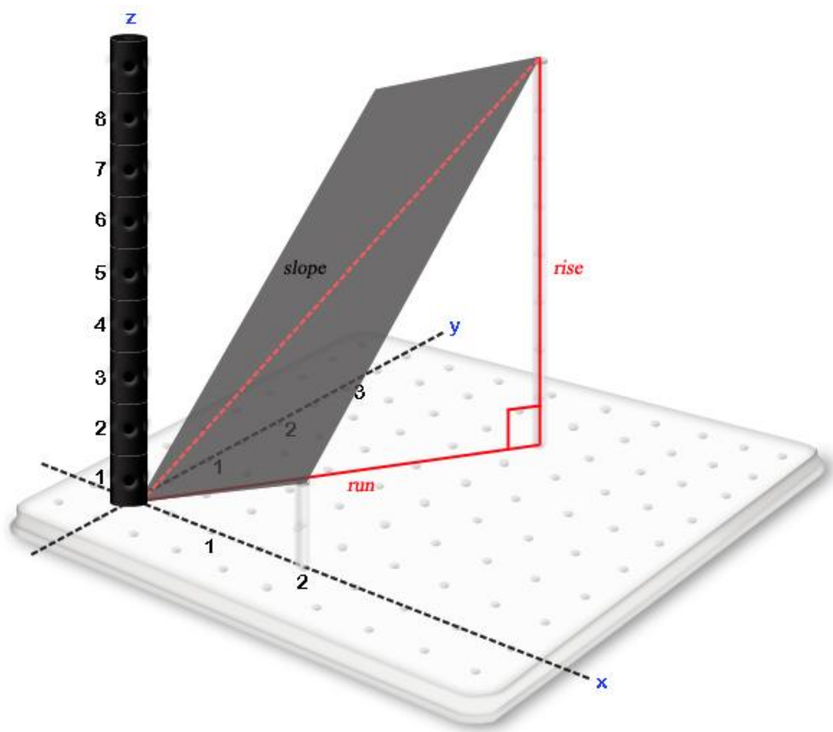
   .



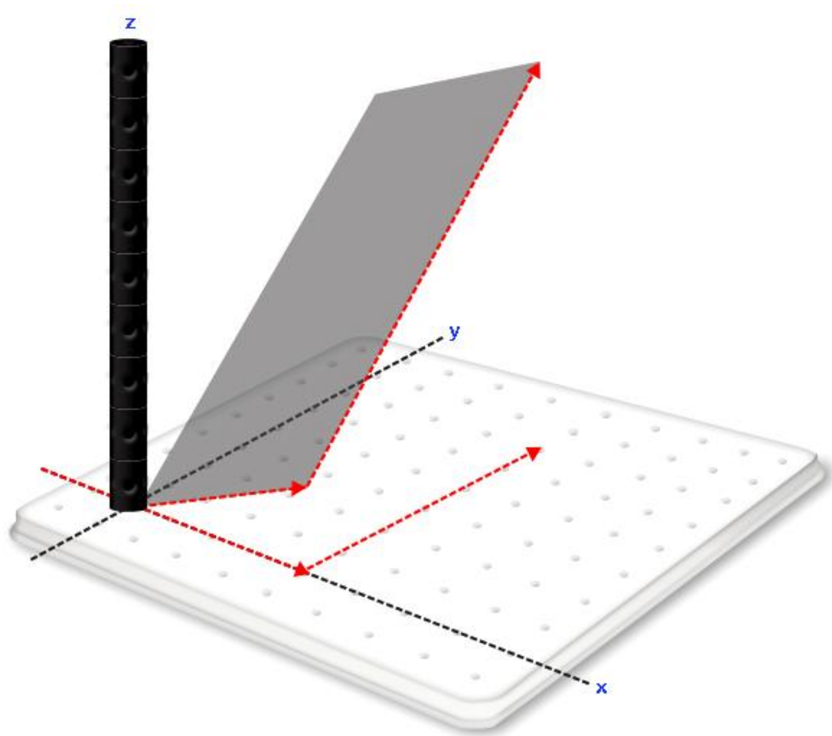
Generalization

If m_x is known and m_x is known then the difference in height between any two points on a plane can be obtained with the following steps.

1. Identify the right triangle whose *rise* and *run* are associated with this slope.

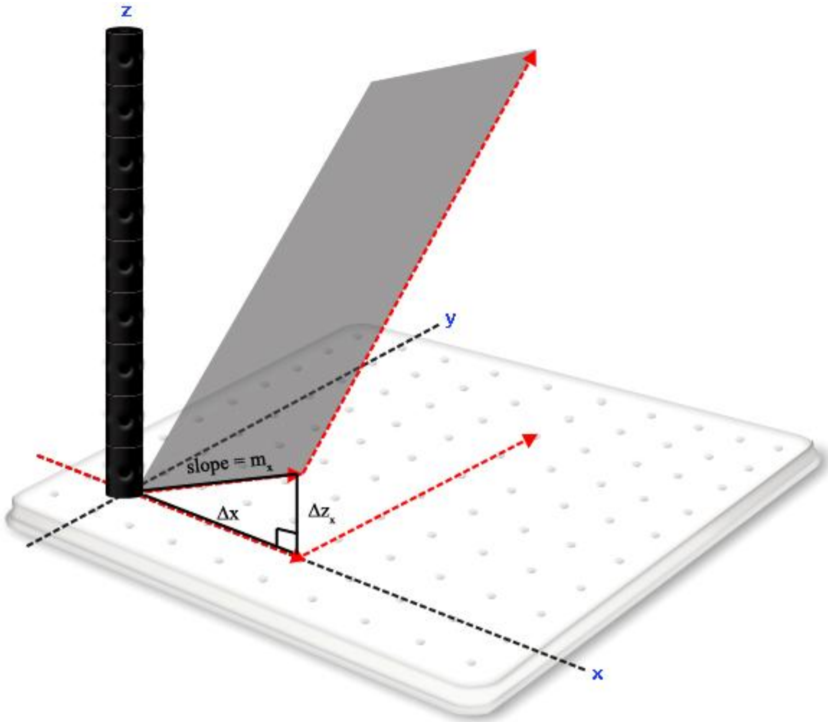


2. Divide the trajectory into movement in the *x* direction and movement in the *y* direction.



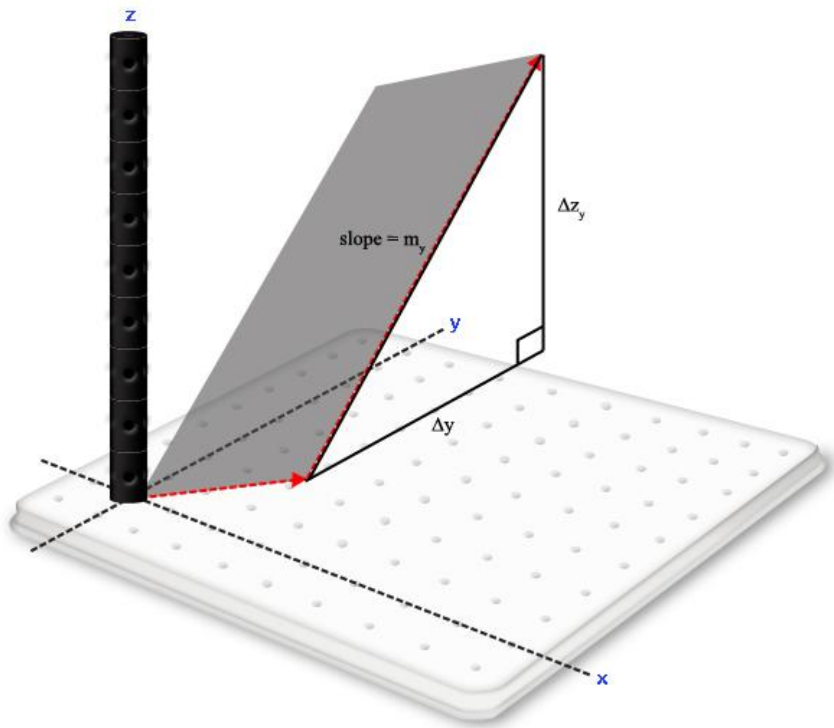
3. Find the change in height associated with each component of the trajectory.

Rise in the *x* direction:



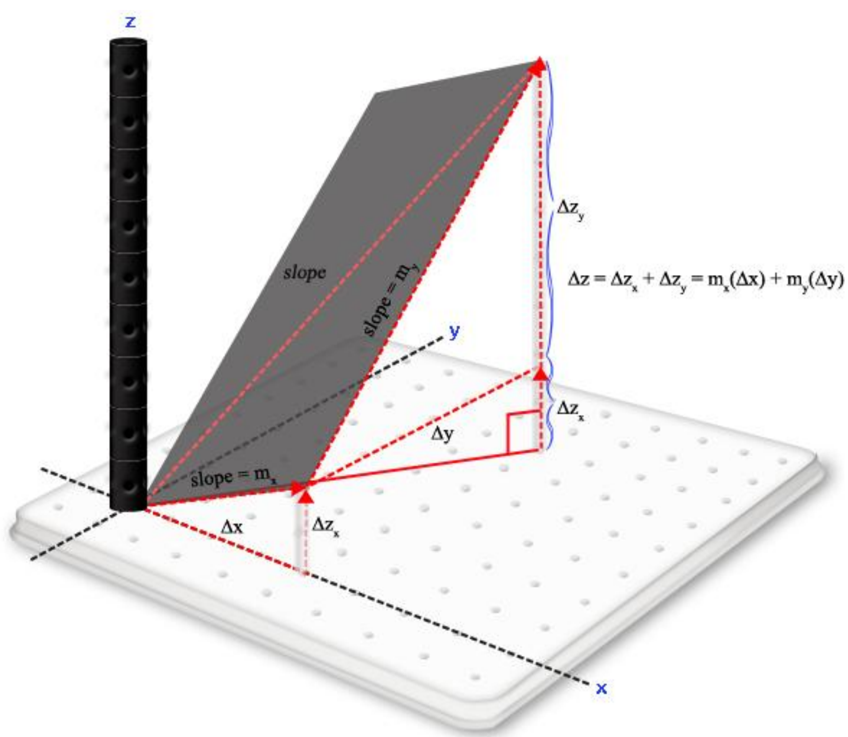
The *rise* in the *x* direction $\Delta z_x = m_x \cdot \Delta x$.

In a similar manner we can find that the *rise* in the *y* direction $\Delta z_y = m_y \cdot \Delta y$.



Conclusion: Taking both components of the change in height, the total difference in height is

$\Delta z = \Delta z_x + \Delta z_y = m_x \cdot \Delta x + m_y \cdot \Delta y$.

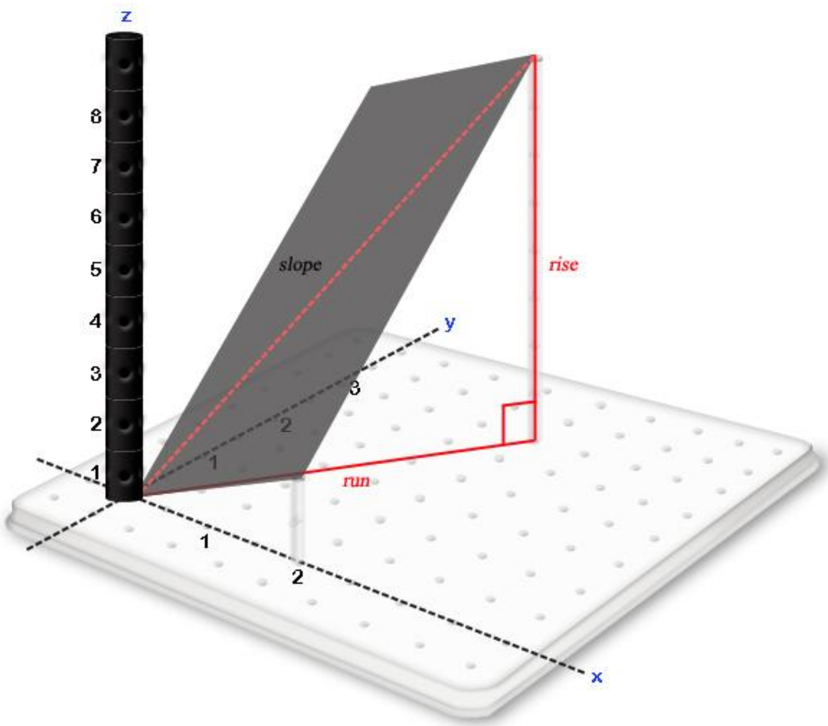


4.5 PLANES AND SLOPES IN VARIOUS DIRECTIONS

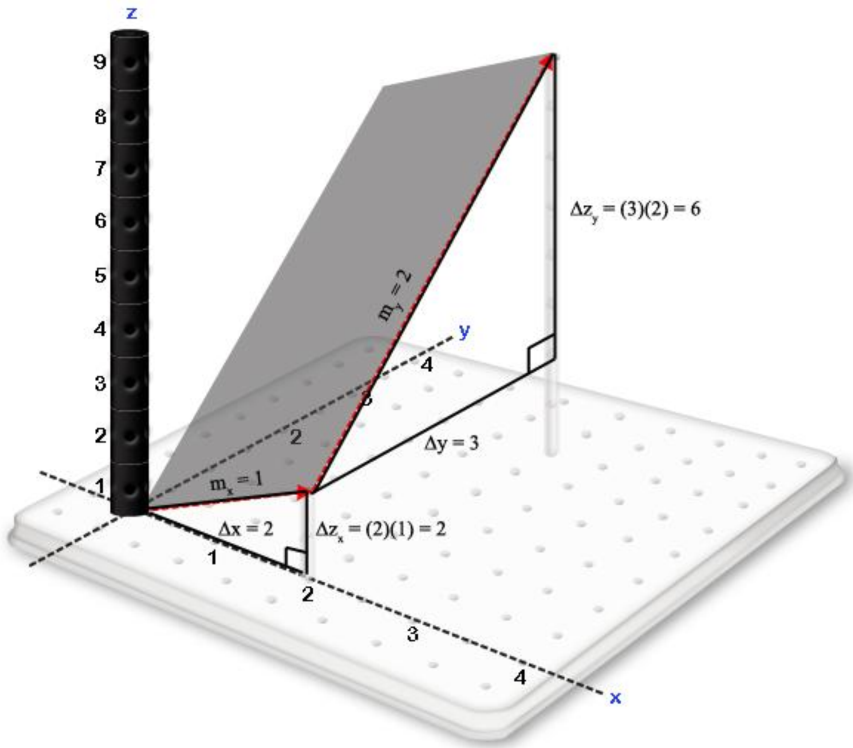
Example Exercise 4.5.1: Given a plane with $m_x = 1$ and $m_y = 2$, find the slope in the direction of the vector <2, 3>. We will refer to this as $m_{<2,3>}$.

Solution:

2. Identify the right triangle whose *rise* and *run* are associated with this slope.



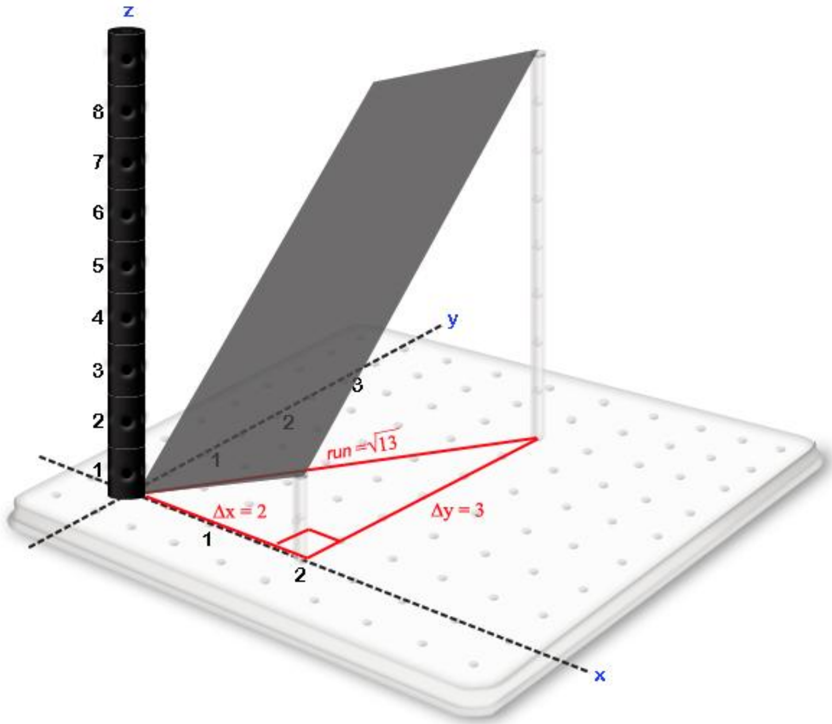
3. Obtain the *rise* for $\Delta x = 2, \Delta y = 3$.



- $\Delta x = 2, m_x = 1 \rightarrow \Delta z_x = 2$
- $\Delta y = 3, m_y = 2 \rightarrow \Delta z_y = 6$
- $\Delta z = \Delta z_x + \Delta z_y = 2 + 6 = 8$

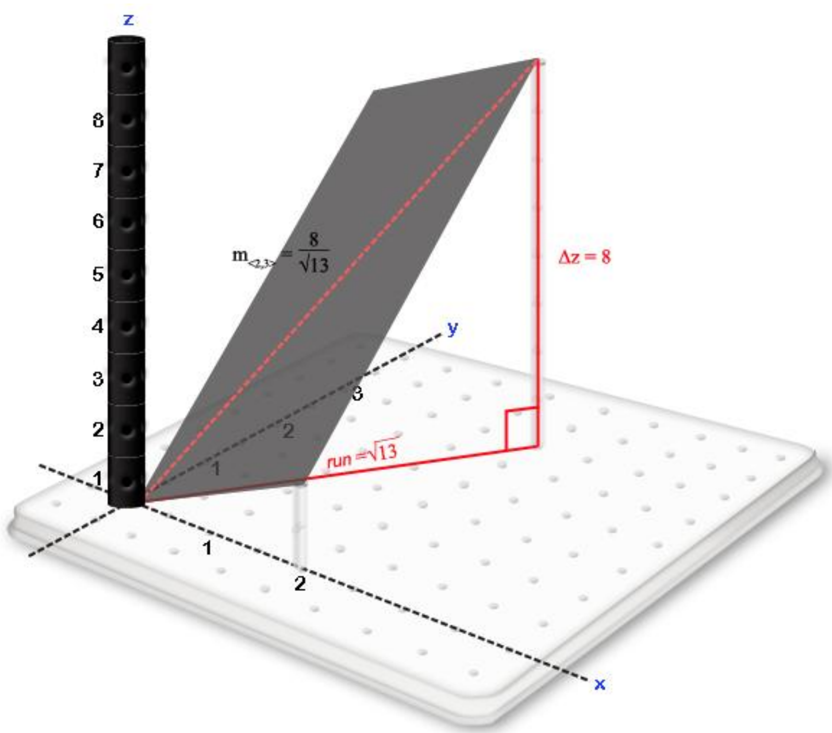
4. Obtain the *run* for, $\Delta x = 2, \Delta y = 3$.

   Using Pythagoras and the above right triangle, $run = \sqrt{2^2 + 3^2} = \sqrt{13}$.



5. Obtain the slope.

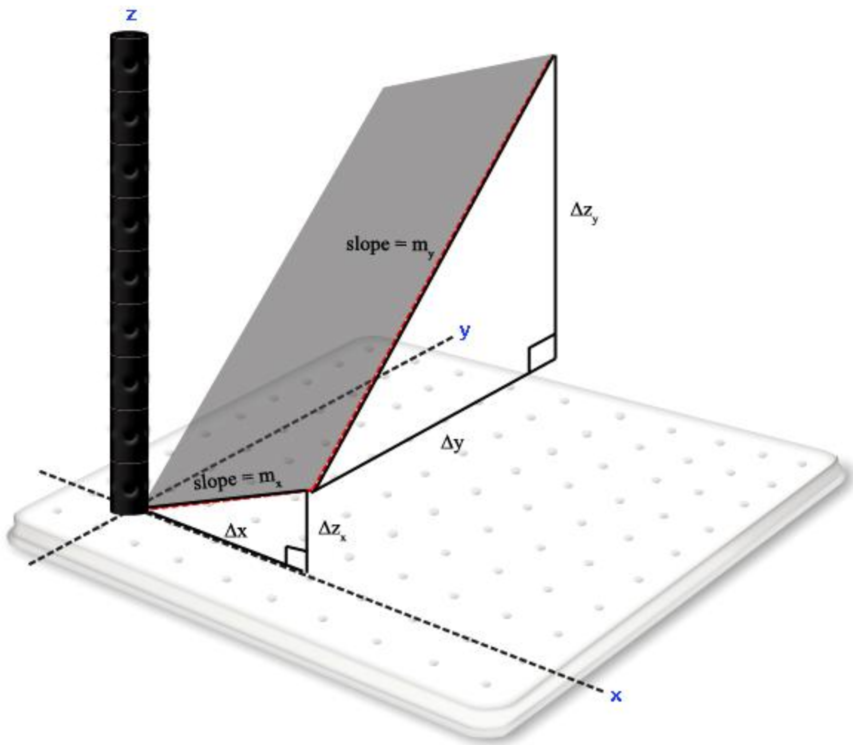
$$m = \frac{rise}{run} = \frac{8}{\sqrt{13}}$$



Generalization

If m_x and m_y are known then the slope in any direction can be obtained with the following steps.

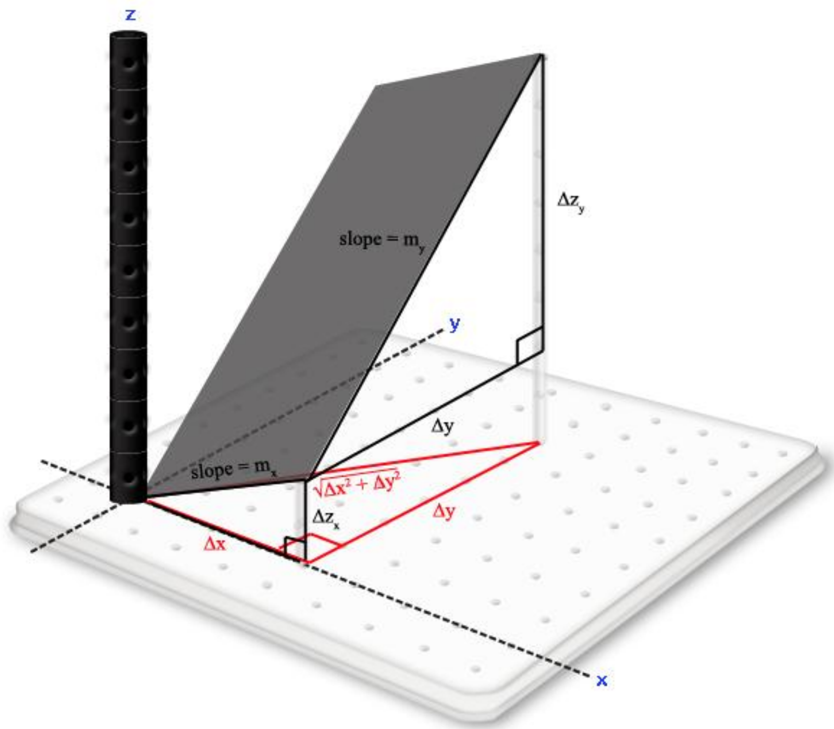
1. Obtain the *rise* using Δx , Δy , m_x, and m_y.



- *Rise* in the *x* direction: $\Delta z_x = m_x \cdot \Delta x$
- *Rise* in the *y* direction: $\Delta z_y = m_y \cdot \Delta y$
- Total *rise*: $\Delta z = \Delta z_x + \Delta z_y = m_x \cdot \Delta x + m_y \cdot \Delta y$

2. Obtain the *run* for Δx and Δy .

$$run = \sqrt{\Delta x^2 + \Delta y^2}$$



3. Obtain the slope:

$$m = \frac{rise}{run} = \frac{(m_x \cdot \Delta x) + (m_y \cdot \Delta y)}{\sqrt{\Delta x^2 + \Delta y^2}}$$

4.6 FORMULAS FOR A PLANE

POINT SLOPE FORMULA

Recall from Precalculus that there are various formulas to represent the same line. One of them is $m = (y - y_0)/(x - x_0)$ where *m* is the slope of the line and (x_0, y_0) is a point on the line. This is usually written as $y - y_0 = m(x - x_0)$ and is called the *point-slope formula*. This formula can be generalized to get a formula for a plane as shown below.

Suppose that we are given a point(x_0, y_0, z_0) on a plane and that the slopes in the *x* direction and in the *y* direction, m_x and m_y respectively, are known. We want a formula that will relate the coordinates of an arbitrary point on the plane (x, y, z) to the rest of the known information. To find this, recall from the previous section that the change in height from the given point (x_0, y_0, z_0) to the generic point (x, y, z) can be obtained by first moving in the *x* direction and then moving in the *y* direction and adding the corresponding changes in height. That is,

$$\Delta z = \Delta z_x + \Delta z_y$$

But we saw in section 4.4 that $\Delta z_x = m_x \Delta x = m_x(x - x_0)$ and $\Delta z_y = m_y \Delta y = m_y(y - y_0)$ so that the change in height may be written as in the following box.

Point-Slopes Formula for a Plane

An equation for a plane that contains the point (x_0, y_0, z_0) and has slopes m_x and m_y in the *x* and *y* directions respectively is

$$z - z_0 = m_x(x - x_0) + m_y(y - y_0)$$

Example

Suppose that in the following table, we are given some values of a linear function. The problem is to find a formula for $f(x, y)$.

$x \setminus y$	1	2	3
1	9	12	15
2	11	14	17
3	13	16	19

We know that since the function is linear the graph must be a plane. Looking at the table we see that the slopes in the *x* and *y* directions are $m_x = \frac{11-9}{2-1} = 2$ and $m_y = \frac{12-9}{2-1} = 3$. Further, the table gives us nine points we can choose from, so that we may use the Points-Slopes Formula. Take for example $(1,1,9)$. Then applying the formula we get:

$$z - 9 = 2(x - 1) + 3(y - 1)$$

which simplifies to

$$z = 2x + 3y + 4.$$

Of course we would have gotten the same formula had we used any other point on the table.

THE SLOPES-INTERCEPT FORMULA FOR A PLANE

Another formula for a line seen in Chapter 1 was the *slope-intercept formula*: $y = mx + b$ where m is the slope and b is the y-intercept of the line. This is a useful formula to represent a line when the slope and the y-intercept of the line are known. This formula can be generalized to get a formula for a plane.

Note that in the Point-Slopes Formula, if the known point happens to be the z intercept $(0,0,c)$ then plugging into the formula produces:

$$z - c = m_x(x - 0) + m_y(y - 0)$$

Simplifying yields the Slopes-Intercept Formula for a plane shown in the box below.

Slopes-Intercept Formula for a Plane

The equation of a plane that crosses the z axis at c and that has slopes m_x and m_y in the x and y directions respectively is

$$z = m_x x + m_y y + c\,.$$

Note the slopes-intercept formula for a plane has the form: $z = ax + by + c$. You may think of this as a generalization of the formula $y = ax + b$ for lines, where now you have two slopes and an intercept instead of one slope and an intercept as it was with lines.

Example: From a Formula to a Geometric Plane

Consider the plane $z = 2x + 3y + 4$.

The cross-section corresponding to $x = 0$ is $z = 2(0) + 3y + 4$. This is a line in the y direction with a slope of 3. The cross-section corresponding to $y = 0$ is $z = 2x + 3(0) + 4$. This is a line in the x direction with a slope of 2. Also, observe that the z intercept must be 4. This may be observed by taking both $x = 0$ and $y = 0$ to get $z = 2(0) + 3(0) + 4 = 4$.

To summarize we have obtained the following three data:

- The slope in the x direction: $m_x = 2$.
- The slope in the y direction: $m_y = 3$.
- The point $(0,0,4)$ satisfies the equation.

We can now geometrically represent this information by placing the point $(0,0,4)$ in 3-space and as we know that for every step in the x direction, our height increases by 2 and for every step in the y direction our height increases by 3, we can place lines in the x direction and in the y direction consistent with this information. The result is illustrated in *Figure 4.6.1*. With this information, we can place the unique plane which will satisfy $m_x = 2$, $m_y = 3$ and have z intercept at $(0,0,4)$ (*see Figure 4.6.2*).

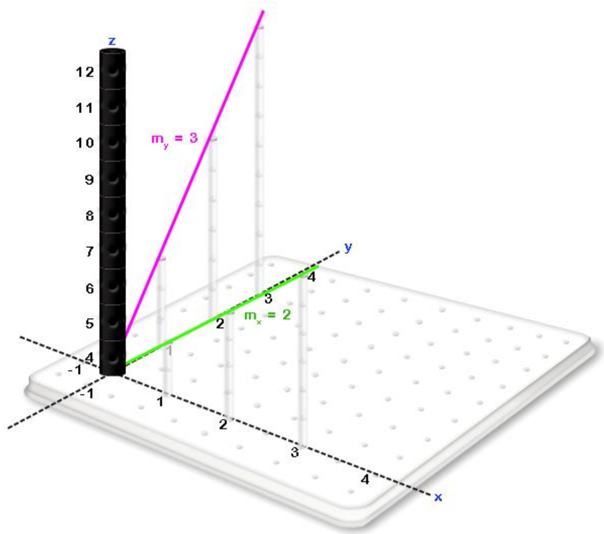


Figure 4.6.1

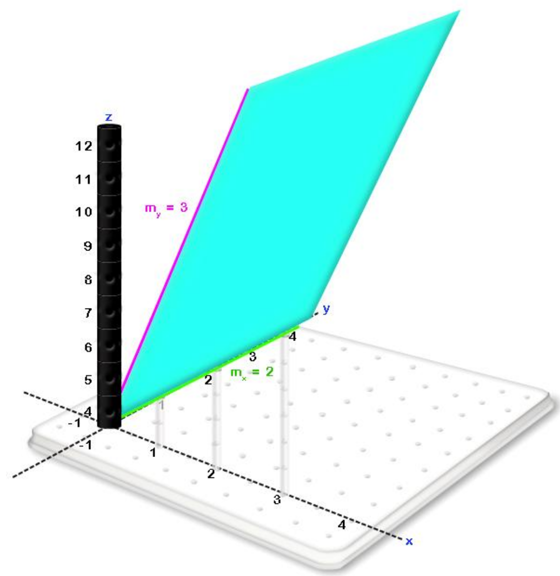


Figure 4.6.2

Example Exercise 4.6.1: Find a formula for the function in following table:

$x \backslash y$	0	1	2	3
0	4	7	10	13
1	6	9	12	15
2	8	11	14	17
3	10	13	16	19

Solution:

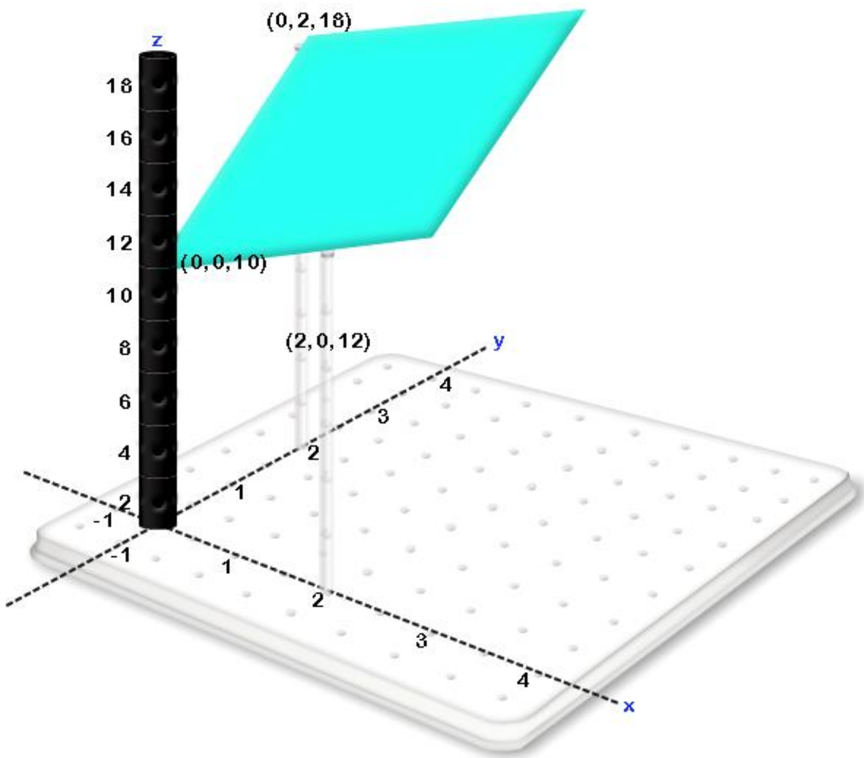
We can easily verify the following three facts:

- For every step we take in the x direction (that is, $\Delta x = 1$ and $\Delta y = 0$), our height increases by 2 irrespective of where we begin. This is the slope in the x direction: $m_x = 2$.
- For every step we take in the y direction (that is, $\Delta x = 0$ and $\Delta y = 1$), our height increases by 3 irrespective of where we begin. This is the slope in the y direction: $m_y = 3$.
- The point $(0,0,4)$ satisfies the equation.

Hence, using the Slopes-Intercept Formula, the equation is $z = 2x + 3y + 4$.

Example Exercise 4.6.2: From a Geometric Plane to the Slopes-Intercept Formula

Consider the following plane:



Solution:

Observe that:

- The point $(0,0,10)$ lies on the plane hence the z intercept is 10.
- Upon moving 2 steps in the x direction we've risen 2 units indicating a *rise* of 1 for each step in the x direction or $m_x = 1$.

- Upon moving 2 steps in the y direction we've risen 8 units indicating a *rise* of 2 for each step in the y direction or $m_y = 4$.

Using the Slopes-Intercept Formula we get that the equation of the plane is $z = x + 4y + 10$.

THE POINT-NORMAL FORMULA OF A PLANE

In *Figure 4.6.3* below, we have a point in 3-space (x_0, y_0, z_0) and a vector $\vec{n} = \langle a, b, c \rangle$. There is only one plane that will pass through the point (x_0, y_0, z_0) and for which $\vec{n} = \langle a, b, c \rangle$ is the perpendicular (also called *normal*). This is illustrated in *Figure 4.6.4*.

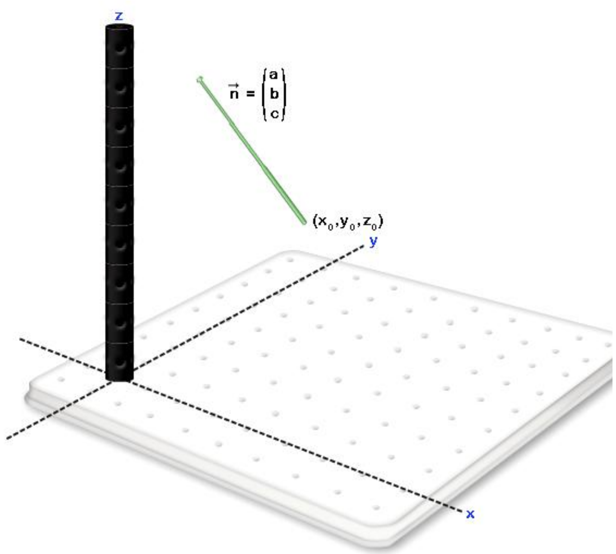


Figure 4.6.3

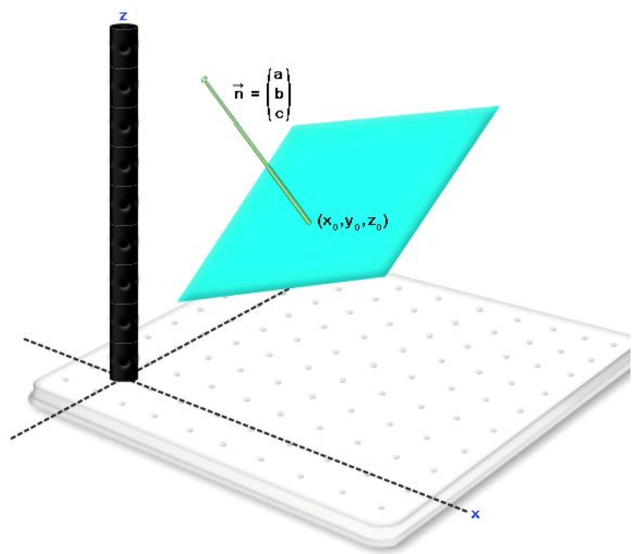


Figure 4.6.4

Hence one point on a plane and a vector perpendicular to the plane is sufficient to determine a unique plane in three-dimensional space. Our goal is to find the formula for this plane.

This formula can be derived through the following steps:

<u>Step 1</u>: As (x_0, y_0, z_0) lies on the plane, if (x, y, z) is any point on the plane other than (x_0, y_0, z_0) then, as Figure 4.6.5 illustrates, the displacement vector from (x_0, y_0, z_0) to (x, y, z), $\langle x - x_0, y - y_0, z - z_0 \rangle$, is parallel to the plane, that is, it can be placed on the plane.

Step 2: If $\langle x - x_0, y - y_0, z - z_0 \rangle$ lies on the plane then it is perpendicular to the normal vector $\vec{n} = \langle a, b, c \rangle$ (see Figure 4.6.6).

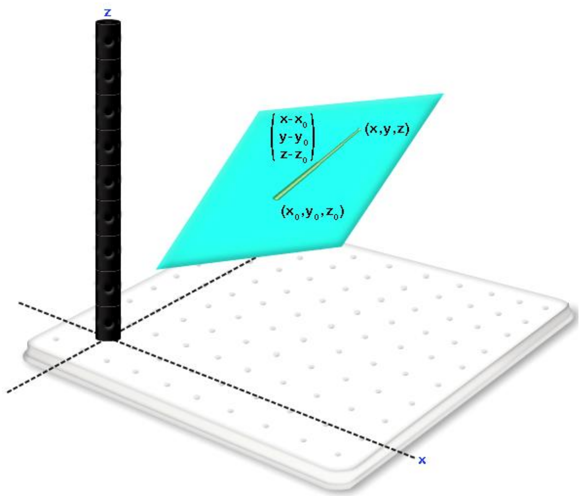


Figure *4.6.5*

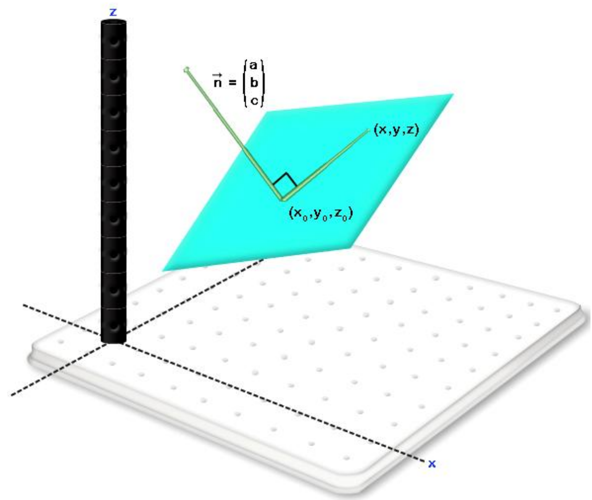


Figure 4.6.6

Step 3: As $\langle x - x_0, y - y_0, z - z_0 \rangle$ is perpendicular to $\langle a, b, c \rangle$ we can conclude that $\langle x - x_0, y - y_0, z - z_0 \rangle \cdot \langle a, b, c \rangle = 0$ or $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

Hence we have:

The Point-Normal Formula of a Plane

The plane that passes through (x_0, y_0, z_0) with normal vector $\vec{n} = \langle a, b, c \rangle$ has equation:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

It is worth noting that the above formula can be rewritten as $ax + by + cz = d$ where $d = ax_0 + by_0 + cz_0$. From here it is not hard to show that if we have the formula for a plane expressed as $ax + by + cz = d$ then the vector $\langle a, b, c \rangle$ is perpendicular to the plane.

Example Exercise 4.6.4: Find the equation of the plane that passes through $(1, 2, -3)$ and that is normal to the vector $\langle 2, -5, 4 \rangle$.

Using the Point-Normal Formula results in: $2(x-1)-5(y-2)+4(z+3)=0$ and so simplifying we get $2x-5y+4z=-20$.

Example Exercise 4.6.5: Find a vector that is perpendicular to the plane $z=2x-3y+4$.

Rewriting the equation of the plane in the form $ax+by+cz=d$ we get $-2x+3y+z=4$. From here we can read off the normal vector $\langle -2,3,1\rangle$.

EXERCISE PROBLEMS:

1. For each of the following planes, find

i. The slope in the *x* direction
ii. The slope in the *y* direction
iii. The *z* intercept
iv. A vector perpendicular to the plane
v. The slope in the direction NE
vi. The slope in the direction $<2,3>$

A. $z = 2x + 3y + 4$
B. $x + 4y - z = 3$
C. $2x + 3y + 4z = 5$
D. $3x = 2y - 5z + 3$
E. $2x - 3y = 7z + 3$
F. $3y = 2x - 4z + 3$

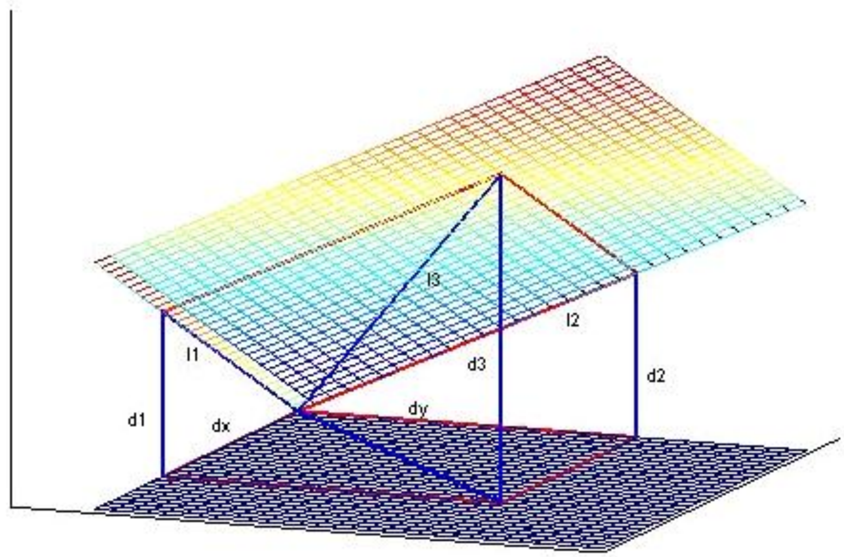
2. For each of the following data associated with a plane, find

i. The slope in the *x* direction
ii. The slope in the *y* direction
iii. The slope in the direction NE
iv. The slope in the direction $<3,4>$
v. The formula for a plane consistent with these data and with *z* intercept 4
vi. The formula for a plane consistent with these data that passes through (3,2,5).

A. The slope towards the east is 3 and the slope towards the north is 2
B. If we move 3 meters east, our altitude rises 6 meters and if we move 2 meters north our altitude rises 10 meters.
C. A normal vector to the plane is <1,2,3>.
D. A normal vector to the plane points directly upwards.
E. The slope towards the west is -3 and the slope towards the north is 4

F. If we move 3 meters north, our altitude falls 6 meters and if we move 2 meters west, our altitude rises 10 meters.

3. The following diagram represents a plane. The values dx and dy are displacements in the direction x and the direction y respectively. The distances, d1, d2, and d3 are vertical line segments. The lines l1, l2, and l3 are lines on the plane.



a. If *dx = 2*, *dy=3*, *d1 = 6* and *d2 = 4*, find the slopes of *l1*, *l2* and *l3*.
b. If *dx = 4*, *dy=2*, *d1 = 16* and *d2 = 10*, find the slopes of *l1*, *l2* and *l3*.
c. If *dx = 3*, *dy=3*, *d1 = 6* and *d3 = 24*, find the slopes of *l1*, *l2* and *l3*.
d. If *dx = 2*, *dy=3*, *d2 = 6* and *d3 = 24*, find the slopes of *l1*, *l2* and *l3*.
e. If *dx = 2*, *dy=3*, the slope in the direction *l1* is 3 and the slope in the direction *l2* is 5, find the distances *d1*, *d2* and *d3* and the slope in the direction *l3*.
f. If *dx = 4*, *dy=2*, the slope in the direction *l1* is 2 and the slope in the direction *l2* is 3, find the distances *d1*, *d2* and *d3* and the slope in the direction *l3*.
g. If *dx = 4*, *dy=3*, the slope in the direction *l1* is 5 and the slope in the direction *l3* is 7, find the distances *d1*, *d2* and *d3* and the slope in the direction *l2*.